# UNIT 3 *Indices and Standard Form* Teaching Notes

*Historical Background and Introduction*

General use of decimal notation for whole numbers and decimal fractions dates from 1585 when *Simon Stevin* (1548–1620) published his book, *Die Thiende*. Stevin used powers of 10 to introduce place value and showed how the algebra of powers (the *index laws*) led to relatively simple ways of doing arithmetic. We write a number such as

three hundred and sixteen and a quarter

in terms of powers of 10 as

$$3\times 10^2 + 1\times 10^1 + 6\times 10^0 + 2\times 10^{-1} + 5\times 10^{-2}$$

and shorten this to 316.25.

Here $10^n = \underset{(n \text{ times})}{10\times 10\times \ldots \times 10}$, when $n>0$, $10^0 = 1$, and $10^{-n} = \dfrac{1}{10^n}$.

When we multiply 316.25 by 10 we use the index law,

$$10^n \times 10 = 10^{n+1}$$

(and the distributive law) to obtain the quick answer 3162.5.

The two basic *index laws*,

$$10^a \times 10^b = 10^{a+b} \quad \text{and} \quad \left(10^a\right)^b = 10^{ab},$$

can be easily checked from the definitions when *a* and *b* are positive integers. A little more thought is needed when *a* and/or *b* are negative integers (or fractions!) The definitions of $10^0\,(=1)$ and $10^{-n}\left(=\dfrac{1}{10^n}\right)$, and later of $10^{\frac{1}{2}}\left(=\sqrt{10}\right)$, are chosen to ensure that the basic index laws

$$x^a \times x^b = x^{a+b}, \quad \left(x^a\right)^b = x^{ab} \quad \text{and} \quad x^a . y^a = (xy)^a$$

remain true.

[Note: some care is needed when $x \le 0$, $0^a$ is not defined when $a \le 0$, and $x^a$ may have no meaning when $x<0$ and *a* is fractional.]

The index laws allow us to write very large numbers in a compact and manageable form. For example, the number of atoms in the universe is frighteningly large but elementary arguments show that this number is approximately $10^{50}$. Scientific notation provides an agreed way of giving in standard form the approximate value of very large numbers which occur in science, e.g.

$$2^{10} = 1024 = 1.024\times 10^3 \approx 1\times 10^3$$
$$2^{20} = 1\,048\,576 \approx 1.05\times 10^6 .$$

Writing numbers in this form makes it easy to do rough calculations. For example,

$$2^{40} = \left(2^{20}\right)^2 \approx \left(1.05\times 10^6\right)^2$$

$$\left(1.05\times 10^6\right)^2 = (1.05)^2 \times \left(10^6\right)^2 = 1.1025 \times 10^{12}$$

$$2^{-20} = \frac{1}{2^{20}} \approx \frac{1}{1.05\times 10^6} \approx 0.95\times 10^{-6}$$

$$0.95\times 10^{-6} = 9.5\times 10^{-7}$$

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In this unit we extend *index notation*, already encountered in Years 7 and 8, to non-positive integer values, e.g. $a^{-1}$, $a^{\frac{1}{2}}$, $a^{\frac{1}{4}}$, noting that the rules of indices still hold and we also introduce *scientific notation*, i.e. standard form, which many pupils will already have experienced with very large or very small numbers, when using their calculators.

| *Routes* | | **Standard** | **Academic** | **Express** |
|---|---|---|---|---|
| 3.1 | Index Notation | ✓ | ✓ | (✓) |
| 3.2 | Laws of Indices | (✓) | ✓ | ✓ |
| 3.3 | Negative Indices | ✕ | (✓) | ✓ |
| 3.4 | Standard Form | ✕ | (✓) | ✓ |
| 3.5 | Fractional Indices | ✕ | ✕ | ✓ |

| *Language* | **Standard** | **Academic** | **Express** |
|---|---|---|---|
| Negative indices | ✕ | (✓) | ✓ |
| Standard form (i.e. scientific notation, standard index notation) | ✕ | (✓) | ✓ |
| Fractional indices | ✕ | ✕ | ✓ |

*Misconceptions*

- pupils need to realise that $a^m \times a^n \neq a^{mn}$ (in general); this is easy to show with a simple example, e.g. $2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2)$

$$= 2^5 \neq 2^6$$

- there may well be problems with $a^{\frac{1}{2}} = \sqrt{a}$ and pupils might confuse $\left(a^n\right)^m$ with $a^n \times a^m$. For these later manipulations, always convince your students by writing out examples fully, e.g.

$$\begin{aligned} \left(a^4\right)^3 &= a^4 \times a^4 \times a^4 \\ &= (a \times a \times a \times a) \times (a \times a \times a \times a) \times (a \times a \times a \times a) \\ &= a^{12} \end{aligned}$$

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- confusion can arise with the inverse of a fractional number; for example:

$$\left(\frac{1}{10}\right)^{-2} \neq \frac{1}{100}$$

as $$\left(\frac{1}{10}\right)^{-2} = \left[\left(\frac{1}{10}\right)^{-1}\right]^{2} = (10)^{2} = 100$$

- $a^0 \neq 0$, but 1; this is often problematic, but you can help pupils to understand why we define $a^0 = 1$ by looking at the sequence, for example, $5^{\frac{1}{n}}$, as $n$ gets larger (on their calculators).

*Challenging Questions*

The following questions are more challenging than others in the same section:

| | *Section* | *Question No.* | *Page* |
|---|---|---|---|
| *Practice Book Y9A* | 3.1 | 11 | 40 |
| " " | 3.2 | 9 | 44 |
| " " | 3.3 | 10 | 49 |
| " " | 3.4 | 8, 11, 12 | 53,55 |
| " " | 3.5 | 7 | 57 |