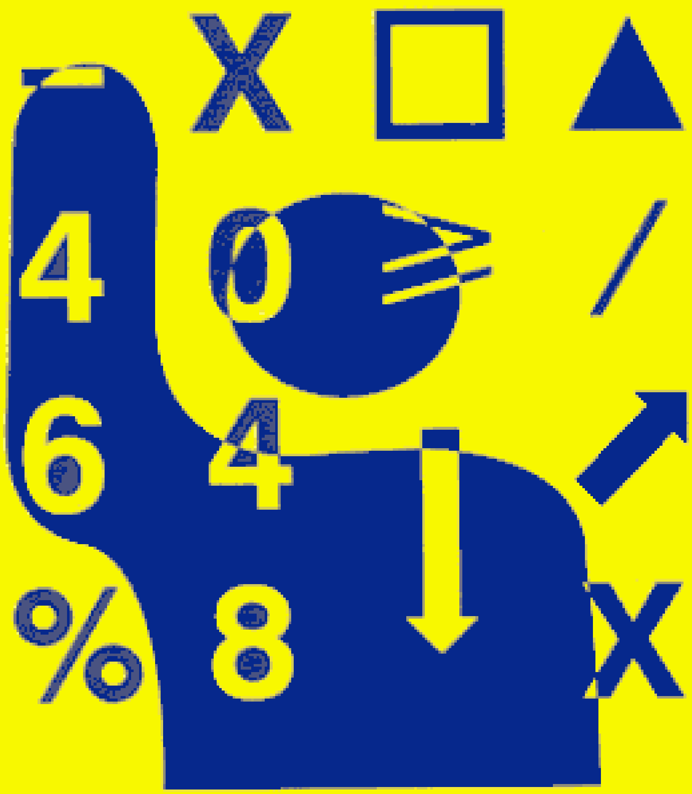
Mathematics Enhancement Programme

Primary Demonstration Project

4A Fractions

Help Booklet



Support for Primary Teachers in Mathematics

Primary Project funded by Pricewaterhouse Coopers

in association with
British Steel
Garfield Weston Foundation

Sponsored by

ESSO

CIMT
School of Education
University of Exeter

Mathematics Enhancement Programme

Help Module 4

FRACTIONS

Part A

Contents of Part A

Contents of Part B

CIMT
University of Exeter

Sponsored by
ESSO

Gatsby Technical
Education Project

PREFACE

This is one of a series of *Help Modules* designed to help you gain confidence in mathematics. It has been developed particularly for primary teachers (or student teachers) but it might also be helpful for non-specialists who teach mathematics in the lower secondary years. It is based on material which is already being used in the *Mathematics Enhancement Programme: Secondary Demonstration Project*.

The complete module list comprises:

1. ALGEBRA
2. DECIMALS
3. EQUATIONS
4. FRACTIONS
5. GEOMETRY
6. HANDLING DATA
7. MENSURATION
8. NUMBERS IN CONTEXT
9. PERCENTAGES
10. PROBABILITY

Notes for overall guidance:

- Each of the 10 modules listed above is divided into 2 parts. This is simply to help in the downloading and handling of the material.
- Though referred to as 'modules' it may not be necessary to study (or print out) each one in its entirely. As with any self-study material you must be aware of your own needs and assess each section to see whether it is relevant to those needs.
- The difficulty of the material in **Part A** varies quite widely: if you have problems with a particular section do try the one following, and then the next, as the content is not necessarily arranged in order of difficulty. Learning is not a simple linear process, and later studies can often illuminate and make clear something which seemed impenetrable at an earlier attempt.
- In **Part B, Activities** are offered as backup, reinforcement and extension to the work covered in Part A. **Tests** are also provided, and you are strongly urged to take these (at the end of your studies) as a check on your understanding of the topic.
- The marking scheme for the revision test includes B, M and A marks. Note that:

M *marks*	are for method;
A *marks*	are for accuracy (awarded only following a correct M mark);
B *marks*	are independent, stand-alone marks.

We hope that you find this module helpful. Comments should be sent to:

Professor D. N. Burghes
CIMT, School of Education
University of Exeter
EXETER EX1 2LU

The full range of Help Modules can be found at

www.ex.ac.uk/cimt/help/menu.htm

4 *Fractions* Introductory Notes

Historical Background

Background and Preparatory Work

Much of the content here is related to that of the *Percentages* module, and some of the key aspects are replicated in both sections. This is in part due to the fundamental importance of these topics and in part in recognition of their equivalence.

In everyday life we almost never think about how we speak or write in terms of 'grammar' – 'parts of speech', tenses', or 'correct sentence construction'. Yet those who never go beyond instinctive, colloquial speech, and who have little feeling for the way the language works, cannot escape from the fact that their possibilities in life are restricted by their limited means of expression.

Mathematics – as Galileo observed – is 'the language in which the Book of Nature is written'. And as our daily lives come to depend more and more on the control we exert on the world around us, it is ever more important for ordinary people to have a deeper understanding for the simple 'grammar' which underpins all mathematics.

Colloquial mathematics is limited to addition. Mathematics proper begins with *multiplication* (and division), and with the associated themes of *ratio*, *fractions* and *proportion* – though this fact could easily be missed by someone reading the English National Curriculum and examining the associated guidance and assessment materials (and their mark schemes)!

