



Mathematics Enhancement Programme

Primary Demonstration Project

8A Numbers

Help Booklet



Support for Primary Teachers
in Mathematics

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Mathematics Enhancement Programme

Help Module 8

NUMBERS IN CONTEXT

Part A

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PREFACE

This is one of a series of *Help Modules* designed to help you gain confidence in mathematics. It has been developed particularly for primary teachers (or student teachers) but it might also be helpful for non-specialists who teach mathematics in the lower secondary years. It is based on material which is already being used in the *Mathematics Enhancement Programme: Secondary Demonstration Project*.

The complete module list comprises:

- | | |
|--------------|-----------------------|
| 1. ALGEBRA | 6. HANDLING DATA |
| 2. DECIMALS | 7. MENSURATION |
| 3. EQUATIONS | 8. NUMBERS IN CONTEXT |
| 4. FRACTIONS | 9. PERCENTAGES |
| 5. GEOMETRY | 10. PROBABILITY |

Notes for overall guidance:

- Each of the 10 modules listed above is divided into 2 parts. This is simply to help in the downloading and handling of the material.
- Though referred to as 'modules' it may not be necessary to study (or print out) each one in its entirety. As with any self-study material you must be aware of your own needs and assess each section to see whether it is relevant to those needs.
- The difficulty of the material in **Part A** varies quite widely: if you have problems with a particular section do try the one following, and then the next, as the content is not necessarily arranged in order of difficulty. Learning is not a simple linear process, and later studies can often illuminate and make clear something which seemed impenetrable at an earlier attempt.
- In **Part B**, **Activities** are offered as backup, reinforcement and extension to the work covered in Part A. **Tests** are also provided, and you are strongly urged to take these (at the end of your studies) as a check on your understanding of the topic.
- The marking scheme for the revision test includes B, M and A marks.

Note that:

- | | |
|----------------|---|
| M marks | are for method; |
| A marks | are for accuracy (awarded only following a correct M mark); |
| B marks | are independent, stand-alone marks. |

We hope that you find this module helpful. Comments should be sent to:

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The full range of Help Modules can be found at

www.ex.ac.uk/cimt/help/menu.htm

8 *Numbers in Context*

Introductory Notes

Historical Background

The historical background to numbers is given in HELP Module 2, *Decimals*, with further information in Modules 4 and 5, *Fractions* and *Geometry*.

One other historical subject which might be of interest and of particular relevance to the section on map scales is that of the development of Ordnance Survey maps. The usual scale for the Main Series of UK maps is

$$1 : 50\,000$$

and the origin for the coordinate system used is near the Scilly Isles, (off the west coast of Cornwall). So, for example, the grid reference (363242) means

36.3 km east of the origin

and

24.2 km north of the origin.

Note also that the 'hundreds' of km are left off the grid reference, so the point with coordinates

$$(236.3, 124.2)$$

would also have the grid reference 363242.

It should also be noted that the scale 1 : 50 000 results in an exact scale of 2 cm to 1 km, but only an approximate $1\frac{1}{4}$ inches to 1 mile. This was decided at a conference held in Paris in December 1913, and is now a world undertaking in modern countries.

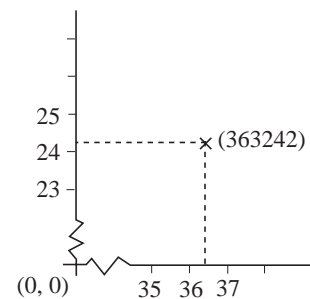
Teaching Points

Introduction

This module brings together a number of threads, starting with negative numbers, leading on to ratio and proportion and then proportional division.

Language / Notation

- For ratio and proportion, we use the notation $1 : n$ or $m : 1$ or $m : n$.
- For map scales, the notation 1 in 2000 (for example) is used to mean the ratio $1 : 2000$.



Ordnance Survey web site
<http://www.ordsvy.gov.uk/>

Misconceptions

There are numerous errors which can easily be made; for example,

- that $6 - (-4) = 2$, instead of 10;
- that $-5 - (-10) = -15$, instead of 5.

Also note that care must be taken with units on map scales, for example, 1 : 5000 means that 1 cm on a map represents 5000 cm in reality (or 50 m).

Note that increasing the map scale actually decreases the map length.

Key Points

1. Rules for multiplication and division:

$$\begin{array}{l}
 + \times + \equiv + \\
 + \times - \equiv - \\
 - \times + \equiv - \\
 - \times - \equiv + \\
 + \div + \equiv + \\
 + \div - \equiv - \\
 - \div + \equiv - \\
 - \div - \equiv +
 \end{array}$$

2. The ratios $a : b$, $1 : \frac{b}{a}$, $\frac{a}{b} : 1$ are all equivalent.
3. To divide a quantity in the ratio $m : n$, you must calculate the fractions $\frac{m}{m+n}$ and $\frac{n}{m+n}$ of the quantity.

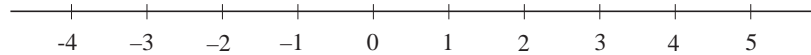
WORKED EXAMPLES and EXERCISES

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8 Numbers in Context

8.1 Negative Numbers

A number line like this is very helpful when dealing with negative numbers.



