Mathematics Enhancement Programme

Primary Demonstration Project

1B Algebra

Help Booklet

Support for Primary Teachers in Mathematics

Primary Project
funded by
Pricewaterhouse
Coopers

in association with
British Steel
Garfield Weston Foundation

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ESSO

CIMT
School of Education
University of Exeter

Mathematics Enhancement Programme

Help Module 1

ALGEBRA

Part B

Contents of Part B

Contents of Part A

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PREFACE

This is one of a series of *Help Modules* designed to help you gain confidence in mathematics. It has been developed particularly for primary teachers (or student teachers) but it might also be helpful for non-specialists who teach mathematics in the lower secondary years. It is based on material which is already being used in the *Mathematics Enhancement Programme: Secondary Demonstration Project*.

The complete module list comprises:

1. ALGEBRA
2. DECIMALS
3. EQUATIONS
4. FRACTIONS
5. GEOMETRY
6. HANDLING DATA
7. MENSURATION
8. NUMBERS IN CONTEXT
9. PERCENTAGES
10. PROBABILITY

Notes for overall guidance:

- Each of the 10 modules listed above is divided into 2 parts. This is simply to help in the downloading and handling of the material.
- Though referred to as 'modules' it may not be necessary to study (or print out) each one in its entirely. As with any self-study material you must be aware of your own needs and assess each section to see whether it is relevant to those needs.
- The difficulty of the material in **Part A** varies quite widely: if you have problems with a particular section do try the one following, and then the next, as the content is not necessarily arranged in order of difficulty. Learning is not a simple linear process, and later studies can often illuminate and make clear something which seemed impenetrable at an earlier attempt.
- In **Part B, Activities** are offered as backup, reinforcement and extension to the work covered in Part A. **Tests** are also provided, and you are strongly urged to take these (at the end of your studies) as a check on your understanding of the topic.
- The marking scheme for the revision test includes B, M and A marks. Note that:

  M *marks* are for method;

  A *marks* are for accuracy (awarded only following a correct M mark);

  B *marks* are independent, stand-alone marks.

We hope that you find this module helpful. Comments should be sent to:

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The full range of Help Modules can be found at

www.ex.ac.uk/cimt/help/menu.htm

ACTIVITIES

ACTIVITY 1.1 *Factors*

Mark out on a large sheet of squared paper a 30 × 30 grid similar to the one below.

Numbers \ Factors	1	2	3	4	5	6	7	8	9	10	11	12	etc
1	✔												
2	✔	✔											
3	✔		✔										
4	✔	✔		✔									
5													
6													
7													
8													
9													
10													
11													
12													
etc													

1. For each column/row, if the number at the top is a factor of the number on the left-hand-side of a row, tick the relevant box (some have been done in the sample grid above).

2. As you fill in your grid you should notice various patterns made by the ticks. Describe and explain these patterns.

ACTIVITY 1.2 *Sieve of Eratosthenes*

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1. Cross out all multiples of 2, e.g. 4, 6, etc.

2. Cross out all multiples of 3, e.g. 6, 9, etc.

4. Continue in this way for all other prime numbers less than 10.

5. What can you say about the numbers **not** crossed out?

ACTIVITY 1.3 *Last Digit*

Note that the last digit of the value of 7^2 is 9.

$$7^2 = 49$$

The last digit of each of the following squares is also 9.

$$\begin{aligned} 27^2 &= 72\ 9 \\ 87^2 &= 7\ 56\ 9 \\ 137^2 &= 18\ 76\ 9 \\ 407^2 &= 165\ 64\ 9 \end{aligned}$$

Look at the left hand side of each of the above equations. Notice that the last digit of each number to be squared ends in 7.

1. Write down the value of each of the following squares:

   (a) 3^2, 83^2, 173^2, 503^2 (b) 9^2, 19^2, 209^2, 699^2

2. What do you notice about the last digit in your answers to 1.(a) and (b)?
   Is there a rule to help you find the last digit of the value of a square?

3. Investigate with other numbers to check if your rule works.

4. Complete the table below by writing down n^2 and the last digit of n^2.
   (One has been done for you.)

n	0	1	2	3	4	5	6	7	8	9
n^2									49	
Last digit of n^2								9		

Extension

Complete a similar table for n^3, n^4, n^5 and n^6. Study the table and answer the following questions:

(a) Which power of n has all its last digits the same as the original number n?

(b) Which numbers do **not** appear among the last digits of square numbers?

(c) Identify and write down other features of the different powers of numbers that you can find.