The idea of counting has arisen naturally in many, if not most, cultures. The act of counting tends to highlight the fact that the counting sequence is based on repetition of a single step. In its crudest form this step is not yet mathematical – being close to the call 'next please' that one hears in queues the world over. However, once one begins to think in terms of *quantity*, it is fairly natural to reinterpret the pure 'sequencing in time' of successive *numbers* in terms of repeatedly 'increasing each quantity by the *first* number in the sequence': that is, 'add one'. From there it is a relatively short step to primitive addition and subtraction – based on the idea of 'adding on' (as used by infants with their fingers, and at check-outs the world over for giving change). These procedures have built-in limitations, which teachers have to help pupils to transcend; but they illustrate the universal 'colloquial' character of addition.

In our desire to encourage those who find mathematics difficult (and perhaps also to ease the lot of those faced with the difficult task of *teaching* mathematics effectively) we have fallen into the trap of ignoring the nature of the discipline. The National Curriculum suggests that it is enough to *'use appropriate methods to solve problems'*; but the associated bureaucracy too often interprets this to mean 'any method that will obtain an acceptable answer'; thus attention shifts from the appropriateness of the *method* to the acceptability of the answer'; thus attention shifts from the appropriateness of the *method* to the acceptability of the answer. This effectively encourages teachers and pupils to be satisfied with any method that seems to work (Since this is enough to obtain almost all the available marks). We have therefore got out of the habit of exercising that judgement, which is an essential part of all good teaching, as to whether the method is really acceptable.

The consequences are now clear for all to see (except for those who prefer to close their eyes). For example, in the *Third International Mathematics and Science Study*, a large random sample of Year 4 primary pupils (aged 9) in 29 countries were asked to write the addition sum '$4+4+4+4+4=20$ as a multiplication'. Despite the fact that pupils in most countries start school later (often considerably later) than they do in England, only 39% of English pupils managed what should have been an automatic response. In contrast, the appropriate response was given by 90% of pupils in the top scoring country and by 63% of pupils in the median country.

Our failure to teach pupils to see the multiplicative structure of so many elementary problems, and our willingness to accept inappropriate 'additive' strategies, has profound consequences. In particular, when we encourage pupils to use primitive addition to solve what should be multiplication problems (simply because this appears to allow more of them to obtain the right answer without having to master any new tricks), we effectively convince them that *all problems can be solved using additive strategies.* Thus when faced with ratio problems, which are unavoidably multiplicative, pupils try using additive strategies and are doomed to fail.

Thus, while it is true that a price increase of 10% may at first be worked out by calculating 10% of the original price and then adding, the goal must be to ensure that pupils understand that the result is bound to be 110% of the original price, and so can be obtained *in one step* by multiplying by 1.1.

The advantage of this way of thinking becomes even more pronounced when trying to analyse a problem such as the following:

> *'One third of the class got As. One quarter of the remainder got Bs. What fraction of the class got Cs or worse?'*

The answer can be obtained in many ways; but these approaches miss the underlying structure of the problem *which is what makes the problem important*! (Since 'one quarter of the remainder got Bs', the required group is '$\frac{3}{4}$ of those who did not get As'. And since 'one third of the whole group got As', precisely $\frac{2}{3}$ of the whole group did not get As. Thus the answer is $\frac{3}{4}$ of $\frac{2}{3}$ – which simplifies to 'one half'.) Once one learns to think this way, all sorts of other similar problems can be solved very quickly.

The lesson here is that mathematics teachers need a clear *mental map* of elementary school mathematics, in which multiplication and its associated themes (fractions, ratio and proportion) are firmly centre-stage. Thus we need to build systematically and purposefully from

- *multiplication* ($4\times 3=12$, $6\times 3=18$, etc.)

via

- *division* with integer answers ($\frac{12}{3} = 4$, $\frac{18}{3} = 6$, etc.)
- with associated *ratio* problems ('12 pies cost £18, what do 4 pies cost?)

to

- the manipulation and simplification of simple ratios and equivalent fractions ($\frac{18}{12} = \frac{3}{2}$)
- and the complete arithmetic of fractions (' $\frac{\left[\frac{2}{3} + \frac{7}{4}\right]}{\left[\frac{5}{6} - \frac{3}{8}\right]} = ?$ ').

In practice, an important ingredient in this sequence is the ability to handle *percentages.* But too often percentages are seen as something separate – almost a subject in their own right – when they are in fact a simple application of the multiplicative principle (albeit with a notation of their own).

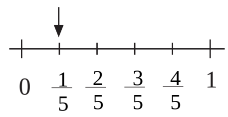
All of these notions are inter-linked, and introduce key ideas which underpin powerful aspects of elementary mathematics. For example,

- the idea that any ratio problem can be solved by the 'unit' method, or by the 'rule of three';
- the fact that ratios and fractions open the door to the exact solution of any linear equation;
- the central notion that *meaning* demands *simplification* (so that one is never satisfied with ' $\frac{18}{36}$ ' as an answer;
- the subtle (but crucial) advantage of using *exact* fraction notation, rather than converting automatically to approximate, ugly decimals;
- the unstated, but important idea, that the *rational numbers* form a number system which is 'closed' under all four operations (with division by zero forbidden);
- the fact that fractions force one to master, to understand, and then to trust, procedures (to evaluate expressions like $\frac{\left(\frac{7}{3}\right)}{\left(\frac{35}{12}\right)}$, or $\left[\frac{\left(\frac{2}{3} + \frac{7}{4}\right)}{\left(\frac{5}{6} + \frac{3}{8}\right)}\right]$);
- the way in which *proportion* underlies (a) all measurement, (b) *similarity* in geometry, (c) the definitions of the *trigonometric* functions sin, cos and tan;
- the way proportion is reflected in formulae for lengths and perimeters, for areas, and for volumes, and so on.

