# UNIT 8 *Algebra: Brackets* Activities

## Activities

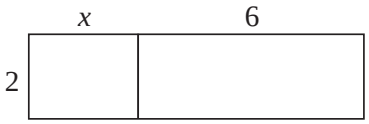
# ACTIVITY 8.1 *Expansions and Area*

You can check expansions of brackets by relating them to areas.

1. (a) Expand $2(x+6)$.

   (b) Calculate the area of each part of the rectangle 2 by $(x+6)$, and hence find the total area.

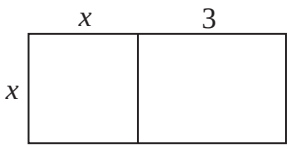
   (c) Compare your answers to (a) and (b).




2. (a) Expand $x(x+3)$.

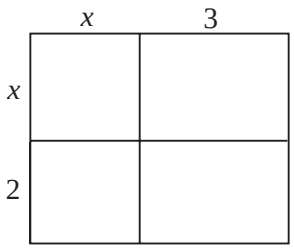
   (b) Calculate the area of each part of the rectangle $x$ by $(x+3)$, and hence find the total area.

   (c) Compare your answers to (a) and (b).




3. (a) Expand $(x+2)(x+3)$.

   (b) Calculate the area of each part of the rectangle $(x+2)$ by $(x+3)$, and hence find the total area.




4. Draw similar diagrams to illustrate the link between area and expansion for each of the following, and hence find the expansions:

   (a) $3(x+5)$ (b) $x(x+7)$

   (c) $(x+5)(x+1)$ (d) $(x+3)^2$

   (e) $(x+a)(x+b)$ (f) $(x+a)^2$

# ACTIVITY 8.2 *Using Algebra to Solve Magic Squares*

Magic squares have a property that, in each row, column and diagonal, the sum of the numbers is always equal to the *magic number* for that square.

1. Here is an example which happens to use 9 consecutive numbers.

   Check that the sum of the numbers in each row, column and diagonal is equal to the magic number, 12.

| | | |
|---|---|---|
| 3 | 2 | 7 |
| 8 | 4 | 0 |
| 1 | 6 | 5 |

*Solving magic squares*

| | | |
|---|---|---|
| | 11 | 7 |
| 9 | | |
| | 5 | 10 |

This magic square is more challenging! The answer may be found by trial and error but really a more systematic method is required.

Let $x$ be the unknown number in *column 1, row 1*;

$y$ be the unknown number in *column 1, row 3*;

$n$ be the *magic* number.

| | | |
|---|---|---|
| | 11 | 7 |
| 9 | | |
| | 5 | 10 |

Then for *row 1*, $n = x + 11 + 7 = x + 18$,

and for *column 1*, $n = x + 9 + y$.

So, $x + 18 = x + 9 + y$ (Subtracting $x$ from each side.)

$18 = 9 + y$ (Subtracting 9 from each side.)

$y = 9$.

From *row 3*, $n = y + 5 + 10$ so $n = 24$. From *row a*, $x + 18 = 24$, so $x = 6$.

The other two missing numbers can then be found to be 8 (*column 2*) and 7 (*column 3*).

2. Use an algebraic approach to solve the following magic squares:

(a)

| | | |
|---|---|---|
| 9 | 2 | |
| 12 | 8 | |
| | | |

(b)

| | | |
|---|---|---|
| 10 | 3 | |
| 5 | | 9 |
| | 11 | 4 |

(c)

| | | |
|---|---|---|
| 14 | | 12 |
| | | |
| 10 | | 8 |

# ACTIVITY 8.3 *Expansion Crazy!*

1. Show that $(x+1)^2 = x^2 + 2x + 1$ by completing the following table:

| × | $x$ | 1 |
|---|---|---|
| $x$ | | |
| 1 | | |

2. The expansion of $(x+1)^3$ can be found using $(x+1) \times (x+1)^2$ and the following table:

| × | $x^2$ | $2x$ | 1 |
|---|---|---|---|
| $x$ | | | |
| 1 | | | |

Complete the table and determine $(x+1)^3$.

3. Calculate the following:

(a) $(x+1)^4$ (b) $(x+1)^5$ (c) $(x+1)^7$

*Extension*

Look carefully at the numbers that multiply the powers of $x$, etc. in your expansions. Can you see how they could be obtained without actually expanding the expressions. Find the expression for $(x+1)^8$ without expanding. Check your answer.

# ACTIVITIES 8.1 - 8.2 *Notes for Solutions*

*Notes and solutions given only where appropriate.*

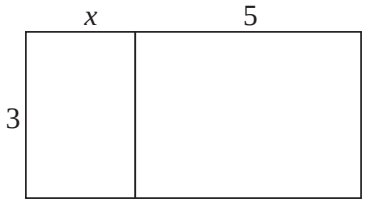
**8.1** 1. (a) $2x + 12$ (b) $2x$, 12, $2x + 12$ (c) Expansion gives total area.

2. (a) $x^2 + 3x$ (b) $x^2$, $3x$, $x^2 + 3x$ (c) Expansion gives total area.

3. (a) $x^2 + 5x + 6$ (b) $x^2$, $3x$, $2x$, 6, $x^2 + 5x + 6$

(c) Expansion gives total area.

4. (a)

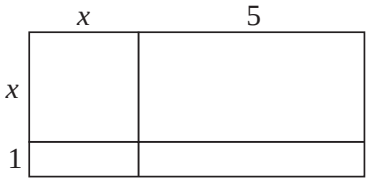



$3x + 15$

(b)

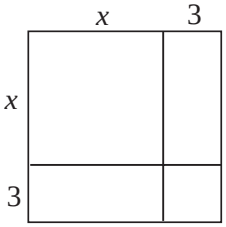

$x^2 + 7x$

(c)



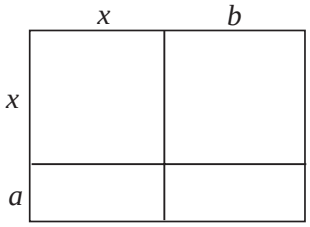

$x^2 + 6x + 5$

(d)



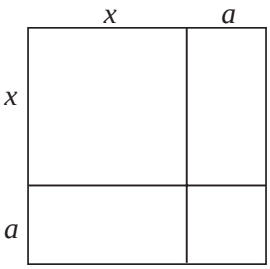

$x^2 + 6x + 9$

(e)




$x^2 + ax + bx + ab$

(f)




$x^2 + 2ax + a^2$

**8.2** 2. (a)

| | | |
|---|---|---|
| 9 | 2 | 13 |
| 12 | 8 | 4 |
| 3 | 14 | 7 |

(b)

| | | |
|---|---|---|
| 10 | 3 | 8 |
| 5 | 7 | 9 |
| 6 | 11 | 4 |

(c)

| | | |
|---|---|---|
| 14 | 7 | 12 |
| 9 | 11 | 13 |
| 10 | 15 | 8 |

# ACTIVITY 8.3 *Notes for Solutions*

**8.3** 2. $(x+1)^3 = x^3 + 3x^2 + 3x + 1$

3. (a) $(x+1)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1$

$(x+1)^5 = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$

$(x+1)^7 = x^7 + 7x^6 + 21x^5 + 35x^4 + 35x^3 + 21x^2 + 7x + 1$

*Extension*

Form Pascal's triangle:

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

Next row is:

1 8 28 56 70 56 28 8 1

so the expansion is

$(1+x)^8 = 1 + 8x + 28x^2 + 56x^3 + 70x^4 + 56x^5 + 28x^6 + 8x^7 + x^8$