

# UNIT 15 *Trigonometry*

# Teaching Notes

## *Historical Background and Introduction*

Very little is known about the life of *Pythagoras*, but he was born on the island of Samos and is credited with the founding of a community at Crotona in Southern Italy by about 530 BC. The community had religious and political purposes, but also dealt with mathematics, especially the properties of whole numbers or positive integers. Mystical attributes, such as that odd numbers were male and even numbers female, were ascribed to numbers. In addition, descriptions of arithmetical properties of integers were found.

The diagram on the right shows that

$$1 = 1^2$$

$$1 + 3 = 2^2$$

$$1 + 3 + 5 = 3^2$$

The Pythagoreans also formulated the idea of proportions in relation to harmonics on stringed instruments. The theorem with which Pythagoras' name is associated was probably only proved later. Specific instances of it were certainly known to the Babylonians. The 'Harpedonaptai', Egyptian rope stretchers, are said to have used the 3, 4, 5 triangle to obtain right angles from equally spaced knots on cords. The ancient Chinese also knew that the 3, 4, 5 triangle was right angled.

The Greeks used 'chord' tables rather than tables of trigonometric functions; the development of trigonometric tables took place around 500 AD, through the work of Hindu mathematicians. In fact, tables of sines for angles up to 90° were given for 24 equal intervals of  $3\frac{3}{4}^\circ$  each. The value of  $\sqrt{10}$  was used for  $\pi$  at that time. Further work a century later, particularly by the Indian mathematician *Brahmagupta* (in 628), led to the sine rule as we know it today.

A useful course book for the historical introduction of these topics is '*Ascent of Man*' by J. Bronowski (BBC publication).

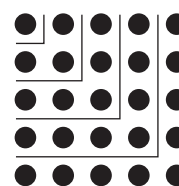
The main applications of trigonometry occur in surveying, geography and architecture, but it is also of crucial importance for navigation, warfare, etc. – although this is of course 3-dimensional rather than 2-dimensional. It is also a topic in which UK students do particularly well when compared to many other countries!

The key building blocks are

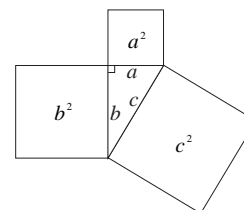
- Pythagoras' Theorem (met before)
- trigonometric relationships (introduced in this unit)
- sine and cosine rules (to come in Y10/11).



The 4th triangle number or 'Holy tetractys' had mystical significance for the Pythagoreans



The sum of consecutive odd numbers, starting at 1, is a square number



Pythagoras' Theorem:  
 $a^2 + b^2 = c^2$

# UNIT 15 Trigonometry

# Teaching Notes

### Routes

	Standard	Academic	Express
15.1 Pythagoras' Theorem	(✓)	✓	(✓)
15.2 Trigonometric Functions	×	✓	✓
15.3 Calculating Sides	×	✓	✓
15.4 Calculating Angles	×	✓	✓

### Language

	Standard	Academic	Express
Adjacent, opposite, hypotenuse ( <i>sides</i> )	×	✓	✓
sine (sin), cosine (cos), tangent (tan) ( <i>trigonometric functions</i> )	×	✓	✓
$\sin^{-1}$ , $\cos^{-1}$ , $\tan^{-1}$	×	✓	✓

### Misconceptions

- care must be taken when using calculators to find the values of trigonometric functions; different calculators have different logic, e.g. to find  $\tan 70^\circ$ , some calculators evaluate this as

$$\boxed{\tan} \quad \boxed{70^\circ}$$

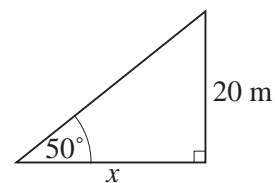
whilst others need

$$\boxed{70^\circ} \quad \boxed{\tan}$$

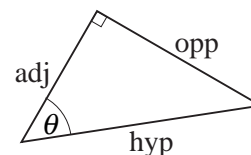
- care must be taken when finding sides; so, for example, from the diagram,

$$\tan 50^\circ = \frac{20}{x}$$

and  $x = \frac{20}{\tan 50^\circ}$  *not*  $20 \tan 50^\circ$



- identifying *adjacent* (adj), *opposite* (opp) and *hypotenuse* (hyp) in a triangle can cause problems (see diagram for correct identification).



### Challenging Questions

The following questions are more challenging than others in the same section:

	Section	Question No.	Page
Practice Book Y9B	15.1	11	152
" "	15.4	10	166
" "	15.4	13	167