Key Issues

Introduction

Much of this module will be revision of what you have, in theory, done before, although it is a topic that seems to give great problems. As with all maths topics though, it should be stressed that there are logical rules to be obeyed at all times and if these rules are followed there should be no difficulties!



Part of the problem may lie in the fact that a fraction (e.g. $\frac{1}{5}$) is both a number in its own right (with a unique place on the number line) and also an operation when written as 'one fifth' of a quantity. As a number it has a decimal equivalent (i.e.. 0.2), and as an operation it is equivalent to a percentage (20%).

It is crucial that you are familiar and confident in moving between fractions, decimals and percentages.

It should also be noted that fractions are a key part of everyday life; for example,

- half price
- price reduced by a third
- gradient of hills
- division of food with quantities.

Language / Notation

Important language used includes

- equivalent fractions
- vulgar fractions.

It is also recommended that fractions are always written as, for example, $\frac{4}{5}$ rather than 4/5; the first version makes it clear that the number is equivalent to 4 divided by 5, whereas the second can so easily lead to errors in calculations.

Key Points

- Equivalent fractions, decimals and percentages.

Fractions	*Decimals*	*Percentages*
$\frac{1}{10}$	0.1	10%
$\frac{1}{8}$	0.125	12.5%
$\frac{1}{5}$	0.2	20%
$\frac{1}{4}$	0.25	25%
$\frac{1}{3}$	$0.\dot{3}$	$33\frac{1}{3}$%
$\frac{1}{2}$	0.5	50%
$\frac{2}{3}$	$0.\dot{6}$	$66\frac{2}{3}$%
$\frac{3}{4}$	0.75	75%
1	1.0	100%

- Equivalent fractions,

  e.g. $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$, etc.

- Percentage increase (decrease)

  $$= \frac{\text{actual increase (decrease)}}{\text{initial value}} = 100$$

- Multiplying fractions,

  $$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

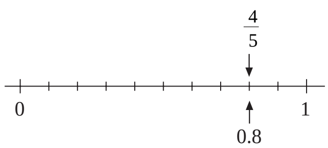
- Dividing fractions,

  $$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

Misconceptions

There are many; for example,

- that $\frac{1}{4} \div \frac{1}{2} = 2$ (instead of the correct answer, $\frac{1}{4} \div \frac{1}{2} = \frac{1}{2}$
- that $\frac{4}{5}$, $4/5$, $4 \div 5$, 0.8 are different (they are all the same number on the number line)



- that 20% is equivalent to $\frac{1}{20}$ (instead of the correct answer, $\frac{20}{100} = \frac{1}{5}$)

WORKED EXAMPLES and EXERCISES

4 Fractions

4.1 Fractions, Decimals and Percentages

Percentages can be converted to fractions because 'percentage' simply means 'per hundred'. They can also be converted very easily to decimals, which can be useful when using a calculator. Fractions and decimals can also be converted back to percentages.

Worked Example 1

Convert each of the following percentages to fractions.

(a) 50% (b) 40% (c) 8%

Solution

(a) $50\% = \frac{50}{100} = \frac{1}{2}$

(b) $40\% = \frac{40}{100} = \frac{2}{5}$

(c) $8\% = \frac{8}{100} = \frac{2}{25}$

Worked Example 2

Convert each of the following percentages to decimals.

(a) 60% (b) 72% (c) 6%

Solution

(a) $60\% = \frac{60}{100} = 0.6$

(b) $72\% = \frac{72}{100} = 0.72$

(c) $6\% = \frac{6}{100} = 0.06$

Worked Example 3

Convert each of the following decimals to percentages.

(a) 0.04 (b) 0.65 (c) 0.9

Solution

(a) $0.04 = \frac{4}{100} = 4\%$

(b) $0.65 = \frac{65}{100} = 65\%$

(c) $0.9 = \frac{9}{10} = \frac{90}{100} = 90\%$

Information

'Per cent' comes from the Latin , 'per centum', which means 'for each hundred'.

4.1

Worked Example 4

Convert each of the following fractions to percentages.

(a) $\frac{3}{10}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$

Solution

To convert fractions to percentages, multiply the fraction by 100%. This gives its value as a percentage.

(a) $\frac{3}{10} = \frac{3}{10} \times 100\%$ $= 30\%$

(b) $\frac{1}{4} = \frac{1}{4} \times 100\%$ $= 25\%$

(c) $\frac{1}{3} = \frac{1}{3} \times 100\%$ $= 33\frac{1}{3}\%$

Exercises

1. Convert each of the following percentages to fractions, giving your answers in their simplest form.

(a) 10%	(b) 80%	(c) 90%	(d) 5%
(e) 25%	(f) 75%	(g) 35%	(h) 38%
(i) 4%	(j) 12%	(k) 82%	(l) 74%

2. Convert each of the following percentages to decimals.

(a) 32%	(b) 50%	(c) 34%	(d) 20%
(e) 15%	(f) 81%	(g) 4%	(h) 3%
(i) 7%	(j) 18%	(k) 75%	(l) 73%

3. Convert the following decimals to percentages.

(a) 0.5	(b) 0.74	(c) 0.35	(d) 0.08
(e) 0.1	(f) 0.52	(g) 0.8	(h) 0.07
(i) 0.04	(j) 0.18	(k) 0.4	(l) 0.3

4. Convert the following fractions to percentages.

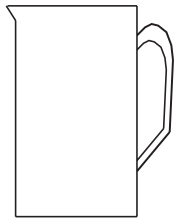
(a) $\frac{1}{2}$	(b) $\frac{7}{10}$	(c) $\frac{1}{5}$	(d) $\frac{3}{4}$
(e) $\frac{1}{10}$	(f) $\frac{9}{10}$	(g) $\frac{4}{5}$	(h) $\frac{4}{50}$
(i) $\frac{8}{25}$	(j) $\frac{7}{20}$	(k) $\frac{7}{25}$	(l) $\frac{2}{3}$

4.1

5. (a) Complete the equation $\frac{2}{3} = \frac{?}{15} = \frac{16}{?}$

(b) Change $\frac{7}{20}$ to a percentage. *(MEG)*

6. (a) Water is poured into this jug.

