UNIT 2 *Factors* Teaching Notes

Historical Background and Introduction

Prime numbers and their properties were first studied extensively by ancient Greek mathematicians. The Pythagorean school (c 400 BC), studied both prime and perfect numbers for their mystical properties (see Activity 2.2). When Euclid's 'Elements' appeared in about 300 BC, several important results concerning primes had been proved. Euclid proved (by contradiction) that there are infinitely many primes; he also gave a proof of the 'Fundamental Theorem of Arithmetic', which states that every integer can be expressed as a product of primes in an essentially unique way (this is assumed in section 2.2).

The Greek, Eratosthenes, in about 200 BC, designed his algorithm for finding prime numbers (dealt with here in Activity 2.1), and his method is still a quick and efficient way of finding small prime numbers.

After the Greek mathematicians had made these advances there was little development in this area until Fermat, at the beginning of the 17th century. He established many results, including:

(i) any prime number of the form $p = 4n + 1$ (n integer) can be expressed uniquely as the sum of two squares, e.g. $5 = 1^2 + 2^2$, $13 = 2^2 + 3^2$.

(ii) if p is a prime number, $2^p - 2$ is divisible by p (this is known as Fermat's Little Theorem), e.g. $2^5 - 2 = 30$ is divisible by 5.

Prime numbers are still the subject of considerable study, and today very large prime numbers are used in the design of codes. For example, the number $n = p_1 \times p_2$, where p_1 and p_2 are both very large primes, is immensely difficult to factorise. In America, in fact, there have been attempts to copyright some of the very large prime numbers in response to their important commercial use both now and in the future.

Routes		**Standard**	**Academic**	**Express**
2.1	Factors and Prime Numbers	✓	✓	✓
2.2	Prime Factors	✓	✓	✓
2.3	Index Notation	✓	✓	✓
2.4	Highest Common Factor and Lowest Common Multiple	(✓)	✓	✓
2.5	Squares and Square Roots	×	×	✓

(✓) *denotes extension work for these pupils*

Language	**Standard**	**Academic**	**Express**
Factors	✓	✓	✓
Prime Numbers	✓	✓	✓
Factor Tree	✓	✓	✓
Prime Factors	✓	✓	✓
Index Notation	✓	✓	✓
Highest Common Factor	(✓)	✓	✓
Lowest Common Multiple	(✓)	✓	✓
Square and Square Root	×	(✓)	✓

Misconceptions

- that 1 is a prime number: the definition of a prime number states that it has *two* factors, 1 and itself. The number 1 has only *one* factor so is *not* a prime number.
- that the factor tree is unique, whereas for most non-prime numbers, this is not true (see, for example, Section 2.2, Example 1 on page 30).

Challenging Questions

The following questions are more challenging than others in the same section:

	Section	*Question No.*	*Page*
Practice Book Y8A	2.1	9, 10	29
" "	2.2	11	33
" "	2.4	9, 10	41
" "	2.5	10	44