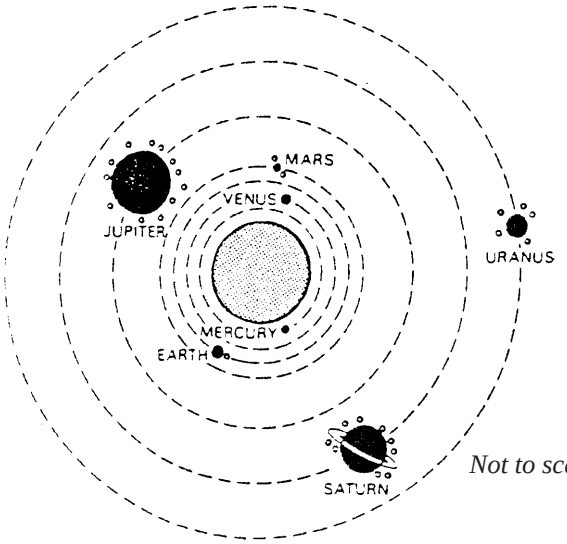
ACTIVITY 1.4 *Bode's Law*

Although we regard our present understanding of the universe as advanced, there are still many mysteries. One of these is known as *Bode's Law*. It is an empirical law, which means it is based on observed data and not on a theoretical understanding.

The law, published in 1772, relates the ratio

$$\frac{R}{R_e} = \frac{\text{Distance of planet from Sun}}{\text{Distance of earth from Sun}}$$ to the number n, where

$n = 0$ - *Venus*
$n = 1$ - *Earth*
$n = 2$ - *Mars*
$n = 3$
$n = 4$ - *Jupiter*
$n = 5$ - *Saturn*
$n = 6$ - *Uranus*
$n = 7$ - *Neptune*
$n = 8$ - *Pluto*



Not to scale

1. For each planet, find the ratio R/R_e to one place decimal and plot a graph of
   R/R_e against n.

2. Bode's Law is given by the formula
   $$R/R_e = 0.4 + 0.3 \times 2^n$$
   Check the accuracy of the values from this formula with the actual values.
   Do they all agree?

3. What value should be taken for n for Mercury?

4. A large number of asteroids are found at distance 433.8×10^6 km from the Sun.
   What can you conjecture from this?

5. Does the data support the view that Neptune and Pluto were once a single planet?

Planet	***Distance from Sun*** *(in millions of km)*
Mercury	57.9
Venus	108.2
Earth	149.6
Mars	227.9
Jupiter	778.3
Saturn	1427.0
Uranus	2870.0
Neptune	4497.0
Pluto	5907.0

ACTIVITY 1.5 *Pendulums*

You can easily observe that the time of swing of a pendulum depends on the length of the pendulum – the longer the length, the longer the swing. But how exactly does the pendulum time, T, (the time for a complete cycle) and its length, L, correspond?

Perform the following experiments, and then analyse the data to answer this question.

1. Time the swing, (a complete cycle) of pendulums of lengths 1 m, 2 m, 3 m, 4 m, etc. (You need to do this as accurately as possible, so it is probably best to time 5 swings and then divide by 5.)

2. Plot a graph of T (on the y-axis) against lengths, L, on the x-axis.
   What sort of relationship does this suggest?

3. Plot a graph of T against $\sqrt{L}$. What shape is this?

Your graph in question 3 should be approximately a straight line, so the formula is of the form

