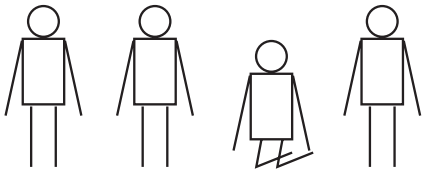
UNIT 1 *Base Arithmetic* Activities

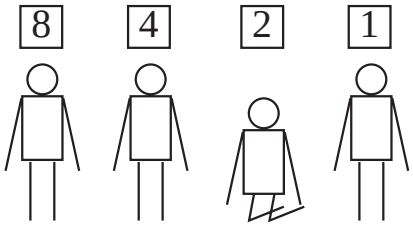
Activities

ACTIVITY 1.1 *Binary Digits*

This activity is based on the idea of using children to represent the digit 1 by *standing*, and 0 by *squatting*.

e.g.  This represents 1101.

It may also be useful to have cards available that can be held up to indicate place value.



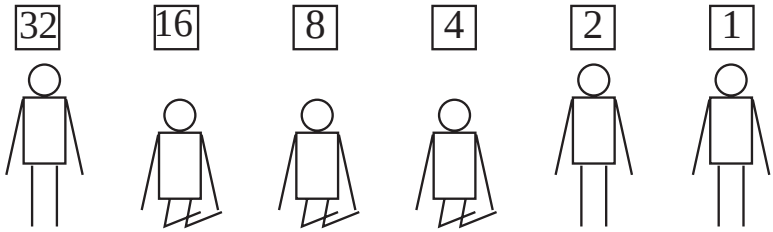
The activity can be used to introduce binary numbers and explain how the place value is used.

A team competition could then be carried out, with each team required to express a base 10 number in binary form.

For example,

"*Express 35 in base 2.*"

should give:



ACTIVITY 1.2 *ASCII Codes*

'ASCII' stands for 'American Standard Code for Information Interchange'.

An ASCII code is a code assigned to every keyboard character and is used extensively in computing. Each character has a code that is normally expressed as a 7-digit binary number.

For example, 'A' has an ASCII code of 1000001 in binary, 65 in base 10 (and 41 in hexadecimal). The table below gives the ASCII codes as binary numbers for all the capital letters.

ASCII System	
A	1000001
B	1000010
C	1000011
D	1000100
E	1000101
F	1000110
G	1000111
H	1001000
I	1001001
J	1001010
K	1001011
L	1001100
M	1001101
N	1001110
O	1001111
P	1010000
Q	1010001
R	1010010
S	1010011
T	1010100
U	1010101
V	1010110
W	1010111
X	1011000
Y	1011001
Z	1011010

1. Use the ASCII code to decode the following messages:

   (a) 1001000 1000101 1001100 1001100 1001111

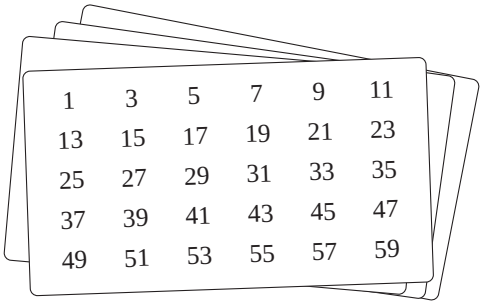
   (b) 1001001 1000011 1000001 1001110 1010111
   1001111 1010010 1001011 1001001 1010100
   1001111 1010101 1010100

2. Now code and decode a short message to a friend.

You can find a full list of ASCII codes at:
www.mindspring.com/~jc1/serial/Resources/ASCII.html

ACTIVITY 1.3 *Choose a Number*

You have probably seen this game in Christmas crackers. You ask someone to choose a whole number between 1 and 60. You then show them each card and ask them to say whether or not their chosen number is on the card. Miraculously you can tell them the number!



Construction

Writing down the page, list all the whole numbers, 1 to 60, and express each one as a base 2 number, as shown below.

