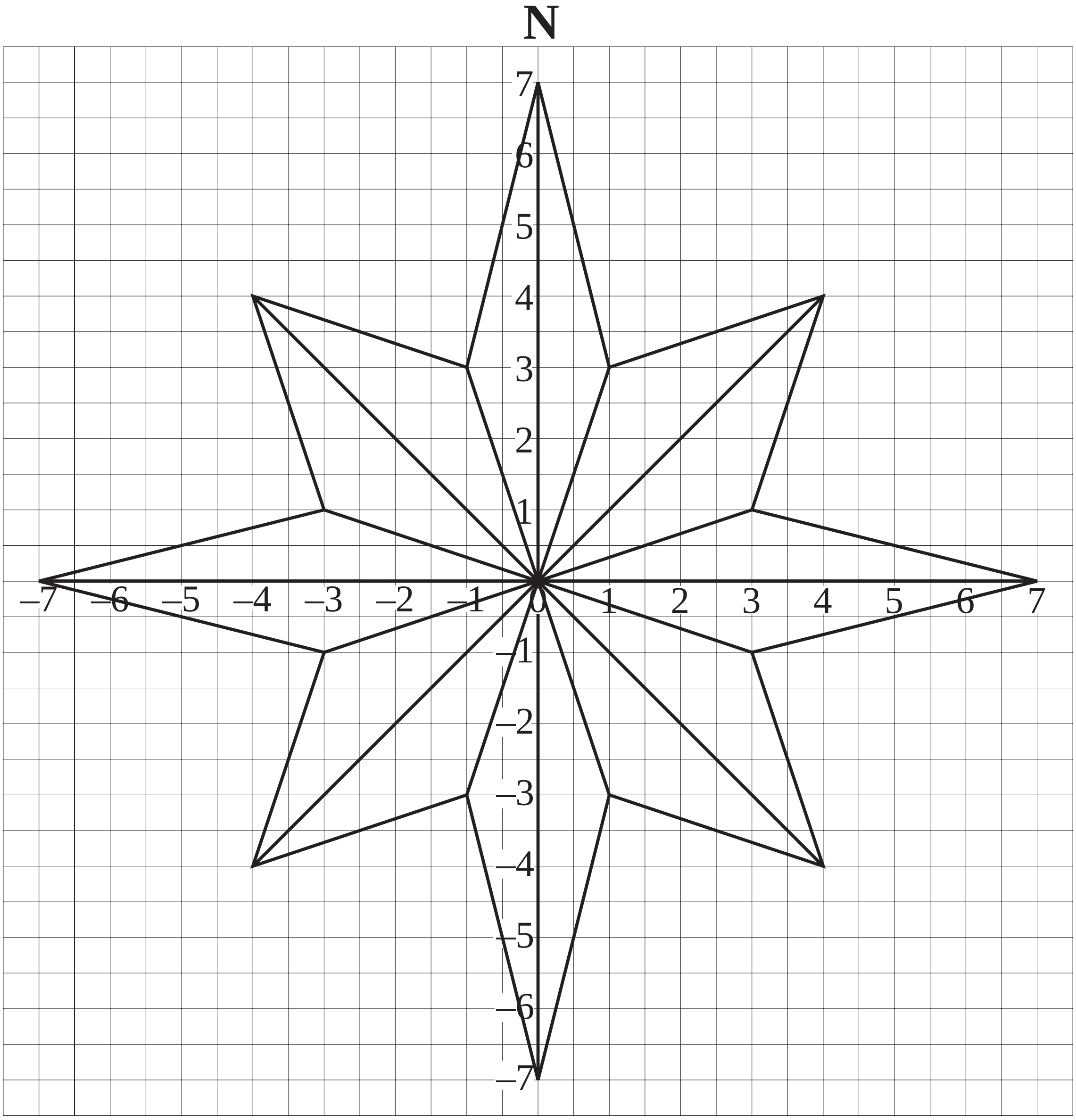
UNIT 5 *Angles* Activities

Activities

ACTIVITY 5.1 *Compass Rose Construction*

Here is a compass rose, symmetric about both the NS and the EW lines, so you can construct it using instructions for any one quarter.



1. Introducing coordinate axes, e.g. x in the E direction, and y in the N direction, as shown, what are the coordinates of the vertices of the rose in the positive quadrant?

2. Give a complete set of instructions for drawing all lines in the positive quadrant.

3. Give directions to now complete the diagram, using reflections.

Extension

Draw your own compass rose, giving a complete set of instructions for completing the drawing.