$$T = k\sqrt{L}$$

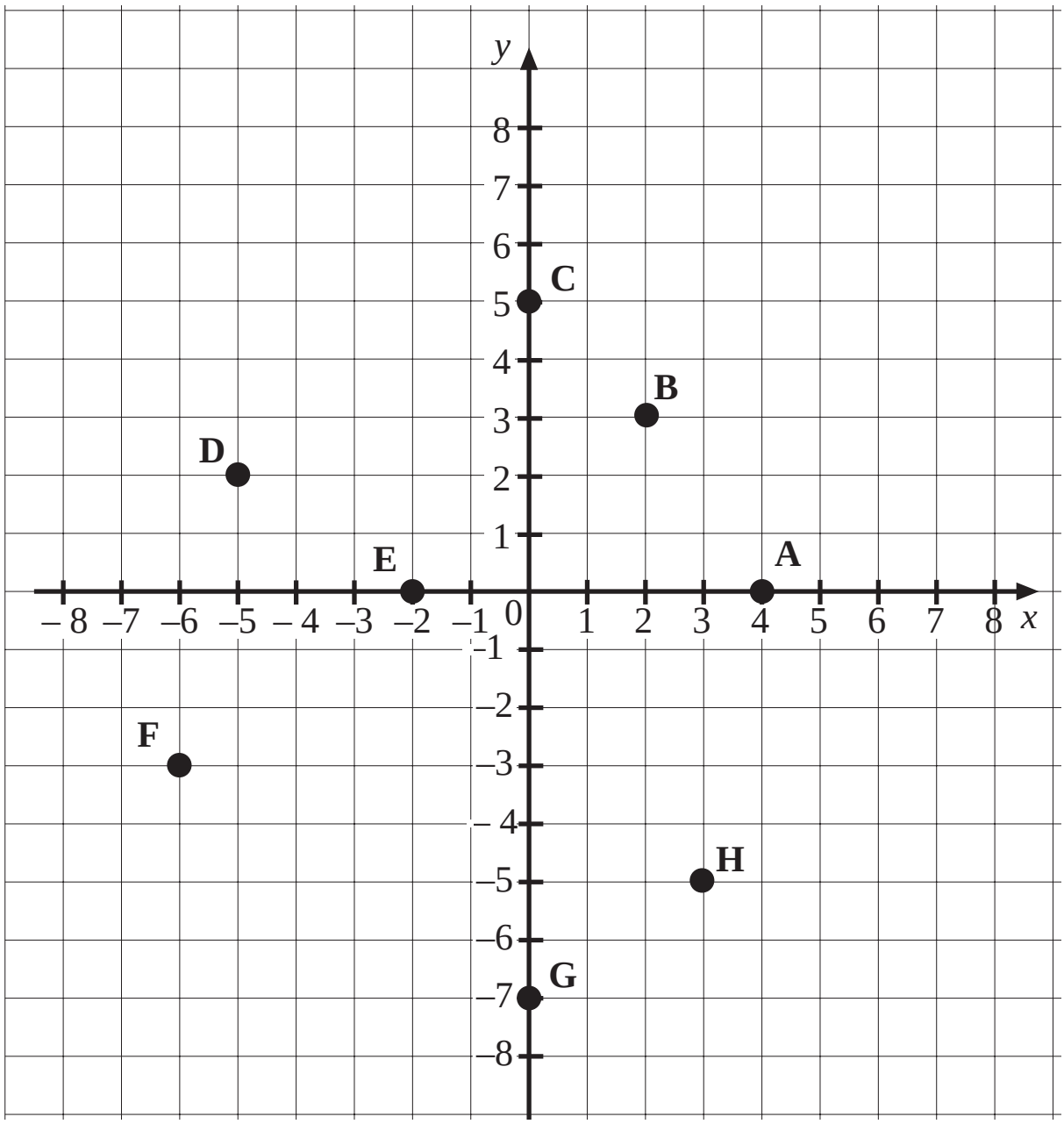
The constant k is the shape of the straight line.

4. Estimate your value of k from your data.

   Use your formula to predict then length of pendulum that has a period of 2 seconds.

   Check your result experimentally.

ACTIVITY 1.6 *Coordinates*



1. What are the coordinates of the points shown?

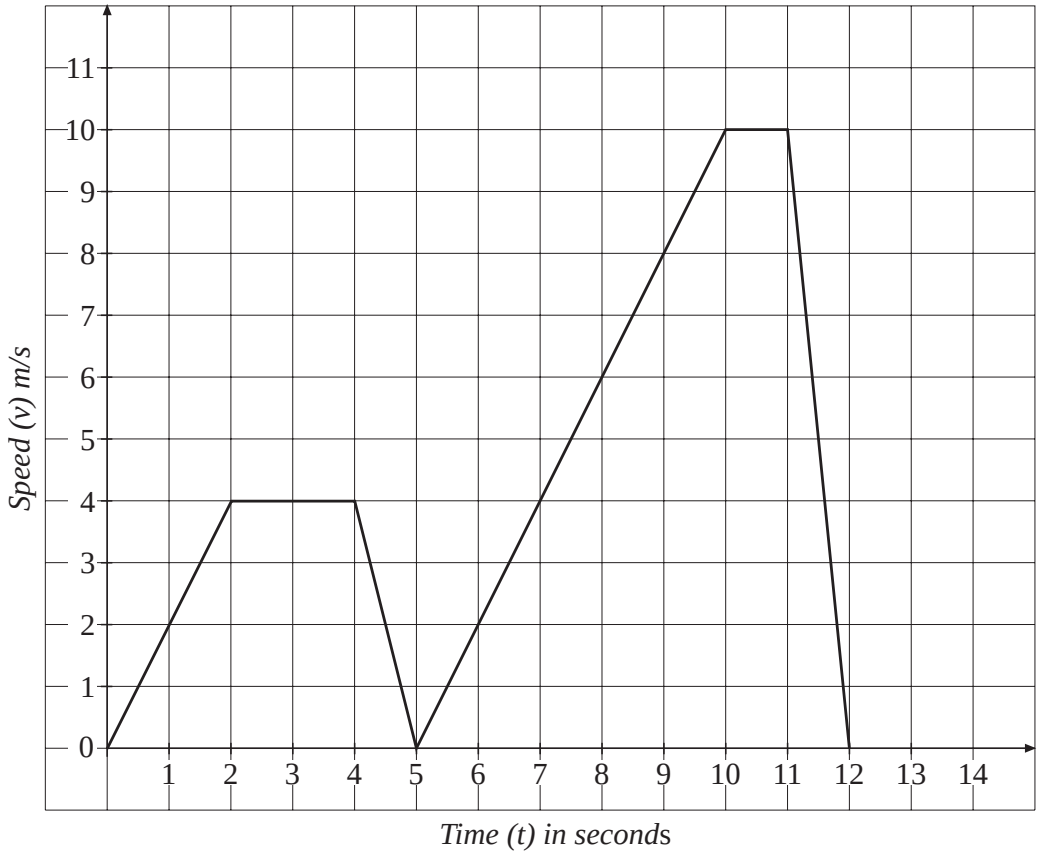
2. Show where the following points are located.

