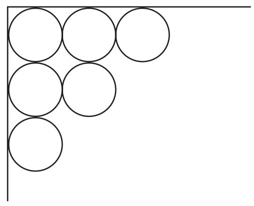
Drinks cans are made by stamping out circular discs from a sheet of tin. Each disc is bent up to make a shallow cup and then stretched to make a can. The tin gets thinner and harder during the process. The top is made separately and fixed on after the can has been made. The cans are usually stamped out from a sheet of tin as shown in the diagram. Given that the sheet is 2 m $\times$1 m, and that the radius of the circle stamped out is 10 cm, what wastage of tin is there and is there a more effective method?

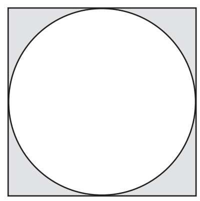


Using the configuration shown opposite you can obtain $10 \times 5 = 50$ such cans from one sheet of metal. Each one gives the same amount of wastage, namely

$$\text{area of square} - \text{area of circle}$$

$$= (20 \times 20 - \pi \times 10 \times 10)\ \text{cm}^2$$

So



$$\%\ \text{wastage} = \frac{(400 - 100\pi)}{400} \times 100$$

$$= \left(1 - \frac{\pi}{4}\right) 100$$

$$\approx 21\%$$

Problem 1

Assuming that the wastage can be recycled, and that can tops are stamped out in a similar way, what will be the percentage change in thickness of the tin in order to provide one top from the wastage in making one can?

Solution

If t_0 is the actual thickness of the wastage from the sheet and t_1 the new thickness, then

$$\text{volume of wastage} = \text{volume of top}$$

$$(400 - 100\pi) t_0 = 100\pi t_1$$

$$t_1 = \left(\frac{1}{\pi} - 1\right) t_0$$

The percentage change is given by

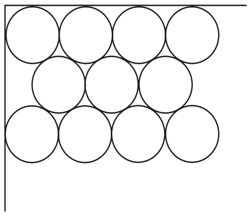
$$\left(\frac{t_1 - t_0}{t_0}\right) \times 100 = \left(\frac{1}{\pi} - 2\right) \times 100$$

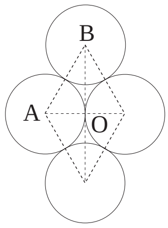
$$\approx -168\%$$

Activity 1

Can you suggest other configurations for stamping out the cans?

Another possible way is shown opposite. Does this have less waste?





Using the dimensions given previously,

$$\cos O\hat{A}B = \frac{OA}{AB} = \frac{10}{10+10} = \frac{1}{2}$$

Hence $O\hat{A}B = 60°$ and the length OB is given by

$$OB = 20 \sin 60° = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$$ (or by Pythagoras).

Activity 2

Using a rectangular sheet 2 m $\times$1 m (as before) and marking first a row of circles along the 2 m length, how many circles can be cut from the sheet using this configuration? Evaluate the total waste, and the percentage waste for the complete sheet.

Exercises

1. What would be the percentage of waste using the second configuration if the sheet was very large (i.e. assume it to be infinite)?

2. Suppose the wastage in using the first design can be recycled to produce further cans. How many would be produced on the next cycle?

3. Investigate other two-dimensional shapes that need to be cut out in bulk from sheets of metal, tin or steel.

4. Investigate three-dimensional packing problems; for example, how can tennis balls be efficiently packed?

Answers to Exercises

1. $\left(1 - \dfrac{\pi}{2\sqrt{3}}\right) \times 100 \approx 9.31\%$

2. 10 cans