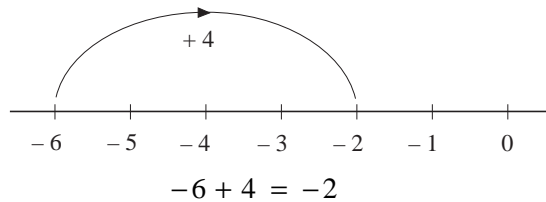
Worked Example 1

An ice cube taken from a deep freeze is at a temperature of -6°C . Its temperature rises by 4 degrees. What is its new temperature?



Solution

The number line can be used as shown.



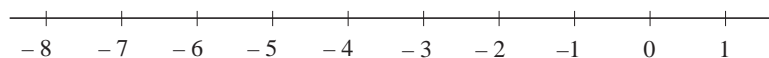
Worked Example 2

Write down numbers which are:

- (a) less than -4
- (b) greater than -5 but less than -2
- (c) less than -3 but greater than -8 .



Solution



- (a) Any number less than -4 must be to the left of -4 on the number line, for example -5 , -6 , -7 , etc.
- (b) A number greater than -5 must be to the right of -5 , and to be less than -2 must also be to the left of -2 , for example -4 and -3 .
- (c) To be less than -3 the number must be to the left of -3 but to be greater than -8 the number must be to the right of -8 , for example -7 , -6 , -5 , -4 .



Investigation

Within 4 consecutive years, Mrs. White gave birth to four lovely children. Today, x years later, Mr and Mrs. White find out that the product of their four children's ages is 3 024. How old is each child now, assuming that all the children are of different ages?

8.1



Exercises

1. Copy and complete the following table of temperature changes.

<i>Original Temperature</i>	<i>Temperature Change</i>	<i>Final Temperature</i>
6° C	-10° C	
-4° C	+2° C	
-5° C	-3° C	
-10° C	-4° C	
2° C	-10° C	
-6° C	+1° C	
-5° C	-2° C	

2. Place each set of numbers in increasing order.

- (a) -10, -4, -6, 2, -5, 3 (b) 3, -1, 0, -4, 7
 (c) 7, -2, -4, -6, 3, 0 (d) -4, 0, -7, 7, 5, -2
 (e) -1, -4, 0, -5, 6, -3

3. Write down all the integers which lie between:

- (a) -6 and -10 (b) -4 and 2 (c) -3 and 0
 (d) -1 and -3 (e) -9 and 0.

4. Write down a whole number which is:

- (a) greater than -10 but less than -6 (b) less than -6
 (c) greater than -5 but less than 0 (d) greater than -10
 (e) less than -6 but greater than -9 (f) less than -4.

5. Insert a < sign or a > sign between each pair of numbers.

- (a) 3 2 (b) -5 -6 (c) 0 -1
 (d) -7 -10 (e) -2 -4 (f) -1 -6
 (g) -5 0 (h) -9 -6 (i) -8 -2



Just For Fun

If $ABCDE \times 4 = EDCBA$, find A, B, C, D and E if none of them is zero.

8.3 Simple Ratios

Ratios are used in many situations to describe how two quantities are related. For example, a cake recipe requires twice as much flour as margarine. The ratio of flour to margarine is 2 : 1. Sometimes ratios can be simplified; for example, the ratio 100 : 50 is the same as the ratio 2 : 1. Ratios are simplified in a very similar way to fractions.



Worked Example 1

Simplify each of the following ratios.

- (a) 48 : 12 (b) 27 : 9 (c) 35 : 49



Solution

- (a) Both numbers in the ratio can be divided by 12. This gives

$$48 : 12 = 4 : 1$$

Alternatively, the ratio can be simplified in a number of steps.

$$\begin{aligned} 48 : 12 &= 24 : 6 \\ &= 12 : 3 \\ &= 4 : 1 \end{aligned}$$

- (b) Here both numbers in the ratio can be divided by 9. This gives

$$27 : 9 = 3 : 1$$

Alternatively, the ratio can be simplified in steps to give

$$\begin{aligned} 27 : 9 &= 9 : 3 \\ &= 3 : 1 \end{aligned}$$

- (c) Here both numbers can be divided by 7 to give

$$35 : 49 = 5 : 7$$



Worked Example 2

A school class contains 12 girls and 20 boys. Find the ratio of:

- (a) girls to boys,
(b) boys to girls.



Solution

- (a) The ratio of girls to boys is:

$$12 \text{ to } 20 \text{ or } 12 : 20$$

This can be simplified by dividing both numbers by 4, to give

$$12 : 20 = 3 : 5$$

8.3

- (b) To find the ratio of boys to girls, reverse the ratio of girls to boys, to give 5 : 3.



Worked Example 3

A glass contains 300 cm^3 of drink. The drink is made by mixing 50 cm^3 of concentrate with water. Find the ratio of concentrate to water.