   P (– 4, – 4) Q (6, – 4) R (5, 3) S (–5, 3)

   What is the name given to the shape PQRS?

ACTIVITY 1.7 *Speed-Time Graph*

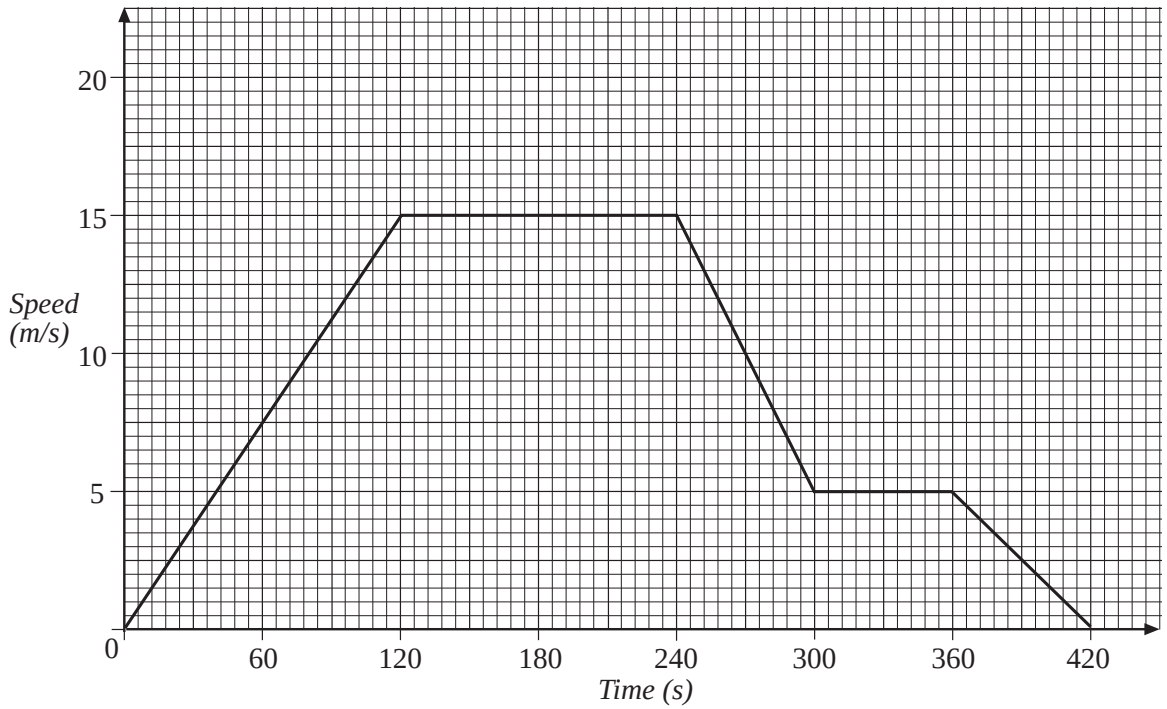
The speed-time graph shows the speeds of a cyclist at different times during a journey.



1. After 1 second what was the speed of the cyclist?
2. From $t = 2$ to $t = 4$, the cyclist was travelling at what speed?
3. After how many seconds did the cyclist make the first stop?
4. When $t = 9$, what was the speed of the cyclist?
5. At what time did the cyclist complete his journey?

ACTIVITY 1.8 *Area Under Speed-Time Graph: Distance*

Here is a speed-time graph for a car.