ACTIVITY 5.2 *Sam Loyd's Dissection*

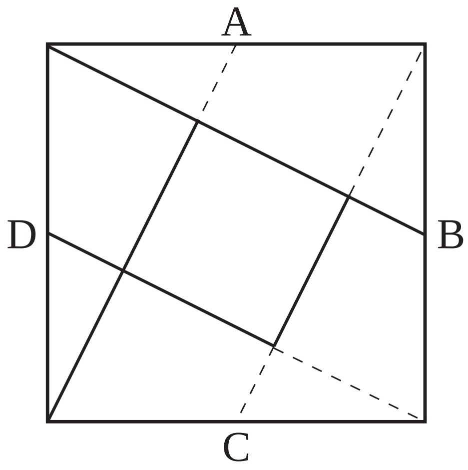
Sam Loyd's famous dissection problem was designed in the 1920s.

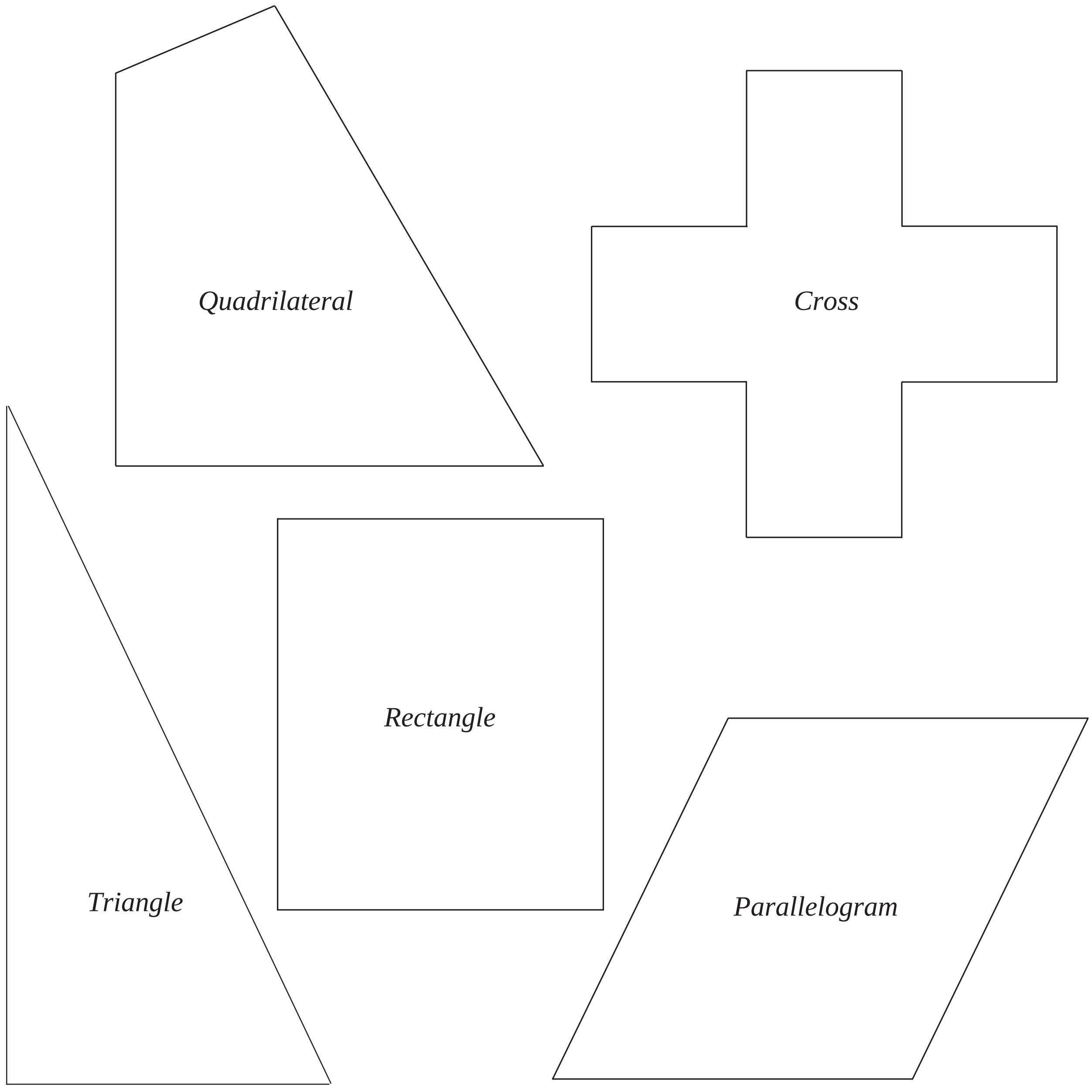
Draw a 5 cm square as shown on the right. Find the midpoints (A, B, C and D in diagram) on each side and join them up as indicated.