Solution

$$\begin{aligned} \text{Amount of water} &= 300 - 50 \\ &= 250 \text{ cm}^3 \end{aligned}$$

The ratio of concentrate to water is

$$50 : 250$$

which simplifies to

$$1 : 5$$



Worked Example 4

The ratio of blue Smarties to other coloured Smarties in one packet is 1 : 12. How many Smarties would there be in the packet if it contained:

- (a) 3 blue Smarties?
 (b) 5 blue Smarties?



Solution

The ratio of 1 : 12 means that for every blue Smartie there are 12 Smarties of other colours.

- (a) This packet contains 3 blue Smarties and $3 \times 12 = 36$ other Smarties.
 In total the packet contains $3 + 36 = 39$ Smarties.
- (b) This packet contains 5 blue Smarties and $5 \times 12 = 60$ other Smarties.
 This give a total of 65 Smarties.



Exercises

1. Simplify each of the following ratios.

- | | | |
|---------------|----------------|----------------|
| (a) 4 : 2 | (b) 8 : 2 | (c) 3 : 6 |
| (d) 9 : 12 | (e) 5 : 30 | (f) 8 : 42 |
| (g) 3 : 18 | (h) 25 : 75 | (i) 8 : 100 |
| (j) 30 : 240 | (k) 64 : 80 | (l) 21 : 15 |
| (m) 81 : 48 | (n) 32 : 100 | (o) 50 : 49 |
| (p) 4.8 : 1.2 | (q) 10.5 : 3.5 | (r) 8.6 : 30.1 |

8.3

2. A school contains 300 girls and 320 boys. Find:
 - (a) the ratio of girls to boys,
 - (b) the ratio of boys to girls.

3. The shape of a room is a rectangle with sides of length 5 m and 3.5 m. Find the ratio of:
 - (a) the length to the width,
 - (b) the width to the length.

4. Two different bus companies have different pricing policies.
For Company A on one route the adult fare is £1.20 and the child fare is 40p.
For Company B on a different route, the adult fare is £1.40 and the child fare is 70p.
 - (a) Find the ratio of the child fare to the adult fare for each company.
 - (b) Which company gives children the better deal?

5. A library contains 720 non-fiction books and 400 fiction books.
 - (a) Find the ratio of fiction books to non-fiction books.
 - (b) Find the new ratio if 40 new fiction books are bought for the library.

6. A fizzy drink contains lemonade and 60 cm^3 of orange juice. Find the ratio of juice to lemonade in:
 - (a) a 100 cm^3 drink,
 - (b) a 300 cm^3 drink.

7. A car park contains 400 parking spaces. Of these spaces, 60 are short term and the rest long term. Find the ratio of short term spaces to long term spaces.

8. In a season a football team played 60 matches. They won 18, lost 20 and the rest were draws. Find the following ratios:
 - (a) number of matches won to number of matches drawn,
 - (b) number of matches won to other matches,
 - (c) number of matches lost to number of matches won.

9. Orange squash is mixed with water in the ratio 1 : 8, i.e. 1 part orange squash to 8 parts water. How much water is mixed with:
 - (a) 100 cm^3 of squash,
 - (b) 20 cm^3 of squash,
 - (c) 5 cm^3 of squash?

8.3

10. In a school the ratio of teachers to pupils is 1 : 20. If there are 12 teachers, how many pupils are there in the school?
11. In a class the ratio of left-handed pupils to right-handed pupils is 1 : 12. There are 2 left-handed pupils in the class. How many pupils are there in the class?
12. In packets of sweets, chocolate covered peanuts are mixed with solid chocolate sweets in the ratio 1 : 3. How many sweets are there in a packet that contains 20 chocolate covered peanuts?
13. In a herd of cattle, the ratio of bulls to cows is 2 : 25. How many cattle would be in the herd of it contained 10 bulls?

8.4 Proportion and Ratio

When solving problems that contain ratios like 4 : 5 or 3 : 7, it is often useful to write the ratios in the form $n : 1$ or $1 : m$.

So $4 : 5$ is equivalent to $1 : \frac{5}{4}$ or $\frac{4}{5} : 1$
 and $3 : 7$ is equivalent to $1 : \frac{7}{3}$ or $\frac{3}{7} : 1$.

**Worked Example 1**

In a fruit drink, orange juice and pineapple juice are mixed in the ratio 3 : 7. Find how much pineapple juice would be mixed with 500 cm³ of orange juice.

**Solution**

The ratio of orange juice to pineapple juice is

$$3 : 7 \text{ or } 1 : \frac{7}{3}$$

So for every 1 cm³ of orange juice, $\frac{7}{3}$ cm³ of pineapple juice is needed.

For 500 cm³ of orange juice, the amount of pineapple juice needed is

$$500 \times \frac{7}{3} = 1166\frac{2}{3} \text{ cm}^3.$$

**Worked Example 2**

Jason buys 8 m of wire netting to make a rabbit run. This costs him £5.04. Ramada is also making a rabbit run. He needs 6.8 m of wire netting. How much will this cost?

