Mathematics Enhancement Programme

Primary Demonstration Project

3A Equations

Help Booklet

Support for Primary Teachers in Mathematics

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CIMT
School of Education
University of Exeter

Mathematics Enhancement Programme

Help Module 3

EQUATIONS

Part A

Contents of Part A

Contents of Part B

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PREFACE

This is one of a series of *Help Modules* designed to help you gain confidence in mathematics. It has been developed particularly for primary teachers (or student teachers) but it might also be helpful for non-specialists who teach mathematics in the lower secondary years. It is based on material which is already being used in the *Mathematics Enhancement Programme: Secondary Demonstration Project*.

The complete module list comprises:

1. ALGEBRA
2. DECIMALS
3. EQUATIONS
4. FRACTIONS
5. GEOMETRY
6. HANDLING DATA
7. MENSURATION
8. NUMBERS IN CONTEXT
9. PERCENTAGES
10. PROBABILITY

Notes for overall guidance:

- Each of the 10 modules listed above is divided into 2 parts. This is simply to help in the downloading and handling of the material.
- Though referred to as 'modules' it may not be necessary to study (or print out) each one in its entirely. As with any self-study material you must be aware of your own needs and assess each section to see whether it is relevant to those needs.
- The difficulty of the material in **Part A** varies quite widely: if you have problems with a particular section do try the one following, and then the next, as the content is not necessarily arranged in order of difficulty. Learning is not a simple linear process, and later studies can often illuminate and make clear something which seemed impenetrable at an earlier attempt.
- In **Part B, Activities** are offered as backup, reinforcement and extension to the work covered in Part A. **Tests** are also provided, and you are strongly urged to take these (at the end of your studies) as a check on your understanding of the topic.
- The marking scheme for the revision test includes B, M and A marks. Note that:

  M *marks* are for method;

  A *marks* are for accuracy (awarded only following a correct M mark);

  B *marks* are independent, stand-alone marks.

We hope that you find this module helpful. Comments should be sent to:

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The full range of Help Modules can be found at

www.ex.ac.uk/cimt/help/menu.htm

3 *Equations* Introductory Notes

Background and Preparatory Work

Primitive algebraic methods began to emerge about 1700 BC in Babylon and in Egypt. However, these methods remained largely dependent on words and so developed very slowly until about 1600 AD, when a suitably flexible symbolism was developed. This flexible use of symbols then gave birth, during the next 70 years, to a truly remarkable flowering of new mathematics – first coordinate geometry (about 1630), leading to the representation of functions and the differential and integral calculus (around 1670).

These few historical details should suffice to underline the fact that

algebra is the natural language of mathematics

a fact that is understood in most countries. In contrast, developments in England in recent years have tended to give the impression that algebra is an 'optional extra' which most students will never need. In reality, algebra holds the key to subsequent progress for anyone who may one day need to use their mathematics – on no matter how lowly a level. This module is important because solving equations provides a very natural way of introducing students to this key skill.

The wording of the National Curriculum, the kind of problems set at KS3 and on GCSE papers, and the associated mark schemes have all conspired to obscure what 'solving equations' is really about. Thus a few words of explanation may be needed before we get down to details.

Suppose a carpenter needs to prepare and fix a strut of a certain (as yet unknown) length. He could 'judge by eye', cut a piece of wood which is roughly the right size, and then test whether it does the job – cutting off a bit more if the strut is too long, or throwing it away and trying again if it is too short. However, it would be better to use the proven method of first carefully measuring the required length, and then cutting the exact length needed. In one sense, there are two possible approaches; *but they are not genuine alternatives*, and one would never *teach* students to use the first method. It is not just that the first method is likely to be very wasteful (of both time and materials). The real difference is that, in this setting, the first (trial-and-improvement) method ignores the underlying 'discipline' of proven methods of carpentry in the hope that 'seat-of-the pants' guesswork is just as good.

In this module, trial-and-improvement can help beginners to get a feel for what is meant by a 'solution' of an equation. However, although guesswork may sometimes succeed in finding one solution, it can never guarantee that all solutions have been found. Moreover, the approach works only in the simplest cases, and is no substitute for a general, systematic method.

Mathematics is *the science of exact calculation.* Its methods are not just the result of accumulated wisdom: they are dictated by the exact, inescapable nature of numbers, symbols and equations. For many youngsters, the art of solving equations should give them their first real glimpse of the power of this 'science of exact calculation'. There should be no guesswork! Also students should understand that there is no approximation involved; thus, for example, fractions should never be replaced by decimals, and when learning the principles of solving equations all calculation should be exact (hence *no calculators*!). It is also important to present successive equations in a calculation in a standard way, with the corresponding justification given in brackets (on the right, say).

If the problem to be solved is given in the form of a 'word problem', then before any of this can be done it is necessary first to translate, or interpret, the information given in the 'word problem' into the form of an *equation*. There are all sorts of potential pitfalls here – not least the fact that one is at the mercy of the students' skill at *comprehension*. We restrict ourselves here to just two comments on this stage of the equation solving process.

(1) There are good reasons for keeping word problems short, and for being aware of the danger that students often respond to particular 'cue' words, rather than thinking about what the problem actually says.

(2) When choosing an unknown (say x) it is important to insist that x should always represent a *pure number* (rather than a length, or a weight, or a distance). Thus the solution should always start:

'Let x be the unknown number of marbles', or
'let the unknown length be x cm', or
'let the unknown weight by x g', or
'let the unknown distance be x km'.

One can then safely calculate with expressions such as '$3x-4$', since these represent *numbers* (though one should remember to re-insert the units at the end of the solution).

Once the unknown has been given a name, and the given information has been written in the form of an initial equation, one is faced with a quite different problem – namely how to 'solve' the equation to find the unknown.

The art of solving equations is rooted in a single fundamental property of real numbers.

The Product Property
The only way that a product, $p \,.\, q \,.\, r \,.\,.\,.\,.\, z$ can be (exactly!) equal to 0 is if one of the factors p, q, r, ..., z is (exactly!) equal to zero.

Examples:

(a) If the unknown number x satisfies the equation

$$(x-1)(x+2) = 0,$$

then either $(x-1) = 0$, so $x = 1$,

or $(x+2) = 0$, so $x = -2$.

Similarly,

(b) if the unknown number x satisfies the equation

$$(3x+2)(7-4x)(x-2) = 0,$$

then one of the factors must equal zero: so

$3x + 2 = 0$, or $7 - 4x = 0$, or $x - 2 = 0$.

Of course, if in the original equation the unknown x represented the number of children in a family, or the width (in cm) of a rectangle, then negative solutions would be discarded.

The *Product Property* implies that our general strategy for solving equations must have two parts:

(1) Given an equation in any form, we must first collect all terms (by doing all sorts of elementary algebra – carefully and correctly) on *one side of the equation* (say LHS), with ZERO on the other side.

(2) We must then write the expression on the LHS as a product; in other words we need to be able to *factorise*.

It is important to note that the usual approach to the solution of *linear* equations appears to ignore the *Product Property*! Instead, it focuses on 'collecting all the x's on one side of the equation and all the numbers on the other side'.

Example:

Solve	$5 - 4x = 3x - 2$	(Add $4x$ to both sides)
$\therefore$	$5 = 3x - 2 + 4x$	
$\therefore$	$5 = 7x - 2$	(Add 2 to both sides)
$\therefore$	$7 = 7x$	(Divide both sides by 7)
$\therefore$	$x = 1$	

Though it is unfortunate that this method does not automatically use (and thus reinforce the importance of) the general *Product Property*, the above approach still has several important advantages.

(1) It gives you a clear sense of what it means to 'solve' an equation, moving systematically from opaque beginnings ($5 - 4x = 3x - 2$) towards the final *coup de grâce* ($x = 1$).

(2) It emphasises, and allows you to practise, the simplest instances of 'doing the same to both sides of an equation'.

However, this method does not generalise to more complex equations, such as solving linear equations.

Key Issues

Introduction

This is an important which is of fundamental importance for further mathematical study.

Unfortunately, many, if not most people find algebra difficult, confusing or intimidating. This is completely unnecessary, as all algebra is based on putting into practice a set of logical steps. Provided you know the rules and stick to them, you cannot go wrong – the key is knowing these rules well.