Using the diagram as a guide, cut your square into 5 pieces along the bold lines.

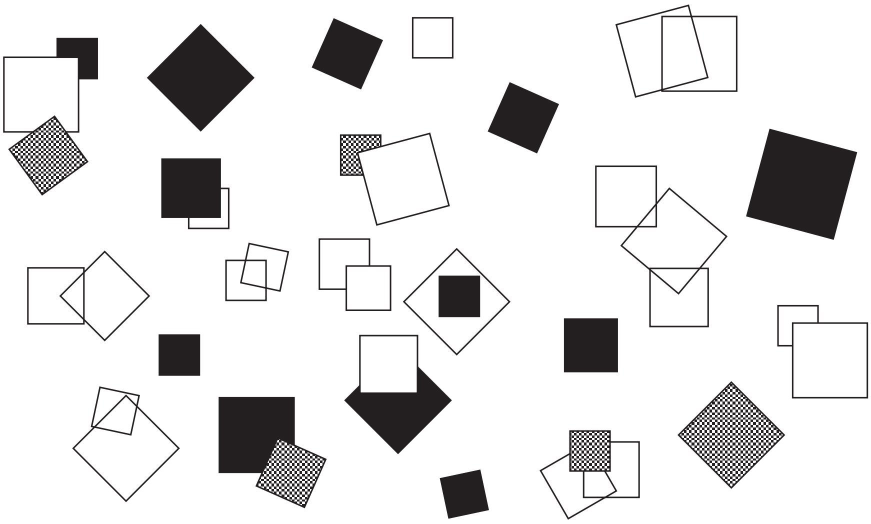
Do not cut along the dotted lines.

Use the 5 pieces to make all the shapes below.





ACTIVITY 5.3 *Overlapping Squares*



Take two squares and put them down on a surface so that they overlap. The squares can be of any size, not necessarily the same.

1. Which of the following shapes can be formed by the overlap:

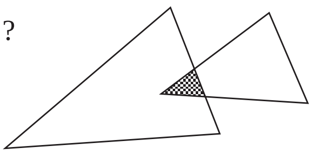
   (a) rectangle (b) square (c) kite (d) rhombus?

2. Can two squares intersect so that a triangle is formed by the overlap?

3. Can two squares intersect so that the overlap forms a polygon of *n* sides for values of *n* equal to

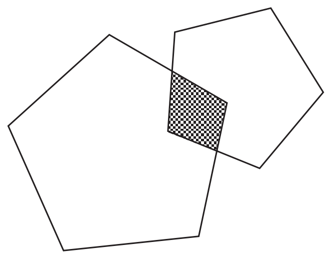
   (a) 5 (b) 6 (c) 7 (d) 8 (e) 9 (f) 10?

4. What happens when two triangles overlap?



Extensions

1. What happens when two pentagons overlap?



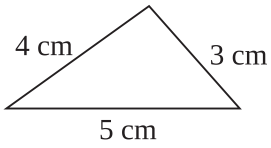
2. What happens when two *different* shapes, e.g. square and triangle, overlap?

ACTIVITY 5.4 *Constructing Triangles*

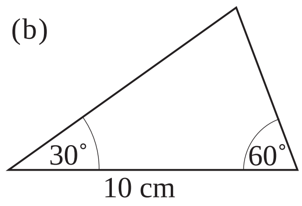
Constructing triangles, using a ruler and protractor, is straightforward when sufficient information is given. Sometimes not enough information is available: at other times you may be given too much information, some of which may be redundant (i.e. not needed).

1. Using the information on the sketches, draw accurately the following triangles.

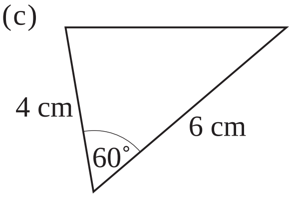
(a)



(b)



(c)

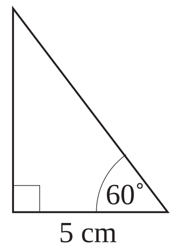


In each case, measure all other sides and angles.

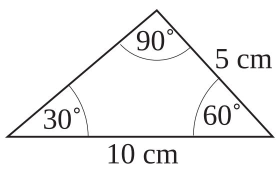
Each of these triangles is exactly defined with sufficient information (but not too much) to enable you to draw the triangle. We refer to these cases as 'SSS' (three side lengths given), 'ASA' (angle, side, angle) and 'SAS' (side, angle, side).