**Solution**

The cost per metre of the wire netting is:

$$\frac{\pounds 5.04}{8} = \pounds 0.63 \text{ or } 63\text{p per m.}$$

8.4

Ramada needs 6.8 m. This will cost

$$\begin{aligned} 6.8 \times 63\text{p} &= 428.4\text{p} \\ &= \text{£}4.28 \quad (\text{to the nearest penny}) \end{aligned}$$

This is equivalent to using the ratio 6.8 : 8; so solving this problem as a ratio, you take

$$\frac{6.8}{8} \times \text{£}5.04 = \text{£}4.28, \text{ to the nearest penny (as before).}$$



Worked Example 3

Three men are digging trenches to install cables to connect computers to the internet. They can dig 12 m of trench each day.

- (a) How long will it take 2 men to dig 120 m of trench?
- (b) How many men will be needed to dig 80 m of trench in 2 days?



Solution

As 3 men dig 12 m each day, 1 man digs 4 m each day.

- (a) As each man digs 4 m per day, 2 men dig 8 m per day. The time taken to dig 120 m is given by

$$\frac{120}{8} = 15 \text{ days}$$

Alternatively, 2 men can dig

$$\frac{2}{3} \times 12 = 8 \text{ m}$$

of trench each day, so it will take

$$\frac{120}{8} = 15 \text{ days}$$

to dig the trench.

- (b) As each man digs 4 m per day, each man digs 8 m in 2 days. The number of men needed is given by

$$\frac{80}{8} = 10 \text{ men.}$$



Exercises

1. The ratio of the length to width of a photograph is 3 : 2. The photograph is enlarged so that the length is 9 inches.
What is the width of the enlarged photograph?
2. The ratio of flour to sugar in a recipe is 5 : 4.
 - (a) How much flour should be mixed with 100 grams of sugar?
 - (b) How much sugar should be mixed with 200 grams of flour?

8.48.3

3. The manager of a music store estimates that the ratio of sales of cassettes to CDs is 4 : 7.
- (a) In one day, 92 cassettes were sold. How many CDs were sold that day?
- (b) On another day, 84 CDs were sold. How many cassettes were sold that day?
4. A 6 m length of rope costs £2.70. Find the cost of the following lengths of rope.
- (a) 1 m (b) 15 m (c) 22 m
5. A greengrocer sells potatoes by the kilogram. She sells 5 kg of potatoes for 60p. Find the cost of:
- (a) 7 kg of potatoes,
- (b) 20 kg of potatoes,
- (c) 2 kg of potatoes.
6. Ribbon is sold from a roll. Rachel buys 3 m for £1.71. Find the cost of each length of ribbon below.
- (a) 5 m (b) 12 m (c) 70 cm
7. A recipe uses 200 grams of flour to make 8 cakes.
- (a) How much flour would be needed for 20 cakes?
- (b) How many cakes could be made with 325 grams of flour?
8. Ben bought 12 litres of petrol for £7.08.
- (a) How much would 40 litres of petrol cost?
- (b) How much petrol could be bought for £10?
9. Shakeel adds the following spices to a recipe for 4 people.
- 2 teaspoons turmeric*
- 1 teaspoons coriander*
- $\frac{1}{2}$ teaspoon hot chilli powder*
- How much of each ingredient should he use for a recipe for:
- (a) 8 people, (b) 10 people, (c) 3 people?
10. Mr Mosafeer employs 5 men who together can build a wall 12 m long in 3 days.
- (a) How long would it take the men to build a wall 20 m long?
- (b) How long would it take 3 men to build the 12 m wall?
- (c) If the 12 m wall must be built in 2 days, how many more men must Mr Mosafeer employ?

8.4

11. A school with 700 pupils employs 25 teachers. Use this ratio of teachers and pupils to answer the following questions.
- How many teachers are needed for a school of 560 pupils?
 - How many pupils are there in a school with 22 teachers?
12. Two people can unload a lorry containing 200 boxes in 5 hours.
- If four people are to unload the lorry instead of 2, how much time will be saved?
 - A larger lorry contains more boxes. If 6 people can unload it in $2\frac{1}{2}$ hours, how many boxes does it contain?
13. Three photocopiers can produce 5400 copies in one hour.
- How long would it take one photocopier to make 16 200 copies?
 - How long would it take two photocopiers to make 1200 copies?
 - How many photocopiers would be needed to make 3600 copies in 30 minutes?
14. A decorator find that 2 people can paint 80 m^2 per day. A new warehouse has 560 m^2 of walls to be painted. The decorator wants to complete the work in a number of whole days and does not want to employ more than 10 people to work on the job. How can the work be completed in the shortest time?

15.



Two pints of milk cost 58p.

What is the cost of 5 pints of milk at the same price per pint?

(SEG)

16. Here is a list of ingredients for making some Greek food.

These amounts make enough for 6 people.

2 cloves of garlic.

4 ounces of chick peas.

4 tablespoons of olive oil.

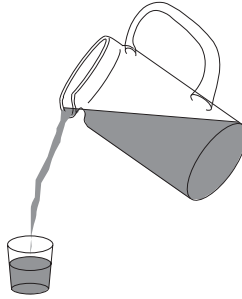
5 fluid ounces of Tahina paste.