Language / Notation

At all levels you need to be familiar with the use of x as an unknown, and that a general linear equation is of the form $ax + b = c$, where a, b and c are numbers.

Key Points

- It is absolutely crucial that at all times equations must balance; that is, the right-hand-side must equal the left-hand-side.
- All manipulation must be logical, for example:

$$x + 5 = 9 \qquad \text{(Take 5 from both sides)}$$
$$\therefore \quad x + 5 - 5 = 9 - 5$$
$$\therefore \quad x = 4.$$

  The middle step can be omitted when you are confident but it is important that you realise the real reason why 'taking +5 to the other side, so it becomes – 5' gives a correct answer.

  Similarly, for example,

$$4x = 12 \qquad \text{(Divide both sides by 4)}$$
$$\therefore \quad \frac{4x}{4} = \frac{12}{4}$$
$$\therefore \quad x = 3.$$

- *Trial and improvement* is a method which is used in place of algebraic manipulation where there is *not* a precise method to solve the equation. Linear and quadratic equations are exceptions to this as they can always be solved algebraically.

Misconceptions

When dealing with this topic there are numerous errors which are often made, usually because of misunderstandings of the logic of the manipulation being undertaken.

So look out for examples, such as the ones below, which often are carried out incorrectly.

$6 - (-4) = 10$ *not* 2

$-5 - (-10) = 5$ *not* 15 or – 5

$$a(bx + c) = abx + ac \qquad \text{not} \quad abx + c$$

It is often not understood that

$$\frac{ax}{b} = \left(\frac{a}{b}\right)x = \frac{x}{\left(\frac{b}{a}\right)}.$$

Two such examples are,

$$\frac{2x}{3} = \left(\frac{2}{3}\right)x = \frac{x}{\left(\frac{3}{2}\right)},$$

and

$$\frac{x}{\left(\frac{1}{2}\right)} = 2x.$$

The reason that this last result is true can readily be seen by multiplying top and bottom by 2.

WORKED EXAMPLES and EXERCISES

3 Equations

3.1 Changing the Subject

Sometimes a formula can be rearranged into a more useful format. For example, the formula

$$F = 1.8C + 32$$

can be used to convert temperatures in degrees Celsius to degrees Fahrenheit. It can be rearranged into the form

$$C = \ldots$$

to enable temperatures in degrees Fahrenheit to be converted to degrees Celsius. We say that the formula has been rearranged to make C the subject of the formula.

Worked Example 1

Rearrange the formula

$$F = 1.8C + 32$$

to make C the subject of the formula.

Solution

The aim is to remove all terms from the right hand side of the equation except for the C. First subtract 32 from both sides, which gives

$$F - 32 = 1.8C$$

Then dividing both sides by 1.8 gives

$$\frac{F - 32}{1.8} = C$$

So the formula can be rearranged as

$$C = \frac{F - 32}{1.8}$$

Worked Example 2

Make v the subject of the formula

$$s = \frac{(u + v)t}{2}$$

Solution

First multiply both sides of the formula by 2 to give

$$2s = (u + v)t$$

Then divide both sides by t, to give

$$\frac{2s}{t} = u + v$$

Finally, subtract u from both sides to give

$$\frac{2s}{t} - u = v$$

3.1

So the formula becomes

$$v = \frac{2s}{t} - u$$

Exercises

1. Make x the subject of each of the following formulae.

(a) $y = 4x$ (b) $y = 2x + 3$ (c) $y = 4x - 8$

(d) $y = \frac{x+2}{4}$ (e) $y = \frac{x-2}{5}$ (f) $y = x + a$

(g) $y = \frac{x-b}{a}$ (h) $y = ax + c$ (i) $y = \frac{ax+b}{c}$

(j) $y = \frac{ax-c}{b}$ (k) $y = a + b + x$ (l) $y = \frac{x-a+b}{c}$

(m) $y = abx$ (n) $y = abx + c$ (o) $y = \frac{4ax-b}{3c}$

(p) $p = \frac{ax-bc}{d}$ (q) $y = (a+x)b$ (r) $y = \frac{(3+x)a}{4}$

(s) $q = \frac{3(x-4)}{2}$ (t) $v = \frac{5(x+y)}{4}$ (u) $z = a + \frac{(x-3)}{4}$

2. Ohm's law is used in electrical circuits and states that

$$V = IR$$

Write formulae with I and R as their subjects.

3. Newton's Second law states that $F = ma$.

Write formulae with m and a as their subjects.

4. The formula $C = 2\pi r$ can be used to find the circumference of a circle.
Make r the subject of this formula.

5. The equation $v = u + at$ is used to find the velocities of objects.

(a) Make t the subject of this formula.

(b) Make a the subject of this formula.

6. The mean of three numbers x, y and z can be found using the formula

$$m = \frac{x+y+z}{3}$$

Make z the subject of this formula.

3.1

7. Make *a* the subject of the following formulae.

   (a) $v^2 = u^2 + 2as$ (b) $s = at + \frac{1}{2}at^2$

8. The formula $V = xyz$ can be used to find the volume of a rectangular box. Make *z* the subject of this formula.

9. The volume of a tin can is given by

$$V = \pi r^2 h$$

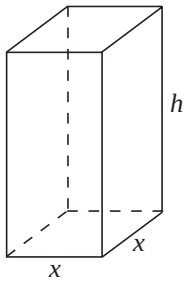
   where *r* is the radius of the base and *h* is the height of the can.

   (a) Make *r* the subject of the equation.

   (b) Find *r* correct to 2 decimal places if $V = 250\ \text{cm}^3$ and $h = 10\ \text{cm}$.

10. A box with a square base has its volume given by

$$V = x^2 h$$



    and its surface area given by

$$A = 2x^2 + 4xh$$

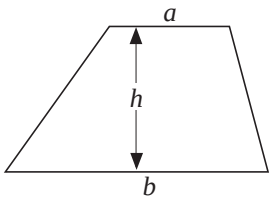
    (a) Make *h* the subject of both formulae.

    (b) Find *h* if $A = 24\ \text{cm}^2$ and $x = 2\ \text{cm}$.

    (c) Find *h* if $V = 250\ \text{cm}^3$ and $x = 10\ \text{cm}$.

11. The area of a trapezium is given by

$$A = \frac{1}{2}(a + b)h$$



    (a) Write the formula with *a* as its subject.

    (b) In a particular trapezium $b = 2a$.
    Use this to write a formula that does not involve *b*, and make *a* the subject.

3.2 Simplifying Expressions

When simplifying expressions you should bring together terms which contain the same letter.

Note

x and x^2 must be treated as if they were different letters. You cannot add an x term to an x^2 term. The + and – signs go with the term which follows.

Worked Example 1

Simplify each expression below.

(a) $4a + 3a + 6 + 2$ (b) $4a + 8b - 2a + 3b$

(c) $x^2 + 5x - 8x + x^2 - 4$ (d) $8x + y - 4x - 6y$

Solution

(a) The terms which involve a can be brought together. Also the 6 and 2 can be added.

$$4a + 3a + 6 + 2 = 7a + 8$$

(b) The terms involving a are considered together, and then the terms involving b.

$$\begin{aligned} 4a + 8b - 2a + 3b &= 4a - 2a + 8b + 3b \\ &= 2a + 11b \end{aligned}$$

(c) Here the x and x^2 must be treated as if they are different letters.

$$\begin{aligned} x^2 + 5x - 8x + x^2 - 4 &= x^2 + x^2 + 5x - 8x - 4 \\ &= 2x^2 - 3x - 4 \end{aligned}$$

(d) The different letters, x and y, must be considered in turn.

$$\begin{aligned} 8x + y - 4x - 6y &= 8x - 4x + y - 6y \\ &= 4x - 5y \end{aligned}$$

When a bracket is to be multiplied by a number or a letter, every term inside the bracket must be multiplied.

Worked Example 2

Remove the brackets from each expression below.