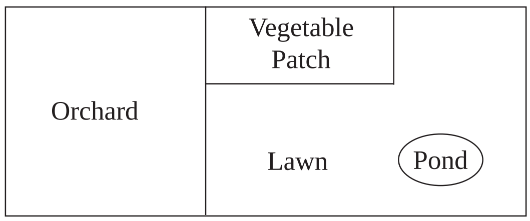


Copy the diagram and show accurately the water level when the jug is three-quarters full.

(b) What percentage of the jug is filled with water?

(SEG)

7. *Plan of a garden*



Not to scale

(a) In the garden the vegetable patch has an area of 46.2 m^2. The orchard has an area of 133.6 m^2.

What is the total area of the vegetable patch and the orchard? Give your answer to the nearest square metre.

(b) The garden has an area of 400 m^2.

(i) The lawn is 30% of the garden. Calculate the area of the lawn.

(ii) A pond in the garden has an area of 80 m^2. What percentage of the garden is taken up by the pond?

(SEG)

4.2 Fractions and Percentages of Quantities

Percentages are often used to describe changes in quantities or prices. For example,

'30% extra free' '10% discount' 'add $17\frac{1}{2}$% VAT'

This section deals with finding fractions or percentages of quantities.

Worked Example 1

Find 20% of £84.

Solution

This can be done by converting 20% to either a fraction or a decimal.

4.2

Converting to a fraction

Note that $20\% = \frac{20}{100} = \frac{1}{5}$

Therefore $20\% \text{ of } £84 = \frac{1}{5} \times £84$

$= £16.80.$

Converting to a decimal

Note that $20\% = 0.2$

Therefore $20\% \text{ of } £84 = 0.2 \times £84$

$= £16.80.$

Worked Example 2

A shopkeeper decides to increase some prices by 10%. By how much would she increase the price of:

(a) a loaf of bread costing 90p (b) a packet of cereal costing £2.00?

Solution

First note that $10\% = \frac{1}{10}.$

(a) $10\% \text{ of } 90\text{p} = \frac{1}{10} \times 90\text{p}$

$= 9\text{p}.$

So the cost of a loaf will be increased by 9p.

(b) $10\% \text{ of } £2 = \frac{1}{10} \times £2$

$= £0.20 \text{ or } 20\text{p}.$

So the cost of a packet of cereal is increased by 20p.

Worked Example 3

A farmer decides to sell 25% of his 500 cows. How many cows does he sell?

Solution

First note that $25\% = \frac{1}{4}.$

$25\% \text{ of } 500 = \frac{1}{4} \times 500$

$= 125.$

So he sells 125 cows.

4.2

Worked Example 4

Natasha invests £200 in a building society account. At the end of the year she receives 5% interest. How much interest does she receive?

Solution

First convert 5% to a fraction. $5\% = \frac{5}{100} = \frac{1}{20}$

$$5\% \text{ of } £200 = \frac{1}{20} \times £200$$
$$= £10.$$

So she receives £10 interest.

Exercises

1. Find

(a) 10% of 200 (b) 50% of £5 (c) 20% of £8

(d) 25% of £100 (e) 40% of £500 (f) 90% of 200

(g) $33\frac{1}{3}\%$ of £12 (h) 75% of 800 (i) 75% of 1000

(j) 80% of 20 kg (k) 70% of 5 kg (l) 30% of 50 kg

(m) 5% of 100 m (n) 20% of 50 m (o) 25% of £30

2. Find

(a) $\frac{2}{5}$ of 80 (b) $\frac{3}{4}$ of 120 (c) $\frac{1}{5}$ of 90

(d) $\frac{1}{4}$ of 360 (e) $\frac{4}{5}$ of 150 (f) $\frac{3}{10}$ of 500

3. A firm decides to give 20% extra free in their packets of soap powder. How much extra soap powder would be given away free with packets which normally contain

(a) 2 kg of powder (b) 1.2 kg of powder?

4. A house costs £30 000. A buyer is given a 10% discount. How much money does the buyer save?

5. John has invested £500 in a building society. He gets 5% interest each year. How much interest does he get in a year?

6. Karen bought an antique vase for £120. Two years later its value had increased by 25%. What was the new value of the vase?

7. Ahmed wants to buy a new carpet for his house. The cost of the carpet is £240. One day the carpet shop has a special offer of a 25% discount. How much money does he save by using this offer?

4.2

8. When Wendy walks to school she covers a distance of 1800 m. One day she discovers a short cut which reduces this distance by 20%. How much shorter is the new route?

9. Chen earns £30 per week from his part-time job. He is given a 5% pay rise. How much extra does he earn each week?

10. Gareth weighed 90 kg. He went on a diet and tried to reduce his weight by 10%. How many kilograms did he try to lose?

11. Kim's mother decided to increase her pocket money by 40%. How much extra did Kim receive each week if previously she had been given £2.00 per week?

12. A new-born baby girl weighed 4 kg. In the first three months her weight increased by 60%. How much weight had the baby gained?

13. Work out

(a) $\frac{7}{10}$ of £8 (b) 20% of £25 (c) $\frac{3}{8}$ of 6 metres.

(LON)

14. (a) Calculate 15% of £600.