Change the amounts so that there will be enough for 9 people.

(LON)

8.4

17. (a)



An orange drink is made by mixing water with concentrated orange juice.

$\frac{3}{4}$ of the orange drink is water.

How many litres of water will be in 12 litres of orange drink?

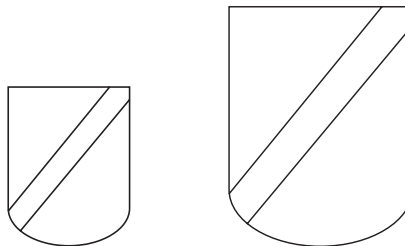
(b) It takes 100 g of flour to make 15 shortbread biscuits.

(i) Calculate the weight of flour needed to make 24 shortbread biscuits.

(ii) How many shortbread biscuits can be made from 1 kg of flour?

(MEG)

18. A school badge is made in two sizes.



Not to scale

The width of the small size is 3 cm.

The large size is an enlargement of the small size in the ratio 2 : 3.

Calculate the width of the large size badge.

(SEG)

19. (a) A shopkeeper orders one hundred 60 g packets of peanuts.

What is the total weight of the order in kilograms?

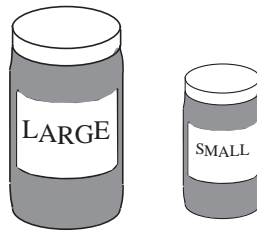
(b) A 60 g packet of peanuts costs 48 pence.

Calculate the cost of an 80 g packet of peanuts at the same price per gram.

(SEG)

8.4

20. Jam is sold in two sizes.



A large pot of jam costs 88p and weighs 822 g.

A small pot of jam costs 47p and weighs 454 g.

Which pot of jam is better value for money?

You **must** show all your working.

(SEG)

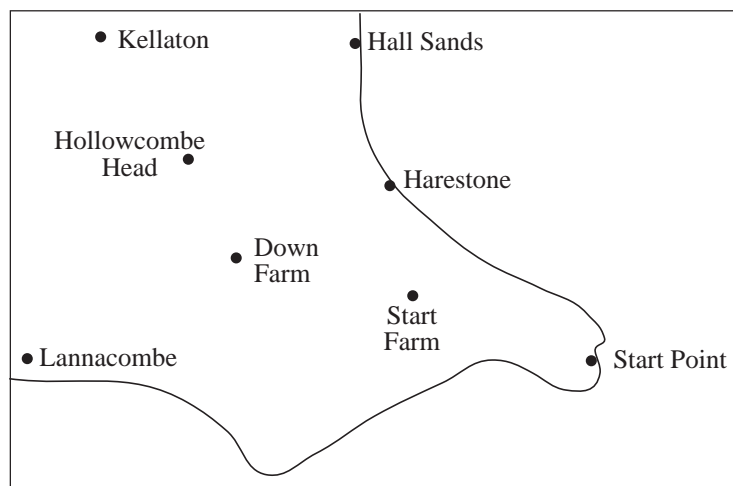
8.5 Map Scales and Ratios

Maps have a scale which relates the distance on the map to the corresponding distance on the ground. For example, on some ordnance survey maps the scale 1 : 50 000 is quoted on the cover. This means that a distance of 1 cm on the map corresponds to 50 000 cm, or 500 m, on the ground.



Worked Example 1

The map shown below has been drawn with a scale of 1 : 50 000.



Find the distances between:

- (a) Down Farm and Start Farm,
- (b) Kellaton and Harestone.

Give your answer in kilometres.

8.5

**Solution**

As the scale is 1 : 50 000, each centimetre on the map represents 50 000 cm in reality.

- (a) From the map the distance between Down Farm and Start Farm can be measured as 2.4 cm.

The actual distance between the two points is given by:

$$\begin{aligned} 2.4 \times 50\,000 &= 120\,000 \text{ cm} \\ &= 1200 \text{ m} \\ &= 1.2 \text{ km} \end{aligned}$$

- (b) The distance between Kellaton and Harestone can be measured as 4.3 cm.

The actual distance can then be calculated:

$$\begin{aligned} 4.3 \times 50\,000 &= 215\,000 \text{ cm} \\ &= 2150 \text{ m} \\ &= 2.15 \text{ km} \end{aligned}$$

**Worked Example 2**

The distance between two places is 12 km. A map scale is 1 : 25 000. Find the distance between the two places on the map, in centimetres.

**Solution**

This map will use 1 cm for every 25 000 cm in reality.

First convert 12 km to centimetres.

$$\begin{aligned} 12 \text{ km} &= 12\,000 \text{ m} \\ &= 1\,200\,000 \text{ cm} \end{aligned}$$

To find the distance on the map divide 1 200 000 by 25 000. This gives

$$\frac{1\,200\,000}{25\,000} = 48$$

So on the map the two places will be 48 cm apart.

**Worked Example 3**

Two places are 4.5 km apart. On a map they are 15 cm apart. What is the scale of the map?