(a) $6(x + 5)$ (b) $3(2x + 7)$ (c) $4(x - 3)$ (d) $x(x - 4)$

Solution

(a) $\begin{aligned} 6(x + 5) &= 6 \times x + 6 \times 5 \\ &= 6x + 30 \end{aligned}$

(b) $\begin{aligned} 3(2x + 7) &= 3 \times 2x + 3 \times 7 \\ &= 6x + 21 \end{aligned}$

(c) $\begin{aligned} 4(x - 3) &= 4 \times x - 4 \times 3 \\ &= 4x - 12 \end{aligned}$

(d) $\begin{aligned} x(x - 4) &= x \times x - x \times 4 \\ &= x^2 - 4x \end{aligned}$

3.2

Exercises

1. Simplify each of these expressions.

(a) $a + 2a + 3a$ (b) $3a + 2 + 4 + 6$

(c) $3a + 2b + 8a + 4b$ (d) $4x + 2y + 8y + y$

(e) $5x + 2y + 8x - 3y$ (f) $6a + 7b + 3b - 4a$

(g) $4 + 6a - 3a + 2 + b$ (h) $p + q + 2p - 8q + 3p$

(i) $x + y - 8x + 2y$ (j) $4x - 3p + 2p - 2x$

(k) $7x - 4z + 8x - 5z$ (l) $3z - 4x + 2z - 10x$

(m) $3q - 4x + 8a - 2x + q$ (n) $x + y + z - p - q - y$

(o) $x + 6 + y + 4 + 2x - 3y$ (p) $4x - 8q + 17x - 24q$

(q) $-x + y + x + y$ (r) $4x + 7y - 3x - 8y + x + y$

(s) $-8x + 7y - 11x + 4y$ (t) $6x - 18y + 17x - 4$

(u) $x + y - 8x - 11y$ (v) $4p + 8q - 8p - 4q$

2. Simplify each of the following expressions.

(a) $2x^2 + 3x + 4x^2 + 5x$ (b) $x^2 + 8x + 5x + 10$

(c) $x^2 + 6x + 4x + x^2$ (d) $x^2 + x + 10 + x + 4x^2$

(e) $5x^2 - x - 6x^2 + 8x$ (f) $4x^2 - 3y^2 - x^2 + y^2$

(g) $x^2 + y^2 - x - y + x^2$ (h) $4x^2 - 7x + 1 + x^2 + 4x - 11$

(i) $x^2 - y^2 - x - y + 2x^2 - 2y^2$ (j) $y^2 + y - 4 + y + 4y^2$

(k) $ab + cd + 4ab$ (l) $xy + xz + xy + 4xz$

(m) $4ab + 7ab - 3ad$ (n) $4pq - 3qr + 5pq$

3. Remove the brackets from each expression below.

(a) $3(x + 5)$ (b) $4(6 + x)$ (c) $7(x + 2)$

(d) $2(x + 6)$ (e) $5(x + 2)$ (f) $4(2x + 3)$

(g) $5(3x + 2)$ (h) $8(5x + 3)$ (i) $7(x - 6)$

(j) $8(5 - x)$ (k) $4(2x - 7)$ (l) $7(5x - 3)$

(m) $6(3x - 5)$ (n) $4(x - 2y)$ (o) $5(x + 2y + 3z)$

(p) $x(5 + x)$ (q) $a(2 - a)$ (r) $4(b - 3)$

(s) $2x(x - 6)$ (t) $4x(2x + 3)$ (u) $3x(7 - 2x)$

(v) $8x(x - 5)$

3.2

4. Simplify each of the following expressions, by first removing all the brackets.

(a) $3(a+2)+4(a+5)$ (b) $2(2x+4)+3(x+5)$

(c) $5(x+2)+3(x+4)$ (d) $x(x+1)+x(x+6)$

(e) $4(x+1)+x(x+5)$ (f) $2(a+b)+5(2a+3b)$

3.3 Simple Equations

To solve simple equations you must carry out the *same* operation (addition, subtraction, multiplication or division) on *both* sides of the equation so that the new equation is still balanced.

Worked Example 1

Solve each of the following equations.

(a) $x+3=8$ (b) $x-8=11$ (c) $4x=32$ (d) $\frac{x}{6}=7$

Solution

(a) To solve this equation, subtract 3 from both sides.

$$\begin{aligned} x+3 &= 8 \\ x+3-3 &= 8-3 \\ x &= 5. \end{aligned}$$

(b) To solve this equation, add 8 to both sides.

$$\begin{aligned} x-8 &= 11 \\ x-8+8 &= 11+8 \\ x &= 19. \end{aligned}$$

(c) To solve this equation divide both sides by 4.

$$\begin{aligned} 4x &= 32 \\ \frac{\cancel{4}^{1}x}{\cancel{4}_{1}} &= \frac{\cancel{32}^{8}}{\cancel{4}_{1}} \\ x &= 8. \end{aligned}$$

(d) To solve this equation multiply both sides by 6.

$$\begin{aligned} \frac{x}{6} &= 7 \\ \frac{x}{\cancel{6}_{1}}\times\cancel{6}^{1} &= 7\times 6 \\ x &= 42. \end{aligned}$$

3.3

Worked Example 2

A packet of sweets is divided equally among 5 children and each child is given 4 sweets. Write down an equation to describe this situation and solve it to find the number of sweets in the packet.

Solution

Let x be the number of sweets in the packet.

Then $$\frac{x}{5} = 4,$$

since the 5 children have 4 sweets each. Now the equation can be solved by multiplying both sides by 5.

$$\frac{x}{\cancel{5}_{1}} \times \cancel{5}^{1} = 4 \times 5$$

$$x = 20.$$

Exercises

1. Solve each of these equations.

(a)	$x + 6 = 10$	(b)	$x - 7 = 3$	(c)	$x + 4 = 7$
(d)	$x - 1 = 11$	(e)	$x - 6 = 8$	(f)	$x + 5 = 3$
(g)	$5x = 45$	(h)	$6x = 24$	(i)	$6x = 108$
(j)	$7x = 56$	(k)	$3x = 102$	(l)	$6x = 42$
(m)	$\frac{x}{2} = 5$	(n)	$\frac{x}{2} = 12$	(o)	$\frac{x}{5} = 10$
(p)	$\frac{x}{3} = 4$	(q)	$\frac{x}{7} = 3$	(r)	$\frac{x}{6} = 11$
(s)	$x - 5 = 3$	(t)	$x + 8 = 14$	(u)	$4x = 104$
(v)	$\frac{x}{2} = 18$	(w)	$x - 3 = 12$	(x)	$x + 7 = 11$

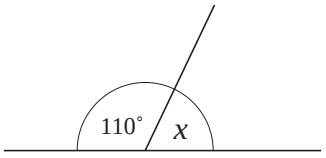
2. Solve each of these equations.

(a)	$x + 6 = 2$	(b)	$x + 8 = 3$	(c)	$x - 5 = -2$
(d)	$x + 2 = -4$	(e)	$x - 2 = -6$	(f)	$x + 4 = -10$
(g)	$2x = -12$	(h)	$3x = -24$	(i)	$5x = -60$
(j)	$\frac{x}{2} = -8$	(k)	$x - 2 = -5$	(l)	$x + 6 = -14$
(m)	$x - 10 = -2$	(n)	$x - 12 = -4$	(o)	$x - 7 = -1$
(p)	$x + 4 = -1$	(q)	$x + 12 = 2$	(r)	$x + 10 = -16$

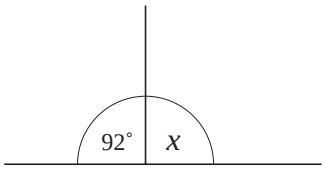
3.3

3. The angles on a straight line add up to $180°$. Write down and solve an equation for each diagram shown below.

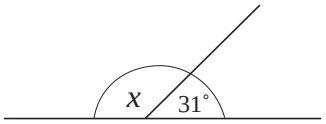
(a)



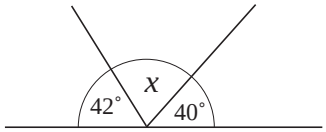
(b)



(c)



(d)



4. Sanjit is two years older than his brother. His brother is 16. Write down an equation which uses x to represent Sanjit's age. Solve the equation for x..

5. To pay for a school trip, 12 children take the same amount of money to school. If the total money collected is £54 and the amount each child takes is x, write down an equation to describe this situation. Solve your equation for x.

6. The train fare for a long journey increases by £3 to £41. If x is the old fare, write down an equation to describe this situation. Solve your equation for x.

7. Majid knows that when a certain number is doubled, the answer is 52. Explain in words, starting with 52, how he can work out the number.

