

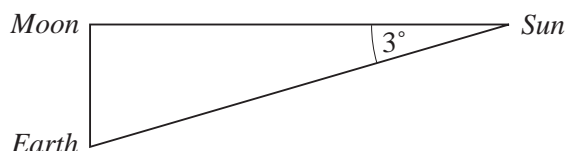
UNIT 5 *Angles*

Teaching Notes

Historical Background and Introduction

An important part of the mathematics syllabus over the next 5 years is geometry, and this first unit deals with angle measures.

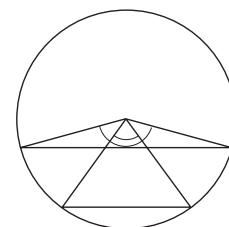
The origins of angle measures are not due to any one person but to a variety of developments in different countries. For example, *Aristarchus* (around 260 BC), in his treatise, *On the Sizes and Distances of the Sun and Moon*, made the observation that when the moon is half full, the angle between the lines of sight of the sun and the moon is less than a right angle by one-thirtieth of a quadrant (the systematic use of the 360° circle came a little later). In today's language, this gave the angle 3° in the diagram below.



In fact, the angle should have been about $0^\circ 10'$.

It is not known just when the systematic use of 360° was established, but it seems likely to have been largely due to *Hipparchus* (180-125 BC), who is thought to have produced the first trigonometric table.

It was firmly established by *Ptolemy* (c 100-178 AD), who used it consistently in his astronomical treatise. He noted that the ratio of arc to chord reduced as the angle subtended at the centre decreased, with a limit of 1 and he actually produced tables giving values for angles varying from 0° to 100° .



Although the use of 360° was adopted by most mathematicians, the idea of using 400° for a circle was developed in Scandinavian countries and is, in fact, still used on a limited basis. (It is even included on most calculators with the 'grad' mode for angles.)

Angle geometry is one topic in mathematics where 'proof' should be emphasised. So, for example, the *verification* that the sum of the angles in a triangle is 180° is not at all the same as a proof, and this should be made clear to pupils. In fact, the difference between

conjecture, proof and verification

should be emphasised, especially for pupils on the *Express Route*; brief dictionary definitions are:

- *conjecture* – an opinion formed without proof, based on slight or no evidence;
- *proof* – that which proves or establishes the truth of anything;
- *verification* – the process of ascertaining, confirming or testing truth or accuracy.

The unit also introduces compass directions; their origin is somewhat obscure and they have, at times, been attributed to the Chinese, Arabs, Greeks, Etruscans, Finns and Italians! What is clear, though, is that they were used, in a primitive way, as early as the 13th century, for navigation of ships, and that their use was widespread among seagoing nations by the 16th century. Even the earliest designs used 32 points (or subdivisions) on the compass, and sophisticated compasses were very much the order of the day by the early 18th century. The compass works on the principle that the earth has a magnetic field so that the needle of the compass indicates the direction of that field – magnetic North. Nowadays, ships (and planes) have sophisticated radar, but compass directions are still of great value to many, including walkers and hikers!

This is also the first unit of the course where accuracy of construction (angles, sides, etc.) is crucial. You must stress the importance of accurate construction and measurements – and ensure that pupils have the right equipment (ruler, protractor, pair of compasses, sharp pencil, etc.). *You* must show the same skills on the board, and you should definitely use board equipment to emphasise the accuracy needed.

Routes

| | Standard | Academic | Express |
|--------------------------------------------|-----------------|-----------------|----------------|
| 5.1 Angles and Turns | ✓ | ✓ | ✓ |
| 5.2 Measuring Angles | ✓ | ✓ | ✓ |
| 5.3 Classifying Angles | ✓ | ✓ | ✓ |
| 5.4 Angles on a Line and Angles at a Point | (✓) | ✓ | ✓ |
| 5.5 Constructing Triangles | ✓ | ✓ | ✓ |
| 5.4 Finding Angles in Triangles | × | (✓) | ✓ |

Language

| | | | |
|------------------------------------------------|---|-----|---|
| • Angles and turns | ✓ | ✓ | ✓ |
| • Compass directions | ✓ | ✓ | ✓ |
| • Clockwise and anticlockwise | ✓ | ✓ | ✓ |
| • Acute, obtuse, reflex and right angles | ✓ | ✓ | ✓ |
| • Isosceles, equilateral and scalene triangles | × | (✓) | ✓ |

(✓) denotes extension work for these pupils

Misconceptions

- It is important to always use the 'degrees' sign for angles to avoid confusion, e.g. 60° , not 60.
- Pupils must know that a right angle is *exactly* 90° (one quarter turn), and is therefore neither reflex nor acute.

Challenging Questions

The following questions are more challenging than others in the same section:

| | <i>Section</i> | <i>Question No.</i> | <i>Page</i> |
|--------------------------|----------------|---------------------|-------------|
| <i>Practice Book Y7A</i> | 5.2 | 6 | 73 |
| " " | 5.4 | 10 (c) | 80 |
| " " | 5.6 | 5 | 87 |
| " " | 5.6 | 11 | 89 |