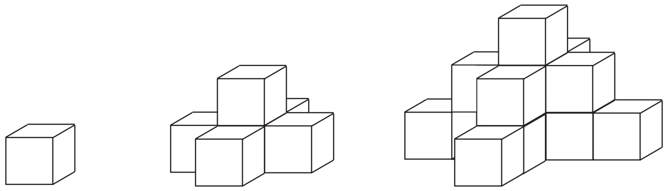
UNIT 3 *Indices and Standard Form* Activities

Activities

ACTIVITY 3.1 *Towers*



How many cubes are needed to build a tower which has 100 steps?

At first sight, this might seem daunting but we will see ways of tackling this kind of problem by trying to find the formula which fits the data.

1. Complete the following table:

No. of Steps	1	2	3	4	5	6
No. of Cubes	1	6				

From your table you can see that the number of cubes needed increases much faster than the number of steps - but how much faster?

2. Compute the value of n^2 for $n = 1, 2, \dots$, and also for $2n^2$ and $3n^2$ by completing the table below:

n	1	2	3	4	5	6
n^2	1	4	9	...	...	...
$2n^2$	2	8	...	...	...	...
$3n^2$	3	12	...	...	...	...

Which of the sequences given appears closest to the sequence found in question 1 ?

3. From questions 1 and 2, complete the table:

No. of Steps, n	1	2	3	4	5	6
No. of Cubes	1	6	...	...	...	...
$2n^2$	2	8				
No. of Cubes $-\ 2n^2$	-1	-2				

Deduce the formula for the number of cubes in the form

$$\text{number of cubes} = 2n^2 - kn$$

for some constant k.

4. Now solve the problem of finding the number of cubes needed for the 100-step tower.

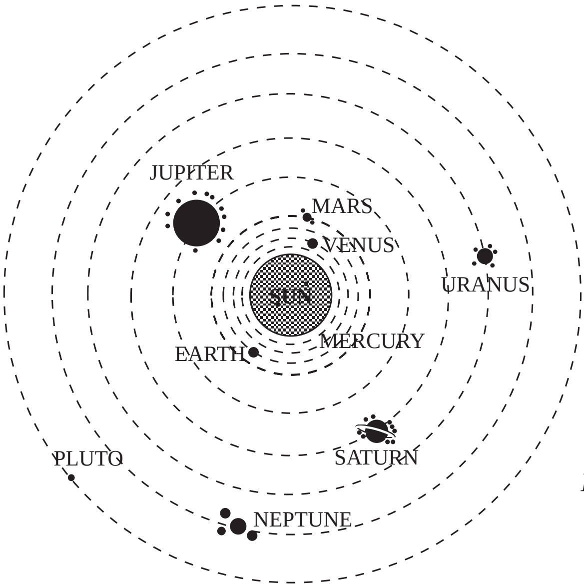
ACTIVITY 3.2 *Bode's Law*

In 1772, the German astronomer, *Johann Bode*, published his law which relates the distance ratio:

$$x_n = \frac{\text{distance of the planet from the Sun}}{\text{distance of the Earth from the Sun}}$$

to n, the number which Bode used to specify each planet, as shown in the following table.

$n = 1$	Venus
$n = 2$	Earth
$n = 3$	Mars
$n = 4$	
$n = 5$	Jupiter
$n = 6$	Saturn
$n = 7$	Uranus
$n = 8$	Neptune
$n = 9$	Pluto



Not to scale

He stated his law as:

$$x_n = 0.4 + 0.3 \times 2^{n-1}$$

1. Use this formula to determine $x_1, x_2, \ldots, x_9$.

2. Find the first and second differences of this sequence. What do you notice?

3. The actual distances are given in the table opposite. Find the actual values of

   $$x_1 \left(= \frac{108.2}{149.6}\right),\ x_2 \left(= \frac{149.6}{149.6}\right),\ x_3 \left(= \frac{227.9}{149.6}\right),\ x_4, \ldots \text{ up to } x_{10},$$

   and compare with predicted values from *Bode's Law*, ignoring the x_4 value.

4. A large number of asteroids are found at about 433.8×10^6 km from the Sun. Does *Bode's Law* provide confirmation that there was once a single planet at this distance from the sun? [*Hint*: consider x_4.]

5. Does the data support the view that *Neptune* and *Pluto* were once a single planet?

Planet	*Distance from Sun* (in millions of km)
Mercury	57.9
Venus	108.2
Earth	149.6
Mars	227.9
Jupiter	778.3
Saturn	1427.0
Uranus	2870.0
Neptune	4497.0
Pluto	5907.0

ACTIVITY 3.3 *Measuring and Standard Form*

We normally measure in units such as cm, m or km, as appropriate to a particular problem. Using different units can be a good opportunity to use standard form.

1. The distance between two towns is 108 km. Convert this distance to m, cm and mm, using standard form for your answers.

2. The area of a plot of land is 42 km^2. Convert this area to m^2, cm^2 and mm^2, using standard form for your answers.

3. The radius of a planet is 5000 km. Calculate the circumference of the planet in m, cm and mm.

4. Calculate the number of seconds in 1 year, giving your answer in standard form.

5. The volume of an adult human is estimated at 100 000 cm^3.
   Convert this volume to:

   (a) mm^3,

   (b) km^3,

   using standard form for your answers.

6. A spaceship travels at a speed of 2000 km/h. Convert this speed to mm/second.

ACTIVITY 3.4 *Standard Index Form*

The object of this game is to calculate numbers in standard index form to see which is the largest or smallest. It is suitable as a whole-class activity.

For each of the two numbers, *a* and *b*, given in standard form, determine which of

$a \times b$	$a \div b$	$b \div a$	$a + b$

is the largest number and which the smallest number.

For example:

A : $a = 4 \times 10^{3}$, $b = 2 \times 10^{-4}$

B : $a = 5 \times 10^{5}$, $b = 2 \times 10^{2}$

C : $a = 3 \times 10^{4}$, $b = 8 \times 10^{5}$

1. Working in pairs, one pupil chooses the values for *a* and *b* and sets the problem for their partner; the problem is then repeated the other way round.

2. Can you find any rules that determine which of the numbers is the largest?