(SEG)

8. Tim thought of a number. He doubled his number. His answer was 24.
What number did Tim think of?

(LON)

9. Ali is twice as old as Sue. Sue is 2 years younger than Philip. Philip is 11 years old.

(a) How old is Sue? (b) How old is Ali?

(NEAB)

3.4 Solving Equations

Most equations require a number of steps to solve them. These steps must be logical so that the new equation still balances. Whatever you do to one side of an equation you *must* do the same to the other side. The following examples illustrate these steps.

Worked Example 1

Solve the following equations.

(a) $3x + 7 = 13$ (b) $5x - 8 = 13$ (c) $\frac{x}{5} - 2 = 3$

(d) $4(x - 3) = 8$

3.4

Solution

(a) First subtract 7 from both sides of the equation.

$$\begin{aligned} 3x + 7 &= 13 \\ 3x + 7 - 7 &= 13 - 7 \\ 3x &= 6 \end{aligned}$$

Next divide both sides of the equation by 3.

$$\begin{aligned} \frac{\cancel{3}^{1}x}{\cancel{3}_{1}} &= \frac{\cancel{6}^{2}}{\cancel{3}_{1}} \\ x &= 2. \end{aligned}$$

(b) First add 8 to both sides of the equation.

$$\begin{aligned} 5x - 8 &= 13 \\ 5x - 8 + 8 &= 13 + 8 \\ 5x &= 21. \end{aligned}$$

Then divide both sides of the equation by 5.

$$\begin{aligned} \frac{\cancel{5}^{1}x}{\cancel{5}_{1}} &= \frac{21}{5} \\ x &= \frac{21}{5} \\ &= 4\tfrac{1}{5}. \end{aligned}$$

(c) First add 2 to both sides of the equation.

$$\begin{aligned} \frac{x}{5} - 2 &= 3 \\ \frac{x}{5} - 2 + 2 &= 3 + 2 \\ \frac{x}{5} &= 5 \end{aligned}$$

Then multiply both sides of the equation by 5.

$$\begin{aligned} \frac{x}{\cancel{5}_{1}} \times \cancel{5}^{1} &= 5 \times 5 \\ x &= 25. \end{aligned}$$

(d) First remove the brackets, multiplying each term inside the bracket by 4.

$$\begin{aligned} 4(x - 3) &= 8 \\ 4x - 12 &= 8. \end{aligned}$$

Then add 12 to both sides of the equation.

3.4

$$4x - 12 + 12 = 8 + 12$$
$$4x = 20.$$

Finally divide both sides by 4.

$$\frac{\cancel{4}^{1}x}{\cancel{4}_{1}} = \frac{\cancel{20}^{5}}{\cancel{4}_{1}}$$
$$x = 5$$

Sometimes equations may contain the letter x on both sides of the equation or a $-x$ term. The next examples show how to deal with these cases.

Worked Example 2

Solve these equations.

(a) $4x + 6 = 3x + 10$ (b) $6 - 2x = 8$ (c) $4x - 2 = 8 - 6x$

Solution

(a) As x appears on both sides of the equation, first subtract $3x$ from both sides.

$$4x + 6 = 3x + 10$$
$$4x + 6 - 3x = 3x + 10 - 3x$$
$$x + 6 = 10.$$

Then subtract 6 from both sides.

$$x + 6 - 6 = 10 - 6$$
$$x = 4.$$

(b) As the left-hand side contains $-2x$, add $2x$ to both sides.

$$6 - 2x = 8$$
$$6 - 2x + 2x = 8 + 2x$$
$$6 = 8 + 2x$$

Then subtract 8 from both sides.

$$6 - 8 = 8 + 2x - 8$$
$$-2 = 2x.$$

Finally divide both sides by 2.

$$\frac{\overset{-1}{\cancel{-2}}}{\cancel{2}_{1}} = \frac{\overset{1}{\cancel{2}x}}{\cancel{2}_{1}}$$
$$-1 = x \quad \text{or} \quad x = -1.$$

(c) As one side contains $-6x$, add $6x$ to both sides.

$$4x - 2 = 8 - 6x$$
$$4x - 2 + 6x = 8 - 6x + 6x$$
$$10x - 2 = 8$$

3.4

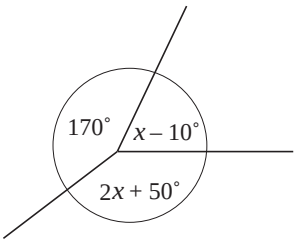
Then add 2 to both sides of the equation.

$$10x - 2 + 2 = 8 + 2$$
$$10x = 10$$

Finally divide both sides by 10.

$$\frac{{}^{1}\cancel{10}x}{{}_{1}\cancel{10}} = \frac{\cancel{10}^{1}}{\cancel{10}_{1}}$$
$$x = 1.$$

Worked Example 3



Use the information in the diagram to write down an equation and then find the value of x.

Solution

The three angles shown must add up to $360°$, so

$$170 + 2x + 50 + x - 10 = 360$$
$$210 + 3x = 360.$$

Subtracting 210 from both sides gives

$$210 + 3x - 210 = 360 - 210$$
$$3x = 150.$$

Then dividing both sides by 3 gives

$$\frac{{}^{1}\cancel{3}x}{{}_{1}\cancel{3}} = \frac{\cancel{150}^{50}}{\cancel{3}_{1}}$$
$$x = 50.$$

Exercises

1. Solve each of these equations.

(a)	$3x + 6 = 48$	(b)	$5x - 6 = 39$	(c)	$2x - 6 = 22$
(d)	$6x - 7 = 41$	(e)	$8x - 3 = 29$	(f)	$6x + 12 = 20$
(g)	$4x + 18 = 2$	(h)	$5x + 10 = 5$	(i)	$3x + 6 = 1$
(j)	$5(x + 2) = 45$	(k)	$3(x - 2) = 12$	(l)	$2(x + 7) = 10$
(m)	$3(2x - 1) = 57$	(n)	$3(2x + 7) = 27$	(o)	$5(5x + 1) = 20$
(p)	$4(2x + 3) = -8$	(q)	$5(3x - 1) = -2$	(r)	$2(8x + 5) = -2$
(s)	$6x - 8 = -26$	(t)	$4(x + 15) = 60$	(u)	$5x - 8 = -10$
(v)	$\frac{x}{4} - 1 = 8$	(w)	$\frac{x}{3} + 2 = 7$	(x)	$\frac{2x}{5} + 1 = 3$

3.4

2. Solve these equations.

(a) $2x + 6 = x + 3$ (b) $4x - 8 = 5x - 2$

(c) $6x + 7 = 2x + 20$ (d) $x + 6 = 2x - 8$

(e) $3x + 7 = 2x + 11$ (f) $10x + 2 = 8x + 22$

(g) $6 - x = 5$ (h) $2 - x = 5$

(i) $3 - x = -10$ (j) $14 - 3x = 5$

(k) $10 - 2x = 2$ (l) $4 - 3x = 2$

(m) $x + 2 = 8 - x$ (n) $x + 4 = 10 - 2x$

(o) $x + 4 = 9 - 2x$ (p) $8 - x = 12 - 2x$

(q) $22 - 4x = 18 - 2x$ (r) $3 - 6x = 2 - 4x$

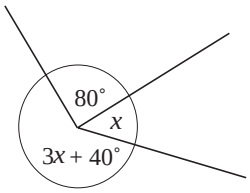
(s) $3(x + 2) = 5(x - 2)$ (t) $4 = 8 - \frac{x}{3}$

(u) $3 - \frac{x}{4} = -5$ (v) $4(x - 2) = 3(x + 2)$

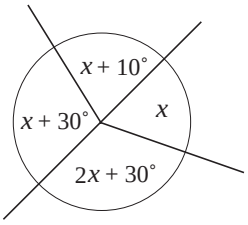
(w) $5 = 18 - \frac{x}{3}$ (x) $2 - \frac{x}{4} = 1 - \frac{x}{6}$

3. For each diagram below, write down an equation involving x and solve it.

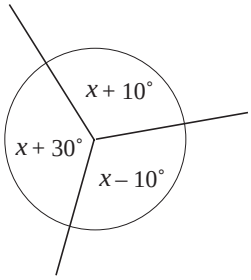
(a)



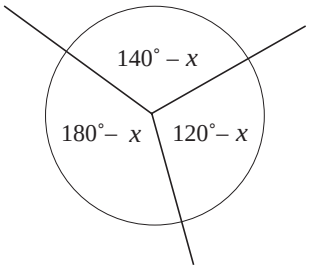
(b)



(c)



(d)



4. A rope of length 10 m is used to mark out a rectangle, so that the two long sides are 1 m longer than the short sides. If x is the length of the short sides, write down an equation to describe this situation and hence find x.