2. (a) Draw accurately the following triangle and then measure all other sides and angles.

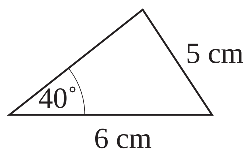
(b) Compare this triangle with triangle (b) in question 1. What do you notice?



So, if you are given the triangle opposite to draw you actually have more information than you need! You will have to decide which information to use.



3. Draw accurately the triangle sketched opposite. Be careful as there are *two* distinct possibilities!

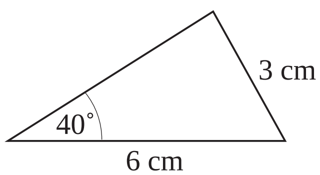


This is the 'ASS' case, and it does not necessarily have a unique (only one) solution. As you saw in question 3, there were two distinct triangles that agreed with the information given, i.e. there was insufficient information given for a unique solution.

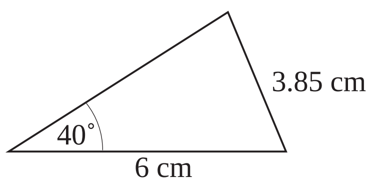
Extension

Can you construct either of the triangles below?

(a)

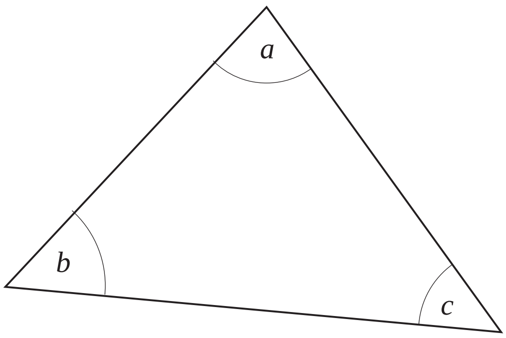
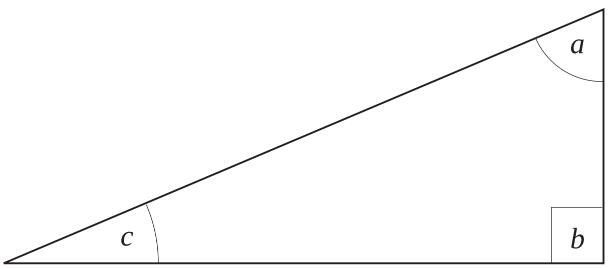
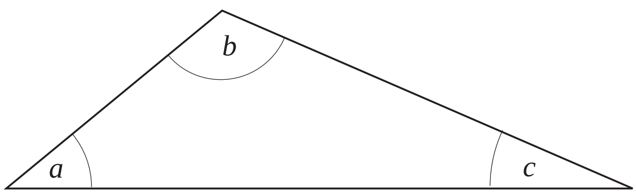


(b)



ACTIVITY 5.5 *Angles in Triangles*

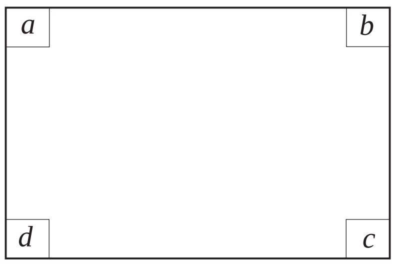
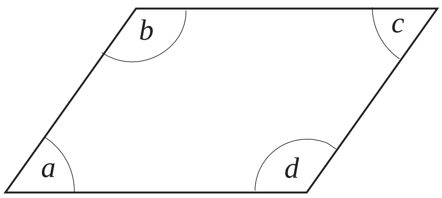
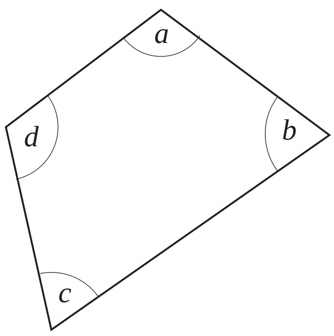
In each of the three triangles, measure all the angles as accurately as possible, and add up the values.

Triangle	*Angles*			
	a	b	c	$a + b + c$
				
				
				

What do you notice? Repeat the exercise with your own triangles.

ACTIVITY 5.6 *Angles in Quadrilaterals*

In each of the three quadrilaterals, measure all the angles as accurately as possible, and add up the values.

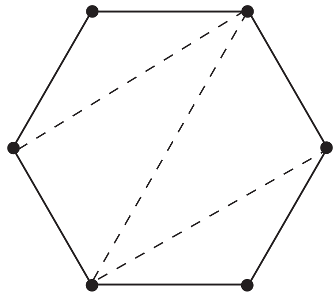
Quadrilateral	*Angles* a	b	c	d	$a + b + c + d$
					
					
					

What do you notice? Repeat the exercise with your own quadrilaterals.