(b) List these fractions in order of size, starting with the smallest.

$$\frac{1}{3}, \quad \frac{2}{9}, \quad \frac{5}{6}, \quad \frac{1}{6}$$

(MEG)

15. A cake weighs 850 grams. 20% of the cake is sugar. Calculate the weight of sugar in the cake.

(MEG)

16. An athletics stadium has 35 000 seats. 4% of the seats are fitted with headphones to help people hear the announcements. How many headphones are there in the stadium?

(NEAB)

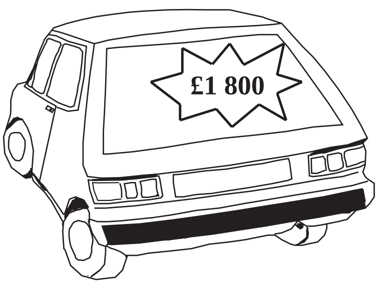
17. Jane wants to buy this car.

The deposit is $\frac{2}{5}$ of the price of the car.

Jane's father gives her 30% of the price.

Will this be enough for her deposit?

You must explain your answer fully.



Investigation

The ancient Egyptians were the first to use fractions. However, they only used fractions with a numerator of one. Thus they wrote $\frac{3}{8}$ *as* $\frac{1}{4}+\frac{1}{8}$, *etc.*

What do you think the Egyptians would write for the fractions $\frac{3}{5}$, $\frac{9}{20}$, $\frac{2}{3}$ *and* $\frac{7}{12}$?

4.3 Addition and Subtraction of Fractions

Note

The *numerator* is the **top** part of a fraction and the *denominator* is the **bottom** part of a fraction.

When adding or subtracting fractions they must have the same *denominator.*

Worked Example 1

$$\frac{4}{7} + \frac{5}{7} = ?$$

Solution

As both fractions have the same denominator (7), they can simply be added to give

$$\begin{aligned} \frac{4}{7} + \frac{5}{7} &= \frac{9}{7} \\ &= 1\frac{2}{7}. \end{aligned}$$

Worked Example 2

$$\frac{3}{4} + \frac{2}{5} = ?$$

Solution

As these fractions have different denominators, it is necessary to find the *lowest common denominator*, that is, the smallest number into which both denominators will divide exactly. In this case it is 20, since both 4 and 5 divide into 20 exactly.

$$\begin{aligned} \frac{3}{4} + \frac{2}{5} &= \frac{15}{20} + \frac{8}{20} \\ &= \frac{15+8}{20} \\ &= \frac{23}{20} \\ &= 1\frac{3}{20} \end{aligned}$$

Worked Example 3

$$\frac{2}{3} + \frac{7}{12} = ?$$

Solution

In this example, 12 is the lowest common denominator.

4.3

$$\frac{2}{3}+\frac{7}{12} = \frac{8}{12}+\frac{7}{12}$$
$$= \frac{8+7}{12}$$
$$= \frac{15}{12}$$
$$= 1\frac{\cancel{3}^{\,1}}{\cancel{12}_{\,4}}$$
$$= 1\frac{1}{4}$$

Worked Example 4

$$\frac{5}{8}-\frac{1}{3} = ?$$

Solution

Here 24 is the lowest common denominator.

$$\frac{5}{8}-\frac{1}{3} = \frac{15}{24}-\frac{8}{24}$$
$$= \frac{15-8}{24}$$
$$= \frac{7}{24}$$

Exercises

1. Give the answers to the following, simplifying them as far as possible.

(a) $\frac{1}{5}+\frac{1}{5}$ (b) $\frac{3}{8}+\frac{1}{8}$ (c) $\frac{5}{7}+\frac{1}{7}$

(d) $\frac{5}{7}-\frac{2}{7}$ (e) $\frac{8}{13}-\frac{5}{13}$ (f) $\frac{7}{9}-\frac{4}{9}$

(g) $\frac{7}{9}+\frac{8}{9}$ (h) $\frac{3}{5}+\frac{4}{5}$ (i) $\frac{6}{7}+\frac{5}{7}$

(j) $\frac{7}{10}-\frac{3}{10}$ (k) $\frac{8}{9}-\frac{5}{9}$ (l) $\frac{4}{15}-\frac{1}{15}$

2. Complete each of the following.

(a) $\frac{2}{5}+\frac{3}{7} = \frac{?}{35}+\frac{15}{35}$

$= \frac{?}{35}$

(b) $\frac{1}{5}+\frac{1}{6} = \frac{?}{30}+\frac{?}{30}$

$= \frac{?}{30}$

4.3

(c) $\frac{1}{2} + \frac{1}{4} = \frac{?}{4} + \frac{1}{4}$

$= \frac{?}{4}$

(d) $\frac{3}{16} + \frac{5}{8} = \frac{3}{16} + \frac{?}{16}$

$= \frac{?}{16}$

(e) $\frac{4}{7} + \frac{2}{3} = \frac{?}{21} + \frac{?}{21}$

$= \frac{?}{21}$

(f) $\frac{3}{5} + \frac{7}{12} = \frac{?}{60} + \frac{?}{60}$

$= \frac{?}{60}$

3. Find the answers to the following, simplifying them if possible.

(a) $\frac{1}{6} + \frac{3}{8}$ (b) $\frac{5}{7} + \frac{2}{5}$ (c) $\frac{1}{8} + \frac{3}{32}$

(d) $\frac{1}{10} + \frac{1}{3}$ (e) $\frac{3}{7} + \frac{5}{8}$ (f) $\frac{1}{2} + \frac{2}{3}$