**Solution**

First convert 4.5 km to cm, so that both distances are given in the same units.

$$\begin{aligned} 4.5 \text{ km} &= 4500 \text{ m} \\ &= 450\,000 \text{ cm} \end{aligned}$$

8.5

The scale can then be expressed as the ratio

$$15 : 450\,000$$

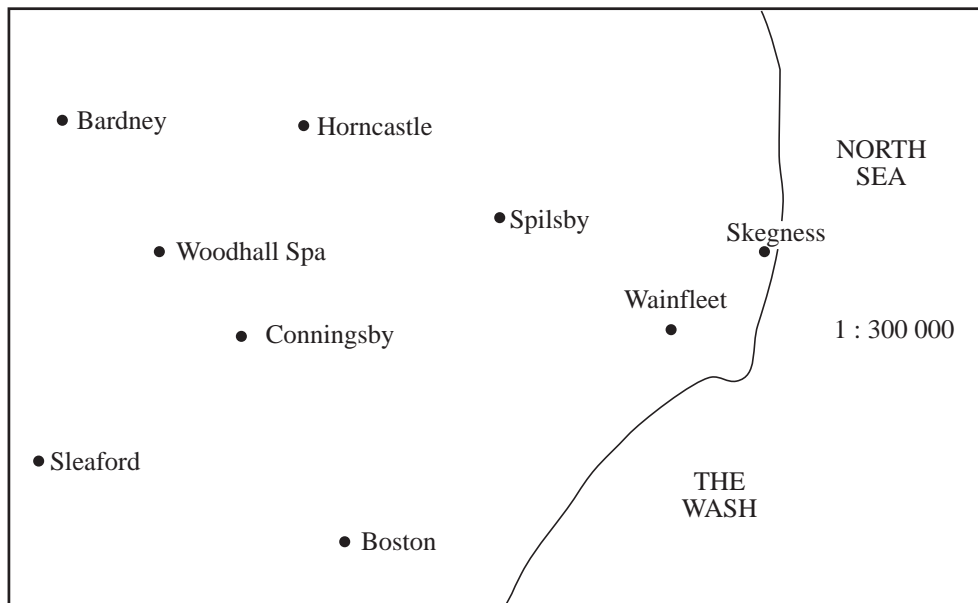
because 15 cm on the map represents 450 000 cm in reality. This ratio can be simplified by dividing both numbers by 15 to give

$$1 : 30\,000$$



Exercises

1. The map below has a scale of 1 : 300 000.



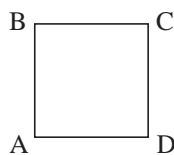
Use the map to find the distances between:

- (a) Woodhall Spa and Boston,
 - (b) Sleaford and Skegness,
 - (c) Horncastle and Boston,
 - (d) Spilsby and Conningsby.
2. A map has a scale of 1 : 400 000. Copy and complete the table below which gives the distances between various towns.

<i>Towns</i>	<i>Distance on map</i>	<i>Actual distance</i>
London and Sheffield	58.5 cm	km
Shrewsbury and Birmingham	15.8 cm	km
Leeds and Hull	21.0 cm	km
Manchester and Newcastle	42.8 cm	km

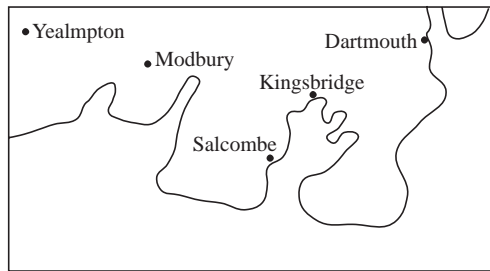
8.5

3. On a map of the Shetland Islands with a scale of 1 : 600 000 the distance between the towns of Scalloway and Lerwick is 0.4 cm. What is the actual distance between the two towns?
4. A ship is to sail from Holyhead in Wales to Dublin in Ireland. On a map with a scale of 1 : 3 000 000, the distance between the two ports is 3.5 cm. Find the actual distance between the ports in kilometres.
5. The direct distance between Dover and Calais is 42 km. What would be the distance between these two ports on maps with scales:
- (a) 1 : 3 000 000, (b) 1 : 1 000 000, (c) 1 : 50 000?
6. On a map with a scale of 1 : 3 000 000 the distance between Bristol and Bath is 0.6 cm. Find the actual distance between the two places and the distance between them on a map with a scale of 1 : 60 000.
7. On a map with a scale of 1 : 300 000 the distance between Burnley and Blackburn is 5.5 cm.
- (a) Find the distance between the two towns in km.
(b) How far apart would the two towns be on a map with a scale of 1 : 50 000?
8. Cambridge and Huntingdon are 24 km apart. Find the scale of a map that represents this distance by:
- (a) 6 cm, (b) 60 cm, (c) 40 cm, (d) 5 cm.
9. A road atlas has maps on each page. The width of each page is 18 cm and this represents a distance of 63 km. Find the scale of the map.
- Some pages of the road atlas are photocopied and their size changed. Find the scale of the map that is produced if the widths of the pages are reduced to:
- (a) 9 cm, (b) 12 cm, (c) 16 cm.
10. A grid square on an ordnance survey map has sides of length 2 cm. The map has a scale of 1 : 5000.
- Four points are marked on the ground at the corners of a grid square. Find the actual distance between AC, correct to the nearest metre.