5. You ask a friend to think of a number, double it and add 10. His answer is 42. If x is the number your friend thought of, write down the relevant equation and find x.

3.4

6. Six teams enter a competition. There are x members in each team. If 8 people drop out and 34 complete the competition, write down an equation and solve it to find the number in each team at the start of the competition.

7. Three people drive a car on a long journey. John drives for 2 hours more than Mary. Philip drives for twice as long as Mary. The whole journey takes 6 hours. Use an equation to find out for how long each person drives.

8. A driver travels 80 miles to a motorway and then travels at a steady 60 mph for x hours. Write down and solve equations involving x if the driver travels a total of

   (a) 290 miles (b) 220 miles.

   Give your answers in hours and minutes.

9. A child was asked to think of a number and follow these instructions. In each case, let x be the number the child thinks of, write down an equation, and find the value of x.

   (a) Think of a number, add 6 and double it. *Answer* 18

   (b) Think of a number, divide by 2 and add 10. *Answer* 16

   (c) Think of a number, divide by 2, add 2 and multiply by 2. *Answer* 9

   (d) Think of a number, subtract 7, divide by 2 and multiply by 10. *Answer* 115

10. Four consecutive numbers, when added together, give a total of 114. If x is the lowest number, write down an equation and solve it.

11. Adrian thinks of a number. He doubles it and then adds 5. The answer is 17. What was his number?

    (SEG)

12. (a) Write, in symbols, the rule,

    To find y, double x and add 1.

    (b) Use your rule from part (a) to calculate the value of x when $y = 9$.

    (LON)

13. Jacob uses ths rule,

    Start with a number, divide it by 2 and then add 3. Write down the result.

    (a) What is the result when Jacob starts with 8?

    (b) What number did Jacob start with when the result is 5?

    (SEG)

14. Solve the equation

    $$11x + 5 = x + 25.$$

    (LON)

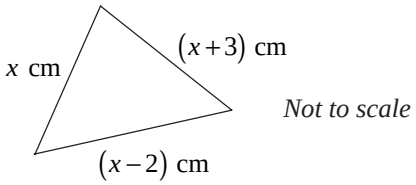
15. Solve the equations

    (a) $4x - 1 = 17$ (b) $11y + 3 = 5y + 27$

    (MEG)

3.4

16. The lengths of the sides of a triangle are x cm, $(x + 3)$ cm and $(x - 2)$ cm.



(a) What is the perimeter of the triangle in terms of x?

(b) The triangle has a perimeter of 22 cm.

(i) Write down an equation in x.

(ii) Use your equation to find the length of each side of the triangle

(SEG)

17. A pint of milk costs x pence.

(a) Write, in terms of x, the cost of two pints of milk.

The cost of a carton of fruit juice is 10 p more than the cost of a pint of milk.

(b) Write in terms of x, the cost of a carton of fruit juice.

Sam pays £1.70 for three pints of milk and two cartons of fruit juice. He writes down the correct equation

$$3x + 2(x + 10) = 170$$

(c) Solve this equation to find the cost of a pint of milk.

(SEG)

3.5 Trial and Improvement Method

The equations which have been solved so far are called *linear* equations,as they contain only x or number terms. However, some equations are *not* linear, that is they contain, for example, x^2 or x^3 terms.

Equations such as these can be solved by a *trial and improvement* method.

Worked Example 1

Solve the equation

$$x^3 + x = 5$$

by using a trial and improvement method.

Solution

First give a value of x to start the process, for example 1.

Start with 1 and substitute 1 into $x^3 + x$, giving

$$1^3 + 1 = 1 + 1$$
$$= 2.$$

3.5

As this is less than 5, x must be greater than 1. So choose a larger value, say 2.

Then substitute 2 into $x^3 + x$ to get

$$2^3 + 2 = 8 + 2$$
$$= 10.$$

As this is greater than 5, x must be less than 2. But x is also greater than 1, so you know that x must lie in the interval

$$1 < x < 2.$$

The next stage is to choose a value between 1 and 2, say 1.5. This is substituted into $x^3 + x$ and the process is repeated.

The results of each trial can be shown in a table, as below.

Trial Value of x	$x^3 + x$ (to 4 d.p.)	*Comment*	*Conclusion*	
1	2	less than 5	$x > 1$	
2	10	greater than 5	$1 < x < 2$	
1.5	5.0625	greater than 5	$1 < x < 1.5$	
1.49	4.9288	less than 5	$1.49 < x < 1.50$	← so $x = 1.5$ to 1 d.p.
1.495	4.9953	less than 5	$1.495 < x < 1.500$	← so $x = 1.50$ to 2 d.p.
1.497	5.0221	greater than 5	$1.495 < x < 1.497$	
1.496	5.0087	greater than 5	$1.495 < x < 1.496$	
1.4955	5.0020	greater than 5	$1.4950 < x < 1.4955$	← so $x = 1.495$ to 3 d.p.

Note

This process could be continued to give the answer correct to any number of decimal places. Usually questions will specify the accuracy required and you must always state the accuracy of your solution. You should ensure that all working is correct to at least two decimal places (or significant figures) more than is required in the final answer; it is a very common mistake to approximate answers too soon in the calculation.

So in this worked example, the answer is 1.495 correct to 3 d.p.

Worked Example 2

Solve the equation

$$\sqrt{x} + 1 - x = 0$$

using the trial and improvement method, giving your answer to 2 decimal places.

Solution

The working is displayed in a table, as in the previous example.

3.5

Trial Value of x	$\sqrt{x}+1-x$ (to 4 d.p.)	*Comment*	*Conclusion*
1	1	greater than 0	$x > 1$
2	0.4142	greater than 0	$x > 2$
3	−0.2679	less than 0	$2 < x < 3$
2.5	0.0811	greater than 0	$2.5 < x < 3$
2.6	0.0125	greater than 0	$2.6 < x < 3$
2.7	−0.0568	less than 0	$2.6 < x < 2.7$
2.65	−0.0221	less than 0	$2.60 < x < 2.65$
2.62	−0.0014	less than 0	$2.60 < x < 2.62$
2.61	0.0055	greater than 0	$2.61 < x < 2.62$
2.615	0.0021	greater than 0	$2.615 < x < 2.62$

So $x = 2.62$ to 2 decimal places.

Exercises

1. Each of the following equations have solutions which are whole numbers. Find the solution of each equation by using a trial and improvement method.

   (a) $x^2 + x = 30$ (b) $x^2 - x = 702$

   (c) $x^3 + 2x = 135$ (d) $x^3 + 2x = 1752$

   (e) $x^5 - x = 240$ (f) $x + \sqrt{x} = 72$

   (g) $x - \sqrt{x} = 90$ (h) $x - x^3 = -1320$.

2. Solve each of the following equations, giving your answers to 1 decimal place.

   (a) $x^3 - 1 = 10$ (b) $x + \sqrt{x} = 7$ (c) $x^3 - x = 4$

   (d) $\sqrt{x} + x = 8$ (e) $x^2 - \sqrt{x} = 5$ (f) $x^3 + x = 16$

3. Trial and improvement can be used to find cube roots of numbers. To find the cube root of 6, you need to solve $x^3 = 6$.

   (a) Solve $x^3 = 6$ to 2 decimal places, starting with a first trial value of 2.

   (b) What would be a good first trial value when solving $x^3 = 1020$? Find x.

   (c) In the same way find the cube root of 350.

Just For Fun

Find a number, the double of which is 99 greater than its half.

3.5

4. Some equations have more than one solution. The equations below all have two solutions, some of which are negative. Find the solutions by using the trial and improvement method.