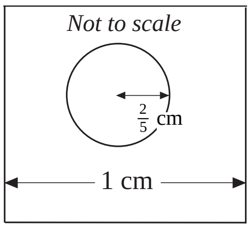
(g) $\frac{1}{7} + \frac{1}{10}$ (h) $\frac{5}{8} + \frac{4}{3}$ (i) $\frac{6}{7} + \frac{2}{3}$

(j) $\frac{4}{7} - \frac{1}{2}$ (k) $\frac{6}{11} - \frac{1}{4}$ (l) $\frac{2}{3} - \frac{1}{6}$

(m) $\frac{3}{4} - \frac{2}{3}$ (n) $\frac{5}{8} - \frac{5}{12}$ (o) $\frac{11}{12} - \frac{3}{8}$

4. A garden has an area of $\frac{2}{5}$ hectare. The owner buys an extra $\frac{1}{3}$ hectare of land to increase the size of the garden. What is the new size of the garden?

5. A large company makes a profit of £ $\frac{3}{4}$ million in one year and £ $\frac{2}{3}$ million the next year. Find the total profits for the two-year period.

6. 

A hole of radius $\frac{2}{5}$ cm is drilled in the middle of a metal sheet of width 1 cm.

How far is it from the edge of the sheet to the hole?

7. A council decides to turn $\frac{1}{3}$ of a park into a dog-free zone. It later bans dogs from the play area which occupies $\frac{1}{10}$ of the park and which was originally outside the dog-free zone. What fraction of the park is now open to dogs?

8. Mike has filled $\frac{3}{5}$ of the space on the hard disc in his computer with software. He wants to keep $\frac{1}{4}$ of the disc free from software. What fraction of the disc is left for extra software?

4.3

9. In a school $\frac{1}{3}$ of the children eat school dinners, $\frac{1}{2}$ bring packed lunches and the rest go home. What fraction of the children go home for lunch?

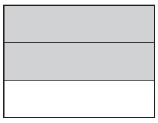
10. A shopper buys $1\frac{1}{4}$ kg of G*olden Delicious* apples and $1\frac{1}{3}$ kg of *Cox's* apples. Find the total weight of the apples bought.

4.4 Multiplication and Division of Fractions

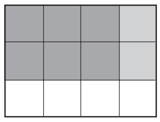
Multiplication

Consider finding $\frac{3}{4}$ of $\frac{2}{3}$ by starting with this rectangle.

First select $\frac{2}{3}$ of the rectangle, as shown by the shaded area.



Then select $\frac{3}{4}$ of the shaded area.



This represents $\frac{3}{4}$ of $\frac{2}{3}$ of the original rectangle, that is, $\frac{6}{12}$ or $\frac{1}{2}$ of the original rectangle.

Now $\frac{3}{4}$ of $\frac{2}{3}$ is the same as $\frac{3}{4}\times\frac{2}{3}$, so

$$\frac{3}{4}\times\frac{2}{3} = \frac{6}{12} = \frac{1}{2}.$$

When multiplying two fractions, the *numerators* (top parts) should be multiplied together to give the numerator of the result. Similarly, the two denominators should be multiplied together.

In general terms,

$$\boxed{\frac{a}{b}\times\frac{c}{d} = \frac{a\times c}{b\times d}}$$

Worked Example 1

$$\frac{3}{4}\times\frac{5}{7} = ?$$

Solution

$$\frac{3}{4}\times\frac{5}{7} = \frac{3\times 5}{4\times 7}$$

$$= \frac{15}{28}$$

4.4

Worked Example 2

$$\frac{3}{5}\times\frac{7}{12} = ?$$

Solution

$$\frac{\overset{1}{\cancel{3}}}{5}\times\frac{7}{\underset{4}{\cancel{12}}} = \frac{1\times 7}{5\times 4}$$

$$= \frac{7}{20}$$

Worked Example 3

$$1\frac{1}{2}\times 3\frac{4}{5} = ?$$

Solution

$$1\frac{1}{2}\times 3\frac{4}{5} = \frac{3}{2}\times\frac{19}{5}$$

$$= \frac{57}{10}$$

$$= 5\frac{7}{10}$$

Division

To understand how to *divide* with fractions, first consider how multiplication and division are related.

Take as an example,

$$3\times 4 = 12.$$

Then it is also true that

$$12\div 4 = 3.$$

So $'\times 4'$ and $'\div 4'$ are *inverse* operations.

Note that

$$12\times\frac{1}{4} = 3,$$

so $\div 4$ is the same as $\times\frac{1}{4}$.

Similarly, because $\div\frac{1}{2}$ is the same as $\times 2$,

$$6\div\frac{1}{2} = 12 \qquad \left(\text{check: } 12\times\frac{1}{2} = 6\right)$$

and, alternatively, $6\times 2 = 12$.

4.4

So $\div \frac{1}{2}$ is the same as $\times 2$.

You can generalise these examples to give

$$\div\, a \quad \text{is the same as} \quad \times \frac{1}{a}$$

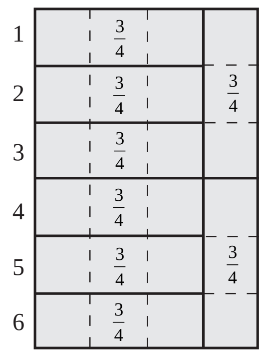
$$\div \frac{1}{b} \quad \text{is the same as} \quad \times\, b$$

and combining the two results gives

$$\div \frac{a}{b} \quad \text{is the same as} \quad \times \frac{b}{a}.$$

For example,

$$6 \div \frac{3}{4} = 6 \times \frac{4}{3}$$
$$= 8.$$



(This result can be seen more easily from the diagram opposite.)

