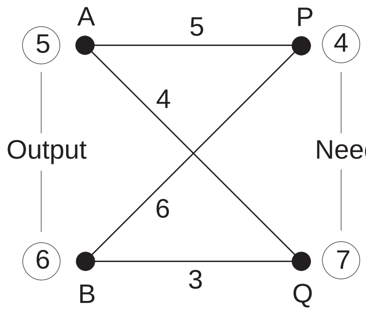
BRITISH STEEL Case Study 4: *Transportation*

The cost of transportation is a key factor for many industries. A particular example is that of British Steel's stainless steel manufacturing plants. They require the raw material, ferrous oxide, which is mined and brought from Australia, South Africa or Japan. The actual cost of the raw material varies in each country, as does the cost of shipment to the UK. The problem faced by British Steel is to decide what quantities to bring from the available sources.

As a more simple example, consider two suppliers of a particular raw material. They can output 5 and 6 units per day respectively. The material is required at two factories, P and Q, which need 4 and 7 units per day. The cost of transporting one unit from the supplier to the factory is shown by the number on each arc in the diagram opposite.

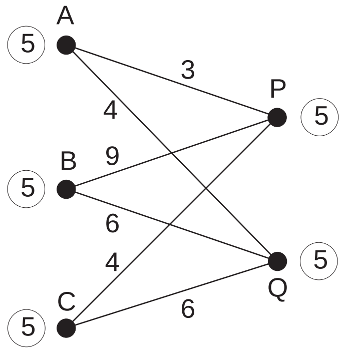


Activity 1

Find a way of supplying the factories, and calculate the total costs. Have you obtained the *minimum* cost solution? If not, find it.

With just two suppliers and two factories it is easy to find the optimum solution, but the problem becomes increasingly difficult to solve when we have more suppliers and factories.

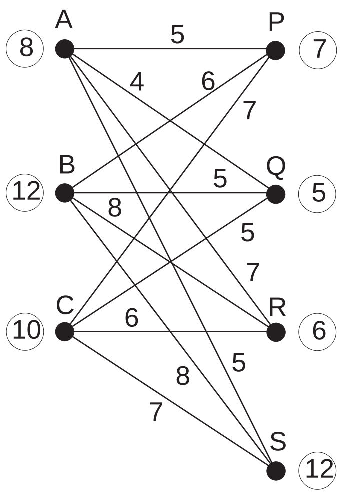
For example, consider *three* suppliers and *two* factories, as shown.



Activity 2

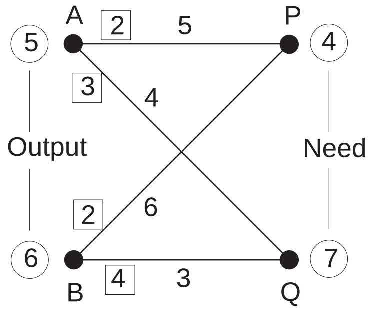
Find the minimum cost solution for the problem above.

An even more complicated example is shown opposite, in which there are *three* suppliers and *four* factories.



Activity 3

Find the minimum cost solution for this problem.



By now it is not quite so easy to see whether you have in fact obtained the *optimum* solution. Returning to the first example, suppose you have so far obtained the solution as shown; actual allocations are shown in squares.

The total cost is given by

$$(2 \times 5 + 3 \times 4) + (2 \times 6 + 4 \times 3)$$

$$= 22 + 24 = 46$$

But is this the minimum cost solution?

Improvements can be made by sending more units from B to Q. For example, if one further unit was sent along BQ, one less would be sent from A to Q, one more from A to P and one less from B to P.

The increment in cost saving would be

$$3 - 4 + 5 - 6 = -2$$

So for each extra unit on the cycle B Q A P B the saving is 2, i.e.

$$\Delta(\text{BQAPB}) = -2$$

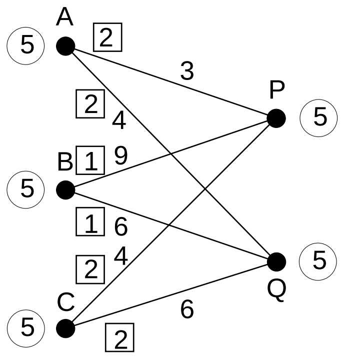
Problem 1

How many more units can be saved on this cycle?

Solution

Since the output of B is just 6 units, only two further units can flow in this way, giving a total saving of 4, and minimum cost of 42.

Check the answer with you answer to Activity 1.



This technique can be applied to more complex problems.

For Activity 2, for example, with the allocation shown (in squares) the total cost is given by

$$(2 \times 3 + 2 \times 4) + (1 \times 9 + 1 \times 6) + (2 \times 4 + 2 \times 6)$$

$$= 14 + 15 + 20$$

$$= 49$$

Sending one further unit from A to P might reduce the cost.

Possible ‘cycles’ are

A P B Q A and A P C Q A

Now

$$\Delta(\text{A P B Q A}) = 3 - 9 + 6 - 4 = -4$$

Problem 2

Show that $\Delta(\text{A P C Q A}) = 1.$

Solution

$\Delta(\text{A P B Q A}) = 3 - 4 + 6 - 4 = 1$

Clearly it is better to send one further unit on the cycle A P B Q A.

Activity 4

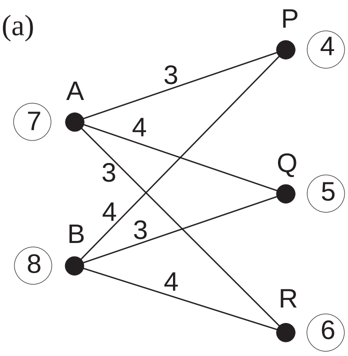
In this problem, what other cycles exist that will reduce the total cost? Use this method to find the minimum total cost.

For the minimum total cost, no cycles with negative increases in cost exist for which units can flow.

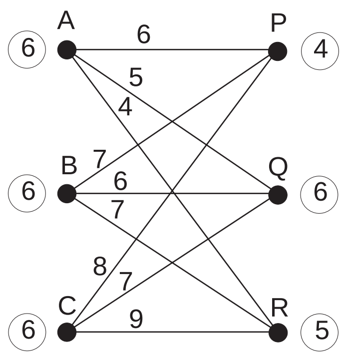
Exercises

1. Use the ‘cycle’ method to check your solution to Activity 3. Can it be improved?
2. For each of the problems illustrated below, find the minimum cost solution.

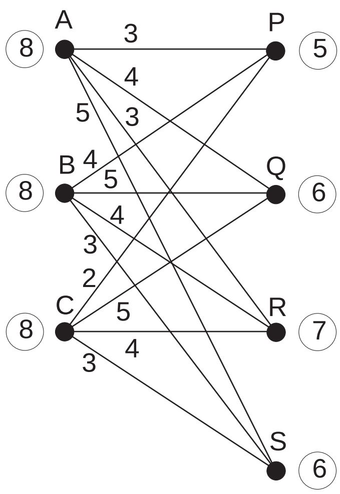
(a)



(b)



(c)



Answers to Exercises

2. (a) 48 (b) 86 (c) 78