(a) $x^2 + x = 56$ (b) $x^2 - x = 72$ (c) $x^4 - 1 = 0$

(d) $x^2 - 2x = 80$ (e) $x^2 - 6x = 7$ (f) $x^2 + 5x - 24 = 0$

5. Find both solutions to the following equations, giving your answers to 2 decimal places.

(a) $x^2 + 5x - 7 = 0$ (b) $x^2 - 6x + 4 = 0$

(c) $x^2 + x - 10 = 0$ (d) $x^2 + 6x = 8$

6. (a) Explain why the equation $\sqrt{x} + x = 8$ cannot have a solution which is negative.

(b) Find the solution correct to 3 decimal places.

7. Solve the following equations, giving your answers to 2 decimal places.

(a) $\frac{1}{x} + x = 8$ (b) $x - \frac{1}{x} = 5$ (c) $\frac{2}{x} + x = 4$

(d) $\frac{4}{x} + \sqrt{x} = 5$ (e) $5x - \frac{2}{x} = 8$ (f) $\frac{5}{x} + x = 7$

8. Jimmy tries to solve this problem using trial and improvement.

$$number \times 3.1 = 5.6$$

Try 2.0	$2.0 \times 3.1 = 6.2$	Too large
Try 1.5	$1.5 \times 3.1 = 4.65$	Too small
...	...	...

Continue Jimmy's working until you know the number correct to one decimal place. Write down this number.

(SEG)

9. Judy is using 'trial and improvement' to solve the equation

$$x^2 + x = 11.$$

Copy and complete her working and find a solution correct to one decimal place.

Try $x = 3.5$	$3.5^2 + 3.5 = 15.75$	Too large
Try $x = 2.5$	$2.5^2 + 2.5 = 8.75$	Too small
Try $x = 3.0$	...	...
...	...	...

(SEG)

3.5

10. (a) Solve the equation $3x + 2 = 17$.

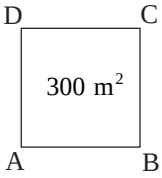
(b) (i) Work out $4^3 = ?$, $5^3 = ?$

(ii) Use a trial and improvement method to solve, correct to 1 decimal place, the equation

$$x^3 = 100.$$

Show all your trials.

(MEG)

11. 

The length, AB, of a side of a square with area $300\ \text{m}^2$ is $\sqrt{300}$ m.

Sabeeha wants to find this length but her calculator does not have a $\sqrt{\ }$ key, so she has to use a trial and improvement method.

Her first 2 trials are as follows:

$18 \times 18 = 324$ (too big)

$17 \times 17 = 289$ (too small)

(a) Showing your working clearly, continue for at least 2 more trials until you have found the length of AB correct to 1 decimal place.

(b) Change your answer to part (a) into centimetres.

(MEG)

12. The number x satisfies the equation

$$x^3 = 20.$$

(a) Between which two consecutive whole numbers does x lie?

(b) Use a trial and improvement method to find this value of x correct to one decimal place. *Show all your working clearly.*

(NEAB)

3.6 Simultaneous Linear Equations

A pair of equations which use both terms at the same time, such as

$$x + 2y = 8$$
$$2x + y = 7,$$

are known as a pair of *simultaneous* equations.

Worked Example 1

Solve the pair of simultaneous equations

$$x + 2y = 8$$
$$2x + y = 7.$$

Solution

First it is helpful to label the equations (1) and (2).

$$x + 2y = 8 \qquad (1)$$
$$2x + y = 7 \qquad (2)$$

Equation (1) is multiplied by 2, so that it contains the same number of x's as equation (2).

Let the new equation be labelled (3).

$$2x + 4y = 16 \qquad (3) \quad [2 \times (1)]$$
$$2x + y = 7 \qquad (2)$$

Equation (2) is now subtracted from equation (3).

$$2x + 4y = 16 \qquad (3)$$
$$2x + y = 7 \qquad (2)$$
$$3y = 9 \qquad (3) - (2)$$

Solving $3y = 9$ gives $y = 3$.

This value of y can now be substituted into equation (1) to give:

$$x + 2 \times 3 = 8$$
$$x + 6 = 8$$

Solving this gives $x = 2$. So the solution to the equation is $x = 2,\ y = 3$.

Worked Example 2

Solve the simultaneous equations

$$3x + 5y = 2$$
$$-4x + 7y = -30.$$

3.6

Solution

First label the equations (1) and (2) as shown below.

$$3x + 5y = 2 \qquad (1)$$
$$-4x + 7y = -30 \qquad (2)$$

Then multiply equation (1) by 4 and equation (2) by 3 to make the number of x's in both equations the same.

$$12x + 20y = 8 \qquad (3) \quad [4 \times (1)]$$
$$-12x + 21y = -90 \qquad (4) \quad [3 \times (2)]$$

Now add together equations (3) and (4) to give

$$12x + 20y = 8 \qquad (3)$$
$$-12x + 21y = -90 \qquad (4)$$
$$41y = -82 \qquad (3) + (4)$$

Solving the equation $41y = -82$ gives $y = -2$.

This value for y can be substituted into equation (1) to give

$$3x + 5 \times (-2) = 2$$

or

$$3x - 10 = 2.$$

Solving this equation gives:

$$3x - 10 = 2$$
$$3x = 12$$
$$x = \frac{12}{3}$$
$$= 4.$$

So the solution is $x = 4$ and $y = -2$.

Note

It is a good idea to check that solutions are correct by substituting these values back into the original equations. Here,

$$3 \times 4 + 5 \times (-2) = 2$$

and

$$-4 \times 4 + 7 \times (-2) = -30$$

You must check *both* equations to make sure that you have the correct answer.

3.6

Worked Example 3

Denise sells 300 tickets for a concert. Some tickets are sold to adults at £5 each and some are sold to children at £4 each. If she collects in £1444 in ticket sales, how many tickets have been sold to adults and how many to children?

Solution

Let x = number of adults' tickets
and y = number of children's tickets.

She has sold 300 tickets, so

$$x + y = 300.$$

The value of the adult tickets sold is £5x, and the value of the children's tickets is £4y.

As the value of all the tickets sold is £1444, then

$$5x + 4y = 1444.$$

The two simultaneous equations

$$x + y = 300 \qquad (1)$$
$$5x + 4y = 1444 \qquad (2)$$

can now be solved. First multiply equation (1) by 5 and subtract equation (2) to give

$$5x + 5y = 1500 \qquad (3) \quad [5 \times (1)]$$
$$5x + 4y = 1444 \qquad (2)$$
$$y = 56 \qquad (3) - (2)$$

This value can then be substituted into equation (1) to give

$$x + 56 = 300$$

or

$$x = 244.$$

So the solution is $x = 244$ and $y = 56$. That is, 244 adults' tickets and 56 children's tickets have been sold.

Investigation

Consider the following simultaneous equations.

$$2x + y = 6 \qquad (1)$$
$$x = 1 - \tfrac{1}{2}y \qquad (2)$$

If (2) is substituted for x into (1), then

$$2\left(1 - \tfrac{1}{2}y\right) + y = 6$$
$$2 - y + y = 6$$
$$2 = 6!$$

Find out where the problem lies.

3.6

Exercises

1. Solve each pair of simultaneous equations.

(a) $x + 2y = 5$, $3x + y = 5$ (b) $3x + 2y = 19$, $x + 5y = 15$ (c) $x - 2y = 4$, $4x + 3y = 49$

(d) $2x + 3y = 14$, $5x + 2y = 24$ (e) $3x + 4y = 2$, $7x - 5y = 9$ (f) $4x + 2y = 16$, $-3x + 2y = -19$

(g) $5x + y = 2$, $-4x + 3y = 44$ (h) $6x - 4y = 12$, $-9x + 2y = -66$ (i) $7x - 2y = 23$, $3x + 4y = 39$

(j) $8x + 4y = 7$, $-12x + 8y = -6$ (k) $4x - 2y = -0.1$, $5x + 2y = 1.5$ (l) $6x - 5y = 41$, $4x + 15y = 31$

(m) $-2x + 5y = 14$, $10x + 7y = 26$ (n) $8x + 5y = -29$, $3x - 7y = -2$ (o) $6x - 5y = -14$, $18x - 4y = 6$

(p) $6x - 8y = -2$, $5x + 2y = 1.8$ (q) $\frac{1}{2}x - \frac{1}{4}y = 0$, $\frac{1}{3}x + \frac{2}{3}y = 10$ (r) $\frac{1}{5}x - \frac{1}{10}y = -1$, $\frac{1}{4}x + \frac{1}{2}y = 10$

2. Find the coordinates of the point of intersection of the lines:

(a) $x + y = 8$ and $y = 2x - 1$

(b) $x + y = 10$ and $y = 2x + 1$

(c) $x + y = 4$ and $y = 2 - \frac{x}{10}$

3. Describe the problems you encounter when you try to solve the simultaneous equations:

$$3x - 2y = 8$$
$$9x - 6y = 2.$$

4. (a) Check that $x = 2$ and $y = 5$ is a solution of both the equations below.

$$x + 2y = 12$$
$$3x + 6y = 36$$

(b) Try to solve the equations. What happens?