Similarly,

$$\frac{6}{20} \div \frac{2}{5} = \frac{6}{20} \times \frac{5}{2}$$
$$= \frac{3}{4}$$

So to divide by a fraction, the fraction should be *inverted*, that is, turned upside down, and then multiplied.

In general terms,

$$\boxed{\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}}$$

Worked Example 4

$$\frac{3}{4} \div \frac{7}{8} = ?$$

Solution

$$\frac{3}{4} \div \frac{7}{8} = \frac{3}{{}_{1}\cancel{4}} \times \frac{\cancel{8}^{\,2}}{7}$$
$$= \frac{3 \times 2}{1 \times 7}$$
$$= \frac{6}{7}$$

4.4

Exercises

1. Find each of the following, cancelling when possible.

(a) $\frac{3}{4} \times \frac{5}{7}$ (b) $\frac{1}{5} \times \frac{7}{8}$ (c) $\frac{4}{5} \times \frac{1}{12}$

(d) $\frac{3}{7} \times \frac{9}{10}$ (e) $\frac{4}{7} \times \frac{5}{8}$ (f) $\frac{6}{7} \times \frac{3}{4}$

(g) $\frac{2}{7} \times \frac{3}{8}$ (h) $\frac{1}{6} \times \frac{4}{7}$ (i) $\frac{3}{5} \times \frac{10}{9}$

(j) $1\frac{1}{2} \times 1\frac{1}{3}$ (k) $4\frac{1}{6} \times 2\frac{1}{2}$ (l) $1\frac{3}{4} \times 2\frac{1}{7}$

(m) $3\frac{3}{7} \times 4\frac{1}{5}$ (n) $5\frac{1}{2} \times 1\frac{3}{4}$ (o) $8\frac{1}{2} \times 3\frac{4}{7}$

(p) $2\frac{3}{4} \times 4\frac{1}{7}$ (q) $5\frac{3}{8} \times 1\frac{5}{6}$ (r) $1\frac{2}{7} \times 1\frac{3}{8}$

2. Find

(a) $\frac{3}{4} \div \frac{1}{2}$ (b) $\frac{6}{7} \div \frac{3}{4}$ (c) $\frac{1}{5} \div \frac{1}{7}$

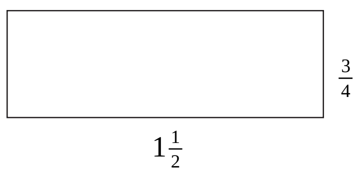
(d) $\frac{3}{8} \div \frac{4}{5}$ (e) $\frac{3}{7} \div \frac{9}{10}$ (f) $\frac{7}{4} \div \frac{2}{5}$

(g) $1\frac{1}{4} \div \frac{3}{4}$ (h) $5\frac{1}{2} \div \frac{1}{4}$ (i) $1\frac{1}{7} \div 2\frac{3}{8}$

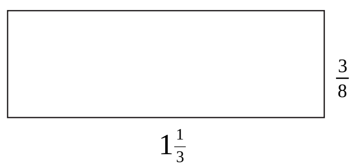
(j) $4\frac{1}{2} \div 1\frac{1}{5}$ (k) $1\frac{3}{4} \div 1\frac{5}{8}$ (l) $3\frac{1}{7} \div 1\frac{7}{8}$

3. Find the area of each rectangle below.

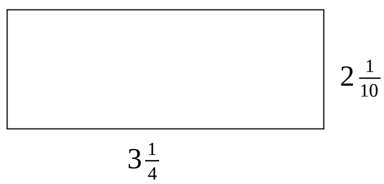
(a)



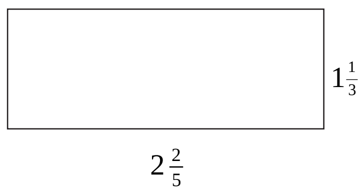
(b)



(c)



(d)

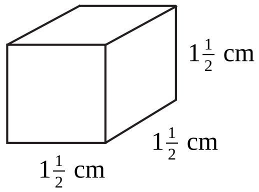


4. In a garden, $\frac{1}{2}$ of it is used for growing vegetables and $\frac{1}{4}$ of this vegetable area for potatoes. What fraction of the garden is used for growing potatoes?

4.4

5. In a school, $\frac{4}{7}$ of the children are boys and $\frac{1}{10}$ of these are colour-blind.
What fraction of the school are colour-blind boys?

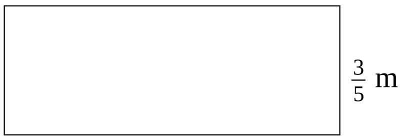
6. A cube is made with sides of length $1\frac{1}{2}$ cm.
Find the volume and surface area of the cube.



7. A petrol can holds $5\frac{1}{2}$ litres when full. How much petrol is in the can if it is $\frac{3}{4}$ full?

8. A car travels at 50 m.p.h. for $\frac{3}{4}$ hour. How far does the car travel?

9. Find the length of the unmarked side of this rectangle if its area is $1\frac{1}{2}$ m^2.



10. A recipe requires $\frac{1}{4}$ kg of sugar for a cake. How many cakes could be made with $1\frac{3}{4}$ kg of sugar?

11. Alison cycles 3 miles in $\frac{2}{3}$ hour. What is her speed?

12. It takes a factory $\frac{3}{4}$ hour to assemble a finished product. How many items could be assembled in an 8 hour day?