1. What is the maximum speed reached by the car?
2. For how long does the car travel at this speed?
3. What distance does the car travel at this speed?
4. What distance does it travel before reaching this speed?
5. What is the total distance travelled on the journey?
6. What is the total time taken?
7. What is the average speed for the journey?

ACTIVITIES 1.1 – 1.8 *Notes for Solutions*

Notes for solutions are given only where appropriate.

1.2 5. All numbers not crossed out are prime numbers.

1.3 1. (a) 9, 6889, 29929, 253009
(b) 81, 361, 43681, 488601

2. Last digit is always either 9 or 1.

4.

n	0	1	2	3	4	5	6	7	8	9
Last digit of n^2	0	1	4	9	6	5	6	9	4	1

Extension (a) 0, 1, 5, 6 (b) 2, 3, 7, 8

1.4 3. $n \to -\infty$, so $R/R_e \to 0.4$

4. They correspond to $n = 3$ so they could have formed from a planet that exploded.

1.5 4. $k \approx 2$

1.6 1. A: (4, 0), B: (2, 3), C: (0, 5), D: (– 5, 2), E: (– 2, 0), F: (– 6, – 3), G: (0, – 7), H: (3, – 5)
2. Parallelogram

1.7 1. 2 m/s 2. 4 m/s 3. 5 seconds 4. 8 m/s
5. 12 seconds after starting

1.8 1. 15 m/s 2. 120 seconds (3 mins) 3. 1800 m 4. 900 m
5. 2745 m 6. 420 seconds (7 mins) 7. 6.54 m/s

TESTS

Test 1.1 Mental Practice

Answer these questions as quickly as you can, but without the use of a calculator.

1. 2^3 ?

2. 7^2 ?

3. $\sqrt{36}$?

4. $\sqrt[3]{64}$?

5. $\sqrt{100}$?

6. List all the factors of 20.

7. Write the number 100 as a product of prime numbers.

8. The volume, V, of a box is given by the formula $V = xyz$.
   What is the value of V when $x = 5,\ y = 6$ and $z = 2$?

9. A rectangle is 3 cm longer than it is wide.
   If x cm is its width, write down a formula for its perimeter, p.

10. The formula for converting from °F to °C is $C = \frac{5}{9}(F - 32)$.
    What is the temperature in °C for 68 °F?

Test 1.2 Mental Practice

Answer these questions as quickly as you can, but without the use of a calculator.

1. 3^4 ?

2. 5^2 ?

3. $\sqrt{64}$?

4. $\sqrt[3]{8}$

5. $\sqrt{400}$?

6. List all the factors of 36.

7. Write down the number 120 as a product of prime numbers.

8. The volume, *V*, of a cube is given by the formula $V = a^3$.
   What is the value of *V* when $a = 5$?

9. A rectangle is 2 cm longer than its width.
   If *x* is its width, write down a formula for its area, *A*.

10. The formula for converting from °C to °F is $F = 32 + \frac{9}{5}\text{C}$.
    What is the temperature in °F for $30\,°\text{C}$?

Test 1.3 Revision

40 minutes are allowed

1. Find the values of:

   (a) 3^2 (b) 4^3 (c) $\sqrt{64}$

   (d) $\sqrt[3]{125}$ (e) $\sqrt[3]{27}$ (f) $3^2 - 2^3$

   (g) $\sqrt{4} + \sqrt{9}$ (h) $\sqrt{16} \times \sqrt{25}$ (i) $3^2 + 4^2 - 5^2$ (*9 marks*)

2. List all the factors of 42. *(2 marks)*

3. Which of there numbers:

   5, 10, 15, 20, 25, 30, 35, 40

   (a) is a multiple of six? *(1 mark)*

   (b) is a square number? *(1 mark)*

   (c) is a prime number? *(1 mark)*

   (SEG)

4. Choose one of these words

   prime, factor, square, multiple, cube

   to complete each of the following sentences about the number sequence

   4, 8, 12, 16, 20, 24, 28

   (a) Each number is a . . . . . of 4. *(1 mark)*

   (b) The numbers 4 and 16 are . . . . . numbers. *(1 mark)*

   (c) Each of the numbers 4 and 8 is a . . . . . of 16. *(1 mark)*

   (LON)

5. A bingo card has the following numbers.

			35		
9		27			51
				47	
	15				60

   (a) Which one of these numbers is a multiple of 17? *(1 mark)*

   (b) Which one of these numbers is prime? *(1 mark)*

   (c) Express the number 60 as a product of its prime factors. *(2 marks)*

   (SEG)

Test 1.3 Revision

6. A removals firm charges

   (i) a fixed charge of £25, plus

   (ii) a variable charge of £3 per mile travelled.