(c) Write both equations in the form $y = \ldots$ and comment on the equations you obtain.

3.6

5. Ian rows a boat in a river where there is a steady current. He travels at 1.3 mph upstream and 3.7 mph downstream.

   Use a pair of simultaneous equations to find v, the speed of the boat in still water, and c, the speed of the current.

6. A machine sells tickets for travel on a tram system. A single ticket costs £1 and a return ticket costs £2. In one day, the machine sells 100 tickets and takes £172. How many of each type of ticket were sold?

7. A shopkeeper takes 200 notes to a bank. They are a mixture of £5 and £10 notes. The shopkeeper claims that there is £1395.

   (a) Find out how many of each type of note there should be.

   (b) In fact the value of the notes turns out to be £1390. How many of each type of note does this mean there should be? What error do you think the shopkeeper made?

8. A bus stops at two places. The fare to the first stop is £3 and the fare to the second stop is £5. When the bus sets out, there are 38 people on the bus and the driver has collected £158 in fares. How many people get off the bus at the first stop?

9. When Isa travels to the USA for a holiday he leaves the UK at 1.00 pm local time and lands at 5.00 pm local time. On the return journey he leaves at 8.00 pm local time and lands at 10.00 am local time the next day.

   Find the length of the flight in hours and the time difference between the UK and the part of the USA that Isa visited.

10. Solve the simultaneous equations,

$$2x + 3y = 23$$
$$x + y = 4$$

(LON)

11. Solve the simultaneous equations,

$$2a + 4c = 13$$
$$a + 3c = 8$$

(NEAB)

12. The height, h metres, of a sky rocket t seconds after being launched is given by the formula

$$h = at^2 + bt + 2$$

where a and b are constants. The heights of the rocket above the ground at two different times are given in the table below.

t (seconds)	1	2
h (metres)	37	62

3.6

(a) At what height above the ground is the rocket launched?

(b) (i) Use the table of values to show that

$$a + b = 35$$
$$\text{and} \quad 4a + 2b = 60.$$

(ii) Solve these simultaneous equations to find the value of a and the value of b.

(c) What was the height of the sky rocket $7\frac{1}{2}$ seconds after it was launched?

(MEG)

Answers to Exercises

3.1 Changing the Subject

1. (a) $x = \frac{y}{4}$ (b) $x = \frac{y - 3}{2}$ (c) $x = \frac{y + 8}{4}$ (d) $x = 4y - 2$

   (e) $x = 5y + 2$ (f) $x = y - a$ (g) $x = ya + b$ (h) $x = \frac{y - c}{a}$

   (i) $x = \frac{yc - b}{a}$ (j) $x = \frac{yb + c}{a}$ (k) $x = y - a - b$ (l) $x = yc + a - b$

   (m) $x = \frac{y}{ab}$ (n) $x = \frac{y - c}{ab}$ (o) $x = \frac{3cy + b}{4a}$ (p) $x = \frac{pd + bc}{a}$

   (q) $x = \frac{y}{b} - a$ (r) $x = \frac{4y}{a} - 3$ (s) $x = \frac{2q}{3} + 4$ (t) $x = \frac{4v}{5} - y$

   (u) $x = 4(z - a) + 3$

2. $I = \frac{V}{R}$; $R = \frac{V}{I}$

3. $m = \frac{F}{a}$; $a = \frac{F}{m}$

4. $r = \frac{C}{2\pi}$

5. (a) $t = \frac{v - u}{a}$ (b) $a = \frac{v - u}{t}$

6. $z = 3m - x - y$

7. (a) $a = \frac{v^2 - u^2}{2s}$ (b) $a = \frac{s}{(t + \frac{1}{2}t^2)}$

8. $z = \frac{v}{xy}$

9. (a) $r = +\sqrt{\frac{V}{\pi h}}$ (only a positive value because r is radius) (b) 2.82

10. (a) $h = \frac{V}{x^2}$; $h = \frac{A - 2x^2}{4x}$ (b) 2 (c) 2.5

11. (a) $a = \frac{2A}{h} - b$ (b) $A = \frac{1}{2} \times 3a \times h = \frac{3ah}{2}$; $a = \frac{2A}{3h}$

Answers

3.2 Simplifying Expressions

1. (a) $6a$ (b) $3a + 12$ (c) $11a + 6b$ (d) $4x + 11y$
   (e) $13x - y$ (f) $2a + 10b$ (g) $3a + b + 6$ (h) $6p - 7q$
   (i) $-7x + 3y$ (j) $2x - p$ (k) $15x - 9z$ (l) $-14x + 5z$
   (m) $8a + 4q - 6x$ (n) $-p - q + x + z$ (o) $3x - 2y + 10$ (p) $21x - 32q$
   (q) $2y$ (r) $2x$ (s) $-19x + 11y$ (t) $23x - 18y - 4$
   (u) $-7x - 10y$ (v) $-4p + 4q$

2. (a) $6x^2 + 8x$ (b) $x^2 + 13x + 10$ (c) $2x^2 + 10x$
   (d) $5x^2 + 2x + 10$ (e) $-x^2 + 7x$ (f) $3x^2 + 4y^2$
   (g) $2x^2 + y^2 - x - y$ (h) $5x^2 - 3x - 10$ (i) $3x^2 - 3y^2 - x - y$
   (j) $5x^2 + 2y - 4$ (k) $5ab + cd$ (l) $2xy + 5xz$
   (m) $11ab - 3ad$ (n) $9pq - 3qr$

3. (a) $3x + 15$ (b) $24 + 4x$ (c) $7x + 14$ (d) $2x + 12$
   (e) $5x + 10$ (f) $8x + 12$ (g) $15x + 10$ (h) $40x + 24$
   (i) $7x - 42$ (j) $40 - 8x$ (k) $8x - 28$ (l) $35x - 21$
   (m) $18x - 30$ (n) $4x - 8y$ (o) $5x + 10y + 15z$ (p) $5x + x^2$
   (q) $2a - a^2$ (r) $4b - 12$ (s) $2x^2 - 12x$ (t) $8x^2 + 12x$
   (u) $21x - 6x^2$ (v) $8x^2 - 40x$

4. (a) $7a + 26$ (b) $7x + 23$ (c) $8x + 22$ (d) $2x^2 + 7x$
   (e) $x^2 + 9x + 4$ (f) $12a + 17b$

3.3 Simple Equations

1. (a) $x = 4$ (b) $x = 9$ (c) $x = 3$ (d) $x = 12$ (e) $x = 14$
   (f) $x = -2$ (g) $x = 9$ (h) $x = 4$ (i) $x = 18$ (j) $x = 8$
   (k) $x = 34$ (l) $x = 7$ (m) $x = 10$ (n) $x = 24$ (o) $x = 50$
   (p) $x = 12$ (q) $x = 21$ (r) $x = 66$ (s) $x = 8$ (t) $x = 6$
   (u) $x = 26$ (v) $x = 36$ (w) $x = 15$ (x) $x = 4$

2. (a) $x = -4$ (b) $x = -5$ (c) $x = 3$ (d) $x = -6$ (e) $x = -4$
   (f) $x = -14$ (g) $x = -6$ (h) $x = -3$ (l) $x = -20$ (m) $x = 8$
   (n) $x = 8$ (o) $x = 6$ (p) $x = -5$ (q) $x = -10$ (r) $x = -26$

3. (a) $x + 110° = 180°$; $x = 70°$ (b) $x + 92° = 180°$; $x = 88°$
   (c) $x + 110° = 180°$; $x = 149°$ (d) $42° + x + 40° = 180°$; $x = 98°$

Answers

3.3

4. $x - 2 = 16$; $x = 18$

5. $12 \times x = 54$; $x =$ £4.50

6. $x + 3 = 41$; $x =$ £38

7. 52 is double Majid's number, so the number is half of 52. It is 26.
   Alternatively, $x \times 2 = 52$, so $x = 26$.