Answers to Exercises

4.1 Fractions, Decimals and Percentages

1. (a) $\frac{1}{10}$ (b) $\frac{4}{5}$ (c) $\frac{9}{10}$ (d) $\frac{1}{20}$ (e) $\frac{1}{4}$ (f) $\frac{3}{4}$
   (g) $\frac{7}{20}$ (h) $\frac{19}{50}$ (i) $\frac{1}{25}$ (j) $\frac{3}{25}$ (k) $\frac{41}{50}$ (l) $\frac{37}{50}$

2. (a) 0.32 (b) 0.5 (c) 0.34 (d) 0.2 (e) 0.15 (f) 0.81
   (g) 0.04 (h) 0.03 (i) 0.07 (j) 0.18 (k) 0.75 (l) 0.73

3. (a) 50% (b) 74% (c) 35% (d) 8% (e) 10% (f) 52%
   (g) 80% (h) 7% (i) 4% (j) 18% (k) 40% (l) 30%

4. (a) 50% (b) 70% (c) 20% (d) 75% (e) 10% (f) 90%
   (g) 80% (h) 8% (i) 32% (j) 35% (k) 28% (l) $66\frac{2}{3}$%

5. (a) $\frac{2}{3} = \frac{10}{15} = \frac{16}{24}$ (b) 35%

6. (a) correct drawing (b) 75%

7. (a) 180 m^2 (to the nearest square metre) (b) (i) 120 m^2 (ii) 20 %

4.2 Simple Fractions and Percentages of Quantities

1. (a) 20 (b) £2.50 (c) £1.60 (d) £25 (e) £200
   (f) 180 (g) £4 (h) 600 (i) 750 (j) 16 kg
   (k) 3.5 kg (l) 15 kg (m) 5 m (n) 10 m (o) £7.50

2. (a) 32 (b) 90 (c) 18 (d) 90 (e) 120 (f) 150

3. (a) 400 g (or 0.4 kg) (b) 240 g (or 0.24 kg)

4. £3 000

5. £25

6. £150

7. £60

8. 360 m

9. £1.50

10. 9 kg

11. 80 p

Answers

4.2

12. 2.4 kg (or 240 g)

13. (a) £5.60 (b) £5 (c) 2.25 metres

14. (a) £90 (b) $\frac{1}{6}, \frac{2}{9}, \frac{1}{3}, \frac{5}{6}$

15. 170 grams

16. 1400 headphones

17. No. $\frac{2}{5}$ of the price is equivalent to 40% of the price, therefore 30% of the price is not enough to pay the deposit.

4.3 Addition and Subtraction of Fractions

1. (a) $\frac{2}{5}$ (b) $\frac{1}{2}$ (c) $\frac{6}{7}$ (d) $\frac{3}{7}$ (e) $\frac{3}{13}$ (f) $\frac{1}{3}$
   (g) $\frac{5}{3}$ (h) $\frac{7}{5}$ (i) $\frac{11}{7}$ (j) $\frac{2}{5}$ (k) $\frac{1}{3}$ (l) $\frac{1}{5}$

2. (a) 14, 29 (b) 6, 5, 11 (c) 2, 3 (d) 3, 4, 7 (e) 12, 14, 26
   (f) 36, 35, 71

3. (a) $\frac{13}{24}$ (b) $\frac{39}{35}$ (c) $\frac{7}{32}$ (d) $\frac{13}{30}$ (e) $\frac{59}{56}$ (f) $\frac{7}{6}$
   (g) $\frac{17}{70}$ (h) $\frac{47}{24}$ (i) $\frac{32}{21}$ (j) $\frac{1}{14}$ (k) $\frac{13}{44}$ (l) $\frac{1}{2}$
   (m) $\frac{1}{12}$ (n) $\frac{5}{24}$ (o) $\frac{13}{24}$

4. $\frac{11}{15}$ hectare

5. £$1\frac{5}{12}$ million

6. $\frac{1}{10}$ cm

7. $\frac{13}{30}$

8. $\frac{3}{20}$

9. $\frac{1}{6}$

10. $2\frac{7}{12}$ kg

Answers

4.4 Multiplication and Division of Fractions

1. (a) $\frac{15}{28}$ (b) $\frac{7}{40}$ (c) $\frac{1}{15}$ (d) $\frac{27}{70}$ (e) $\frac{5}{14}$ (f) $\frac{9}{14}$

   (g) $\frac{3}{28}$ (h) $\frac{2}{21}$ (i) $\frac{2}{3}$ (j) 2 (k) $10\frac{5}{12}$ (l) $3\frac{3}{4}$

   (m) $14\frac{2}{5}$ (n) $9\frac{5}{8}$ (o) $30\frac{5}{14}$ (p) $11\frac{11}{28}$ (q) $9\frac{41}{48}$ (r) $1\frac{43}{56}$

2. (a) $1\frac{1}{2}$ (b) $1\frac{1}{7}$ (c) $1\frac{2}{5}$ (d) $\frac{15}{32}$ (e) $\frac{10}{21}$ (f) $4\frac{3}{8}$

   (g) $1\frac{2}{3}$ (h) 22 (i) $\frac{64}{133}$ (j) $3\frac{3}{4}$ (k) $1\frac{1}{13}$ (l) $1\frac{71}{105}$

3. (a) $1\frac{1}{8}$ (b) $\frac{1}{2}$ (c) $6\frac{33}{40}$ (d) $3\frac{1}{5}$

4. $\frac{1}{8}$

5. $\frac{1}{8}$

6. Volume = $3\frac{3}{8}$ cm^3, Surface area = $13\frac{1}{2}$ cm^2

7. $4\frac{1}{8}$ litres

8. $37\frac{1}{2}$ miles

9. $2\frac{1}{2}$ m

10. 7 cakes

11. $4\frac{1}{2}$ miles per hour

12. 10 items