   If you wish to move a distance of *d* miles, find the formula for the total cost in pounds, *c*.

   Use your formula to determine the total cost when

   (a) $d = 10$ (b) $d = 40$ *(4 marks)*

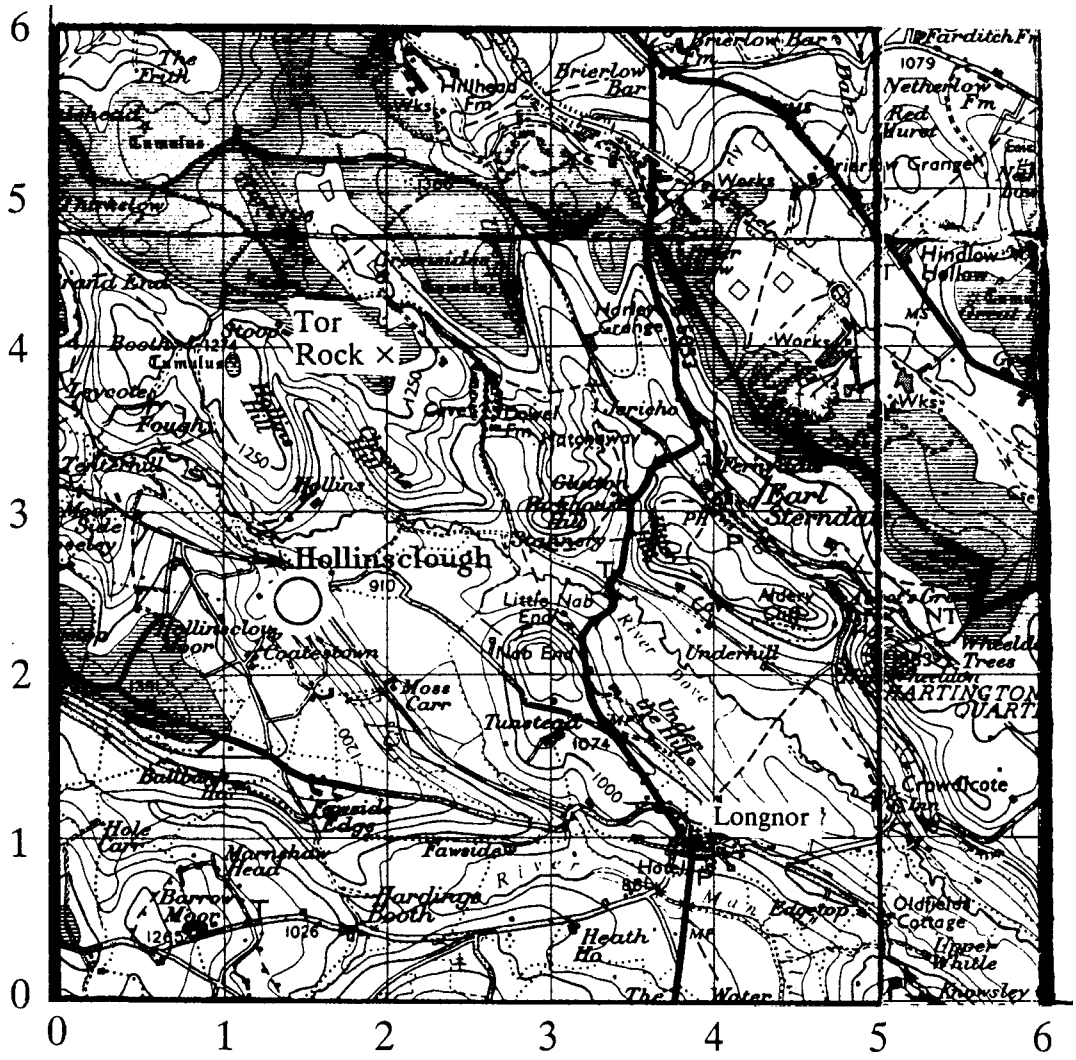
7. Use the formula

$$A = \frac{1}{4}c\sqrt{4a^2 - c^2}$$

   to calculate the value of *A* given that $c = 7.23$ and $a = 8.76$.

   Give your answer correct to 1 decimal place. (*3 marks*)
   (LON)

8. 1. Part of the Derbyshire Peak District is shown in the Ordnance Survey map below.