8. 24 is double Jim's number, so his number is half of 24. It is 12.
   Alternatively, $x \times 2 = 24$, so $x = 12$.

9. Phillip is 11 years old. Sue is $11 - 2 = 9$ years old.
   Ali is twice as old as Sue, so Ali is $9 \times 2 = 18$ years old.

3.4 Solving Equations

1. (a) $x = 14$ (b) $x = 9$ (c) $x = 14$ (d) $x = 8$ (e) $x = 4$
   (f) $x = 7$ (g) $x = 5$ (h) $x = -1$ (i) $x = \frac{-5}{3}$ (j) $x = 7$
   (k) $x = 6$ (l) $x = -2$ (m) $x = 10$ (n) $x = 1$ (o) $x = \frac{3}{5}$
   (p) $x = -2\frac{1}{2}$ (q) $x = \frac{1}{5}$ (r) $x = \frac{-3}{4}$ (s) $x = -3$ (t) $x = 0$
   (u) $x = \frac{-2}{5}$ (v) $x = 36$ (w) $x = 15$ (x) $x = 5$

2. (a) $x = -3$ (b) $x = -6$ (c) $x = \frac{13}{4}$ (d) $x = 14$
   (e) $x = 4$ (f) $x = 10$ (g) $x = 1$ (h) $x = -3$ (i) $x = 13$
   (j) $x = 3$ (k) $x = 4$ (l) $x = \frac{2}{3}$ (m) $x = 3$ (n) $x = 2$
   (o) $x = \frac{5}{3}$ (p) $x = 4$ (q) $x = 2$ (r) $x = \frac{1}{2}$ (s) $x = 8$
   (t) $x = 12$ (u) $x = 32$ (v) $x = 14$ (w) $x = 39$ (x) $x = 12$

3. (a) $(3x + 40) + 80 + x = 360$; $x = 60°$
   (b) $(x + 10) + (x + 30) + (2x + 30) + x = 360$; $x = 58°$
   (c) $(x + 10) + (x + 30) + (x - 10) = 360$; $x = 110°$
   (d) $(140 - x) + (120 - x) + (180 - x) = 360°$; $x = 26\frac{2}{3}°$

4. $x + x + (x + 1) + (x + 1) = 10$; $x =$ 2 m

5. $2x + 10 = 42$; $x = 16$

Answers

3.4

6. $6x - 8 = 34$; $x = 7$

7. $4m + 2 = 6$; $m = 1$; Mary drives for 1 hour.

8. (a) $80 + (60 \times x) = 290$; $x = 3\frac{1}{2}$ (3 hours and 30 mins).

   (b) $80 + (60 \times x) = 220$; $x = 2\frac{1}{3}$ (2 hours and 20 mins).

9. (a) $(x + 6) \times 2 = 18$; $x = 3$ (b) $(x \div 2) + 10 = 16$; $x = 12$

   (c) $[(x \div 2) + 2] \times 2 = 9$; $x = 5$ (d) $[(x - 7) \div 2] \times 10 = 115$; $x = 30$

10. $x + (x + 1) + (x + 2) + (x + 3) = 114$; $x = 27$

11. 6

12. (a) $y = 2x + 1$ (b) $x = 4$

13. (a) 7 (b) 4

14. $x = 2$

15. (a) $x = 4\frac{1}{2}$ (b) $y = 4$

16. (a) $(3x + 1)$ cm (b) (i) $3x + 1 = 22$ (ii) 7 cm, 5 cm and 10 cm.

17. (a) $2x$ pence (b) $(x + 10)$ pence (c) 30 pence

3.5 Trial and Improvement Method

1. (a) $x = 5$ (b) $x = 27$ (c) $x = 5$ (d) $x = 12$ (e) $x = 3$
   (f) $x = 64$ (g) $x = 100$ (h) $x = 11$

2. (a) $x = 2.2$ (to 1 decimal place) (b) $x = 4.8$ (to 1 d.p.)
   (c) $x = 1.8$ (to 1 d.p.) (d) $x = 5.6$ (to 1 d.p.)
   (e) $x = 2.6$ (to 1 d.p.) (f) $x = 2.4$ (to 1 d.p.)

3. (a) $x = 1.82$ (to 2 d.p.) (b) 10.1 is a good starting number; $x = 10.07$ (2 d.p.)
   (c) 7.05 (to 2 d.p.)

4. (a) $x = 7$ or -8 (b) $x = 9$ or -8 (c) $x = 1$ or -1
   (d) $x = 10$ or -8 (e) $x = 7$ or -1 (f) $x = 3$ or -8

5. (a) $x = 1.14$ or 6.14 (b) $x = 5.24$ or 0.76 (c) $x = 2.32$ or -4.32
   (d) $x = 1.12$ or -7.12

Answers

3.5

6. (a) x cannot be negative because, so far, we have no meaning for the square root of a negative number.
   (b) $x = 5.628$ to 3 d,p,

7. (a) $x = 7.87$ or 0.13 (b) $x = 4.24$ or -0.24 (c) $x = 3.41$ or 0.59
   (d) $x = 23.31$ (e) $x = 6.46$ or 0.06 (f) $x = 6.19$ or 0.81

8. $x = 1.8$ to 1 d.p.

9. $x = 2.9$ gives the result nearest 11.

10. (a) $x = 5$
    (b) (i) $4^3 = 64$; $5^3 = 125$ (ii) $x = 4.6$ (to 1 d.p.) is the nearest value.

11. (a) 17.3 m (b) 1730 cm (to the nearest 10 centimetres)

12. (a) 2 and 3 (b) 2.7

3.6 Simultaneous Linear Equations

1. (a) $x = 1, y = 2$ (b) $x = 5, y = 2$ (c) $x = 10, y = 3$
   (d) $x = 4, y = 2$ (e) $x = \frac{46}{43}, y = \frac{-13}{43}$ (f) $x = 5, y = -2$
   (g) $x = -2, y = 12$ (h) $x = 10, y = 12$ (i) $x = 5, y = 6$
   (j) $x = \frac{5}{7}, y = \frac{9}{28}$ (k) $x = \frac{7}{45}, y = \frac{13}{36}$ (l) $x = 7, y = \frac{1}{5}$
   (m) $x = \frac{1}{2}, y = 3$ (n) $x = -3, y = -1$ (o) $x = \frac{43}{33}, y = \frac{48}{11}$
   (p) $x = 0.2, y = 0.4$ (q) $x = 10, y = 20$ (r) $x = -20, y = 30$

2. (a) (3, 5) (b) (3, 7) (c) $\left(\frac{20}{9}, \frac{16}{9}\right)$

3. It is not possible to eliminate either x or y because of their coefficients. If the equations had been used to produce graphs it would be seen that the two straight lines produced are parallel. Therefore, there can be no point of intersection and it is impossible to solve the equations simultaneously.

4. (b) There is no way of eliminating x or y in order to find the value of the other variable.

   (c) $x + 2y = 12 \quad (1)$ gives $y = 6 - \frac{x}{2}$

   $3x + 6y = 36 \quad (2)$ gives $y = 6 - \frac{x}{2}$

   So equations (1) and (2) are different versions of the same equation.

Answers

3.6

5. $v + c = 3.7$; $v - c = 1.3$; $v = 2.5$, $c = 1.2$.

6. 72 return tickets and 28 single tickets were sold.

7. (a) 79 ten-pound notes; 121 five-pound notes
   (b) 78 ten-pound notes; 122 five-pound notes

8. 16

9. The length of the flight is 9 hours; the time difference is 5 hours.

10. $x = -11$, $y = 15$

11. $a = \frac{7}{2}$, $c = \frac{3}{2}$

12. (a) 2 metres
    (b) (i) $t = 1$ when $h = 37$, so $37 = (a \times 1) + (b \times 1) + 2$ and $35 = a + b$
    $t = 2$ when $h = 62$, so $62 = (a \times 4) + (b \times 2) + 2$ and $60 = 4a + 2b$
    (ii) $a = -5$; $b = 40$
    (c) 20.75 metres