8.5

11. (a) Find the scale of the map shown below if the actual distance between Kingsbridge and Salcombe is 5.4 km.



- (b) Find the actual distance between Modbury and Dartmouth.

12. On a map with a scale of 1 : 25 000 a plot of land is represented by a rectangle 1.5 cm by 1.2 cm. Find the area of the plot of land.

8.6 Proportional Division

Sometimes a quantity has to be divided in a certain ratio. For example, two waiters may divide their tips in the ratio 2 : 3 because one has worked longer than the other. If they had £5 of tips, one would get £2 and the other £3. If they had £20 of tips, one would get £8 and the other £12.



Worked Example 1

Medrine and Nikki earn £285 by making curtains. Because Medrine did more of the work they decide to divide the £285 in the ratio 3 : 2. How much do they earn each?



Solution

This problem is solved by dividing the £285 into 5 parts and giving 3 parts to Medrine and 2 parts to Nikki. It is divided into 5 parts because the ratio is 3 : 2.

$$\frac{285}{5} = \text{£}57$$

$$\begin{aligned} \text{Medrine's share} &= 3 \times 57 \\ &= \text{£}171 \end{aligned}$$

$$\begin{aligned} \text{Nikki's share} &= 2 \times 57 \\ &= \text{£}114 \end{aligned}$$



Worked Example 2

Pineapple, orange and apple juices are mixed in the ratio 2 : 3 : 5 to make a new drink. Find the volume of each type of juice contained in 250 cm³ of the new drink.

8.6

**Solution**

Adding the terms of the ratio gives

$$2 + 3 + 5 = 10$$

So the volume of the drink must be divided into 10 parts.

$$\frac{250}{10} = 25 \text{ cm}^3$$

Now the volume of each type of juice can be calculated.

$$\begin{aligned} \text{Volume of pineapple juice} &= 2 \times 25 \\ &= 50 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Volume of orange juice} &= 3 \times 25 \\ &= 75 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Volume of apple juice} &= 5 \times 25 \\ &= 125 \text{ cm}^3 \end{aligned}$$

**Exercises**

- The ratio of the volume of oxygen to nitrogen in the air is 1 : 4. Find the volume of oxygen and the volume of nitrogen in 10 litres of air.
- The ratio of boys to girls in a school play is 2 : 5. Find the numbers of boys and girls in the play if there were 63 children in the play.
- Ben and Emma collect 140 conkers. They share them out in the ratio 4 : 3. How many conkers do they get each?
- Claire and Laura work as waitresses. Each week, Claire works on 5 evenings and Laura on 4 evenings. They share any tips in the ration 5 : 4 at the end of each week. How much do they get each if the total of tips for the week is:
 - £12.69,
 - £33.57,
 - £24? (Give the answer to the nearest penny.)
- In packs of *Fruit and Nut*, raisins and peanuts are mixed. The ratio of the weight of nuts to the weight of raisins is 5 : 3. Find the weight of nuts and the weight of raisins in:
 - 800 grams,
 - 200 grams,
 - 300 grams of the mix.

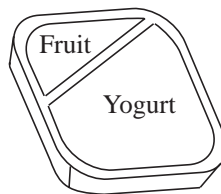
8.6

6. A football team arranges for £1000 to be divided among its players in the ratio of the goals scored. Goals are scored by the three players listed below.

Tim Nicholson	5
Ben Townsend	6
Kris Penk	9

How much cash does each player get?

7. Hannah, Adam, Lucy and Jacob are left £20 000 by a long lost relative. They divide the money in the ratio 4 : 3 : 2 : 1. How much does each of the children get?
8. Ahmed and Afzal win a jar containing 500 sweets in a competition. They divide the sweets in the ratio 3 : 7. Ahmed shares his in the ratio 3 : 2 with his wife and Afzal shares his in the ratio 3 : 4 with his girlfriend.
How many sweets do Ahmed and Afzal have each?
9. Apples, bananas and oranges are mixed in the ratio 5 : 6 : 4 respectively by weight, to make a fruit salad. What weight of each type of fruit would be needed to make 6 kg of fruit salad?
10. A music shop sells a total of 240 albums in one day. Some are on cassette and some on CD. The ratio of CDs to cassettes is 1 : 2. For the cassettes the ratio of pop to classical sales is 7 : 1. For CDs the ratio of classical to pop is 1 : 3. Find the total number of classical albums sold.
11. A fruit corner yogurt weighs 175 g altogether.



The ratio of the **weight of fruit** to the **weight of yogurt** is 2 : 5.

Calculate the weight of the fruit.

(SEG)



Just For Fun

A cashier of a bank was given one million one pound coins to count. How long will he take if he can count five coins in one second?