Test 1.3 Revision

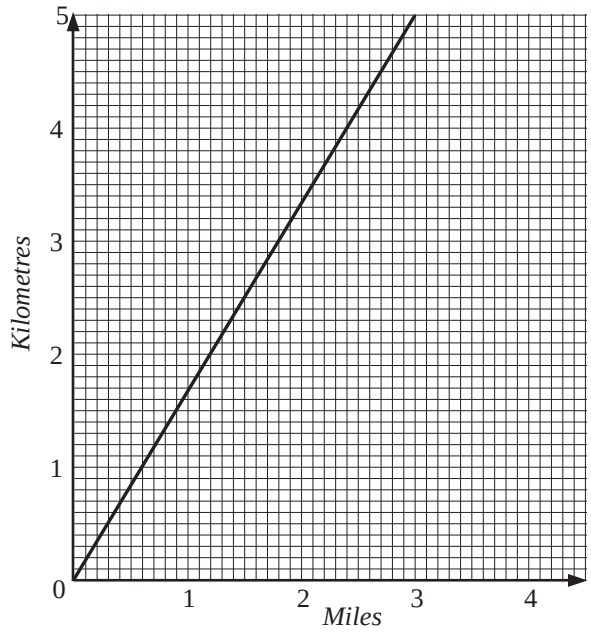
(a) Which village is at the point (4, 1)? *(1 mark)*

(b) Tor Rock is marked with a cross. Write down the co-ordinates of Tor Rock. *(1 mark)*

(c) Hollinsclough is marked with a circle. Estimate the co-ordinates of Hollinsclough. (*2 marks*)

(SEG)

9.



Use the graph to convert

(a) 2.5 miles to km. *(1 mark)*

(b) 3 km to miles. (*2 marks*)

(SEG)

10. When Edna was on holiday in Holland in 1991 the exchange rate was £1 = 3.25 guilders.

(a) What was the equivalent cost, in £s, of a pair of clogs which Edna bought for 65 guilders? *(2 marks)*

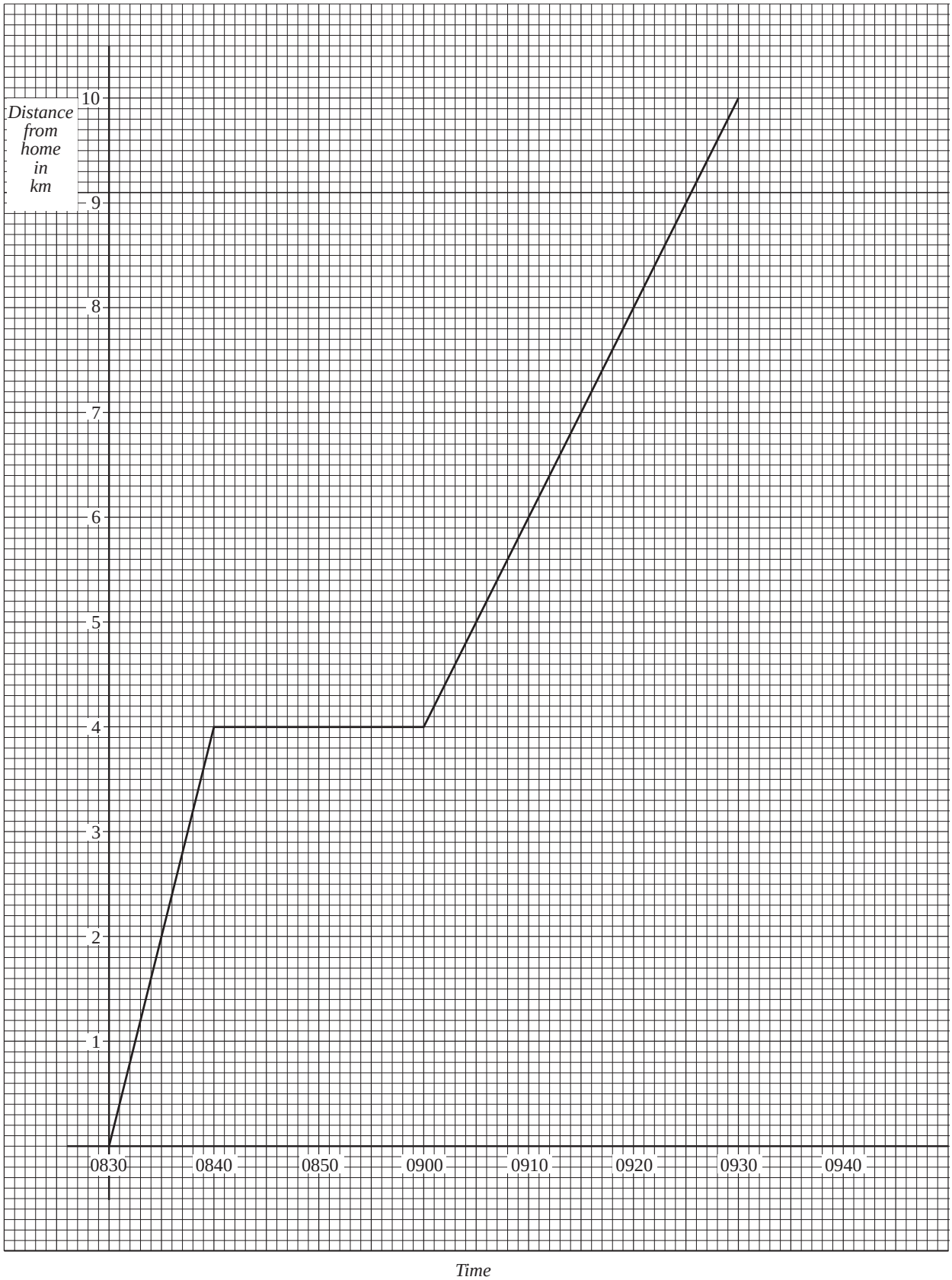
(b) (i) On a copy of the graph on the next page, draw the conversion graph to illustrate this exchange rate. (*2 marks*)