Extension

Given that the interior angles of a triangle sum to $180°$, show that the interior angles of a quadrilateral sum to $360°$.

ACTIVITY 5.7 *Interior Angles in Polygons*



You can find the sum of the interior angles in any polygon, by dividing it up into triangles with lines connecting the vertices.

For example, the hexagon shown opposite has been divided into 4 internal triangles.

The sum of all the interior angles of the hexagon is equal to the sum of all the angles in each triangle, so:

$$\text{sum of interior angles} = 4 \times 180° = 720°$$

1. Repeat the same analysis for the following shapes:

   (a) quadrilateral (b) pentagon (c) heptagon

   (d) octagon (e) nonagon (f) dodecagon.

2. Copy and complete the table.

Name of Polygon	**Number of Sides**	**Number of Triangles**	**Sum of Interior Angles**
Triangle	3	1	180 °
Quadrilateral			
Pentagon			
Hexagon	6	4	720 °
Heptagon			
Octagon			
Nonagon			
Dodecagon			

Extension

What is the formula for the sum of the interior angles of a polygon with *n* sides?

ACTIVITIES 5.1 - 5.4 *Notes and Solutions*

Notes and solutions are given only where appropriate.

5.1 1. (7, 0), (3, 1), (4, 4), (1, 3), (0, 7)

2. Join (7, 0) to (3, 1) to (4, 4) to (1, 3) to (0, 7) and
(0, 0) to (7, 0) ; (0, 0) to (3, 1) ; (0, 0) to (4, 4) ;
(0, 0) to (1, 3) : and (0, 0) to (0, 7).

3. Reflect shape in x -axis, and then reflect new shape in y-axis (or the other way round).

5.3 1. (a) Yes (b) Yes (c) No (d) No

2. Yes

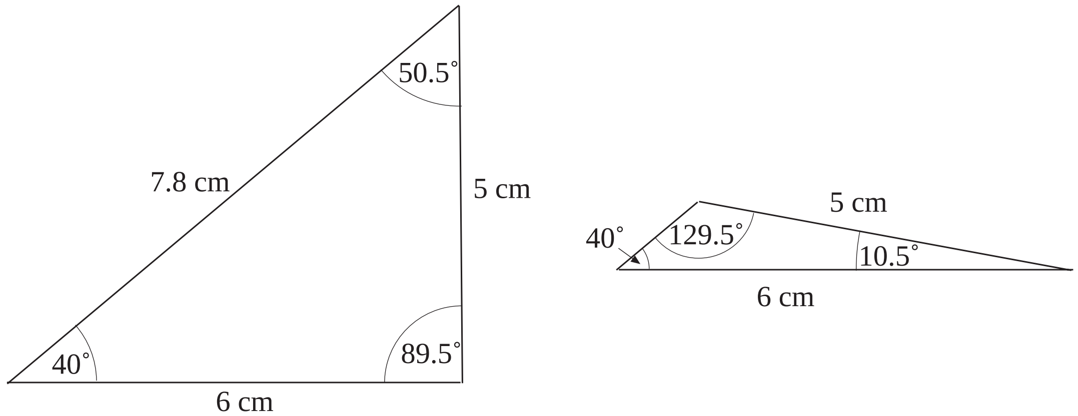
3. 5-8 : Yes : 9-10 : No

4. Overlap forms polygon of n sides with $n \le 6$.

5.4 1. (a) 90°, 37°, 53° (b) 90°, 5 cm, 8.7 cm (c) 5.3 cm, 79°, 41°

2. (a) 10 cm, 8.7 cm, 30° (b) identical triangles (congruent)

3. There are two distinct triangles as shown below.



Extension

(a) not possible

(b) unique answer (with angle of 90° at vertex)

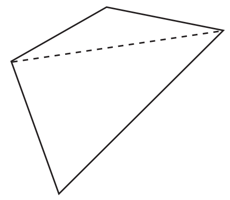
ACTIVITIES 5.5 - 5.7 *Notes and Solutions*

Notes and solutions are given only where appropriate.

5.5 angles add to $180°$ (but note that this is *not* a proof!)

5.6 angles add to $360°$ (but, again, note that this is *not* a proof!)

Extension
Any quadrilateral can be divided into two triangles.



5.7 For a polygon with n sides, the sum of the interior angles is $180(n-2)$; hence the values in the table should be $180°$, $360°$, $540°$, $720°$, $900°$, $1080°$, $1160°$, $1340°$.