Answers to Exercises

8.1 Negative Numbers

- -4°C ; -2°C ; -8°C ; -14°C ; -8°C ; -5°C ; -7°C .
- (a) $-10, -6, -5, -4, 2, 3$ (b) $-4, -1, 0, 3, 7$
(c) $-6, -4, -2, 0, 3, 7$ (d) $-7, -4, -2, 0, 5, 7$
(e) $-5, -4, -3, -1, 0, 6$
- (a) $-7, -8, -9$ (b) $-3, -2, -1, 0, 1$ (c) $-2, -1$
(d) -2 (e) $-8, -7, -6, -5, -4, -3, -2, -1$
- (a) $-7, -8$ or -9 (b) $-7, -8, \dots$ (c) $-1, -2, -3$, or -4
(d) $-9, -8, \dots$ (e) -7 or -8 (f) $-5, -6, \dots$
- (a) $3 > 2$ (b) $-5 > -6$ (c) $0 > -1$ (d) $-7 > -10$ (e) $-2 > -4$
(f) $-1 > -6$ (g) $-5 < 0$ (h) $-9 < -6$ (i) $-8 < -2$

8.2 Arithmetic with Negative Numbers

- (a) 12 (b) -14 (c) -12 (d) -20 (e) -7 (f) -6
(g) -1 (h) 28 (i) 1 (j) 32 (k) -2 (l) 7
(m) -10 (n) 2 (o) -13 (p) -12 (q) 56 (r) 1
(s) -6 (t) -15 (u) -8 (v) -8 (w) 5 (x) -2

8.3 Simple Ratios

- (a) $2 : 1$ (b) $4 : 1$ (c) $1 : 2$ (d) $3 : 4$ (e) $1 : 6$ (f) $4 : 21$
(g) $1 : 6$ (h) $1 : 3$ (i) $2 : 25$ (j) $1 : 8$ (k) $4 : 5$ (l) $7 : 5$
(m) $27 : 16$ (n) $8 : 25$ (o) $50 : 49$ (p) $4 : 1$ (q) $3 : 1$ (r) $2 : 7$
- (a) $15 : 16$ (b) $16 : 15$
- (a) $10 : 7$ (b) $7 : 10$
- (a) $1 : 3$ and $1 : 2$ (b) Company A
- (a) $5 : 9$ (b) $11 : 18$
- (a) $3 : 2$ (b) $1 : 4$
- $3 : 17$
- (a) $9 : 11$ (b) $3 : 7$ (c) $10 : 9$
- (a) 800 cm^3 (b) 160 cm^3 (c) 40 cm^3

Answers

8.3

10. 240
11. 24
12. 80
13. 135

8.4 Proportion and Ratio

1. 6 inches
2. (a) 120 gms (b) 160 gms
3. (a) 161 (b) 48
4. (a) 45p (b) £5.75 (c) £9.90
5. (a) 84p (b) £3.40 (c) 24p
6. (a) £2.85 (b) £6.84 (c) £23.94
7. (a) 500 gms (b) 13
8. (a) £23.06 (b) about 16.95 litres
9. (a) 4, 2, 1 (b) $5, \frac{5}{2}, \frac{5}{4}$ (c) $\frac{3}{2}, \frac{3}{4}, \frac{3}{8}$ (teaspoonsful)
10. (a) 5 days (b) 5 days (c) 3 men (2.5)
11. (a) 20 teachers (b) 616 teachers
12. (a) $2\frac{1}{2}$ hours (b) 300 boxes
13. (a) 9 hours (b) 20mins (c) 4
14. Employ 7 people and it takes 2 days
15. £1.45
16. 3, 6, 6, $7\frac{1}{2}$
17. (a) 9 litres (b) (i) 16 (c) (ii) 150
18. 4.5 cm
19. (a) 6 kg (b) 64p
20. Small pot - unit cost = 0.1035p (< 0.1071p)

Answers

8.5 Map Scales and Ratios

- (a) 13.5 km (b) 30 km (c) 16.5 km (d) 11.25 km
- 234 km, 63.2 km, 84 km, 171.2 km
- 2.4 km
- 105 km
- (a) 1.4 cm (b) 4.2 cm (c) 84 cm
- 18 km; 30 cm
- (a) 16.5 km (b) 33 cm
- (a) 1 : 400 000 (b) 1 : 40 000 (c) 1 : 600 000 (d) 1 : 480 000
- 1 : 350 000 (a) 1 : 700 000 (b) 1 : 525 000 (c) 1 : 393 750
- 141 m
- (a) 1 : 540 000 (b) about 19.5 km
- 0.1125 km²

8.6 Proportional Division

- 2 litres of oxygen, 8 litres of nitrogen
- 18 boys, 45 girls
- Ben - 80, Emma - 60
- (a) £7.05, £5.64 (b) £18.65, £14.92 (c) £13.33, £10.67
- (a) 500 g, 300 g, (b) 125 g, 75 g (c) 187.5 g, 112.5 g
- £250, £300, £450
- £8000, £6000, £4000, £2000
- Ahmed: 90, Afzal: 150
- apples: 2kg; bananas: 2.4 kg; oranges: 1.6 kg
- 50
- 50 g