(ii) By drawing a line on your graph change 70 guilders into £. *(2 marks)*

(SEG)

Test 1.3 Revision

11. The graph represents the journey made by a lecturer travelling from home to college on a Monday morning.



Test 1.3 Revision

(a) Use the graph to find

(i) the distance travelled by the lecturer, *(1 mark)*

(ii) the time taken for his journey. *(1 mark)*

(b) Explain what might have happened between 0840 and 0900 on this journey, *(2 marks)*

(c) On Tuesday he left home at 0840 and travelled at a constant speed to arrive at college at 0910.

(i) On a copy of the graph draw a line to represent his journey on Tuesday. *(2 marks)*

(ii) Calculate in kilometres per hour the average speed for his journey on Tuesday. *(3 marks)*

(SEG)

Tests 1.1 and 1.2 Answers

Test 1.1

1. 8
2. 49
3. 6
4. 4
5. 10
6. 1, 2, 4, 5, 10, 20
7. $5^2 \times 2^2$ or $5 \times 5 \times 2 \times 2$
8. 60
9. $p = 4x + 6$
10. 20 °C

Test 1.2

1. 81
2. 25
3. 8
4. 2
5. 20
6. 1, 2, 3, 4, 6, 9, 12, 18, 36
7. $2^3 \times 3 \times 5$ or $2 \times 2 \times 2 \times 3 \times 5$
8. 125
9. $A = x(x + 2)$ or $A = x^2 + 2x$
10. 86 °F

Test 1.3 Answers

1. (a) 9 (b) 64 (c) 8
   (d) 5 (e) 3 (f) 1
   (g) 5 (h) 20 (i) 0 B1 for each *(9 marks)*

2. 1, 2, 3, 6, 7, 14, 21, 28 [B1 for all correct except 1 and 42] B2 *(2 marks)*

3. (a) 30 (b) 25 (c) 5 B1 B1 B1 *(3 marks)*

4. (a) multiple (b) square (c) factor B1 B1 B1 *(3 marks)*

5. (a) 51 (b) 47 (c) $2^2 \times 3 \times 5$ B1 B1 B2 *(4 marks)*

6. $c = 25 + 3d$; (a) £55 (b) £145 B2 B1 B1 *(4 marks)*

7. $A = \frac{1}{4} \times 7.23 \sqrt{4 \times (8.76)^2 - (7.23)^2}$ M1

   $\approx \frac{1}{4} \times 7.23 \times 15.96 \approx 28.8$ A1 A1 *(3 marks)*

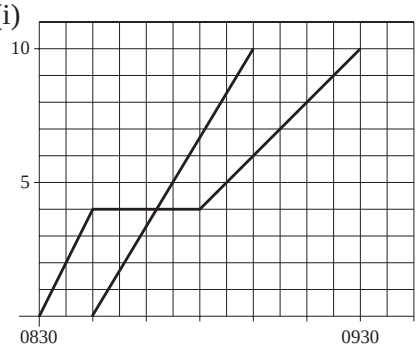
8. (a) Longnor (b) (2, 4) (c) (1.4, 2.4) (Allow (1.5, 2.5) B1 B1 B1 B1 *(4 marks)*

9. (a) 4 km (b) 1.9 miles B1 B2 *(3 marks)*

10. (a) £20 (b) (i) graph (ii) £21.5 B2 B2 M1 A1 *(6 marks)*

11. (a) (i) 10 km (ii) 60 mins (b) stopped (shopping?) (any reasonable explanation) B1 B1 B2

    (c) (i)



    (ii) 20 km/hour B2 M2 A1 *(9 marks)*

(TOTAL MARKS 50)