The 1s in the first *column correspond to numbers on the* first *card.*

$1 = 2^0$	=	1
$2 = 2^1$	=	1 0
$3 = 2^1 + 2^0$	=	1 1
$4 = 2^2$	=	1 0 0
$5 = 2^2 + 2^0$	=	1 0 1
$6 = 2^2 + 2^1$	=	1 1 0
. . .	=	. . .
. . .	=	. . .
$60 = 2^5 + 2^4 + 2^3 + 2^2$	=	1 1 1 1 0 0

The 1s in the second *column correspond to numbers on the* second *card.*

- You will need 6 cards, each with a possible 6×5 array of numbers.
- Each card corresponds to a power of 2. The numbers on the card are those for which there is a 1 entry in the corresponding column, as shown opposite.
- So the first card has on it 1, 3, 5, 7, . . ., 59 and the second card has on it 2, 3, 6, 7, 10, 11, . . ., 59. The third card has 4, 5, 6, etc.
- *Complete all six cards.*

How it works

Ask someone to choose a number, and show them each card in turn. If the number is on the card, note the *power* of 2 on the card (this will be the smallest number).

For example, the number 51 will occur on the four cards corresponding to

$2^5 (= 32)$, $2^4 (= 16)$, $2^1 (= 2)$, $2^0 (= 1)$.

Add up these numbers to give the chosen number.

Why it works

Taking 51 as an example, we see that

$$51 = 32 + 16 + 2 + 1 = 2^5 + 2^4 + 2^1 + 2^0$$

and 51 will occur on the 1st, 2nd, 5th and 6th cards. Adding the powers of 2 uniquely gives the number.

Extension

Design cards to cope with the numbers 1 to 120.

How many cards would be needed to cope with numbers 1 to 1000 ? How many numbers would be on each card?

ACTIVITY 1.4 *Hexadecimal Numbers*

'Hexadecimal' numbers are numbers in base 16. In order to deal with hexadecimal numbers we require extra digits and so use the letters A to F.

Base 10	Hexadecimal
10	A
11	B
12	C
13	D
14	E
15	F

So the hexadecimal number 4AC is:

$$4 \times 16^2 + 10 \times 16 + 12 = 1196$$

Hexadecimal numbers are used in computing because each hexadecimal digit is a byte of 4 bits.

1. Convert the following base 10 numbers to hexadecimal:

   (a) 5 (b) 15 (c) 17

   (d) 47 (e) 69 (f) 149

2. Convert the following hexadecimal numbers to base 10:

   (a) AA (b) 2AF (c) BF3

   (d) 4AF2 (e) 36B9 (f) ABFD

3. Calculat, giving your answers in base 10:

   (a) $A + B$ (b) $E + 11$ (c) $F \times 2A$

   (d) $F - 9$ (e) $D2 \times F2$ (f) $26 \times 1C4$

ACTIVITY 1.5 *Roman Numerals*

The Romans developed a system of numbers based on the letters listed below:

I	1
V	5
X	10
L	50
C	100
D	500
M	1000

The values of the consecutive letters are *added* unless a letter with a lower value appears in front of a letter with a higher value. In this case, the lower value letter is *subtracted* from the higher value.

For example,

VII = 5 + 1 + 1 = 7

XXV = 10 + 10 + 5 = 25

IV = 5 – 1 = 4

IM = 1000 – 1 = 999

1. Convert the following Roman numerals to base 10:

(a) III (b) VIII (c) IX (d) CD

(e) DI (f) DVI (g) LIX (h) XXD

2. Convert the following base 10 numbers to Roman numerals:

(a) 14 (b) 6 (c) 27 (d) 56

(e) 84 (f) 109 (g) 1010 (h) 1499

Extension

You can perform calculations using Roman Numerals. Calculate:

(a) VIII + XXV (b) XIX – IV (c) LIX × V (d) CDLXXX ÷ XXIV

Design your own questions and use them with a friend.