

UNIT 14 *Time and Timetables*

Activities

Activities

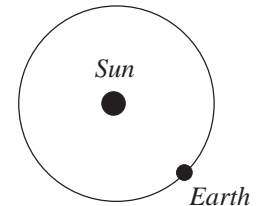
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ACTIVITY 14.1

Gregorian Calendar

When the Roman general and statesman, *Julius Caesar*, introduced his JULIAN calendar in 45 B.C., it was based on an average year-length of $365\frac{1}{4}$ days – there was a *leap year* (in which February had an extra day) every 4 years.

The actual length of a year is the time it takes for the earth to make one complete revolution of the sun; in days, this is 365.2422.



1. What is the difference between the actual time of revolution and the average length of a Julian year?

This difference is the ERROR that occurs each year.

2. After 1000 years, what is the error, in days, between the Julian calendar and the actual number of revolutions?
3. Why does this error matter?

By the sixteenth century, these errors had accumulated to about 12 days. The Pope at that time, Gregory X111, decreed that: 5 October 1582 should be followed by 15 October 1582; from then on, only century years that are divisible by 4 should be leap years (e.g. 16 is divisible by 4 so 1600 was a leap year).

4. Were the following years leap years:

1600, 1700, 1800, 1900

Is the year 2000 a leap year?

Extension

What is the average length of a Gregorian Year? What is the error introduced each year?

ACTIVITY 14.2

Days of the Week

Here is a method for finding the day of the week (i.e. Sunday, Monday, . . .), for any date in either this century, or the next century. The method is illustrated by finding the date of the first day of August 1999.

<i>Method</i>	<i>Example</i>
1. Put $Y = \text{year}$	$Y = 1999$
2. Work out the day of the year, D for the date, using $D = 1$ for 1 January, $D = 31$ for 31 January $D = 32$ for 1 February, etc.	$D = 31 + 28 + 31 + 30 + 31 + 30 + 31 + 1$ i.e. $D = 213$ (Note that there are 28 days in February, 1999)
3. Work out $\left[\frac{Y-1}{4} \right]$, and ignore the remainder.	$\frac{Y-1}{4} = \frac{1998}{4} = 499$ remainder 2 so we take $\left[\frac{Y-1}{4} \right] = 499$
4. $S = Y + D + \left[\frac{Y-1}{4} \right]$	$S = 1999 + 213 + 499$ $= 2711$
5. Work out $S \div 7$, and note the remainder.	$\frac{S}{7} = \frac{2711}{7} = 387$ remainder 2
6. The remainder gives the day of the week, using the key 0 = Friday, 1 = Saturday, 2 = Sunday, . . .	Remainder 2 gives SUNDAY as the day of the week for 1 August 1999.

- Use this method to work out on which day of the week you were born.
- Use this method to work out the day of the week for each of the following :
(a) 31 December 1999, (b) 1 January 2000, (c) 1 August 2000.
- Explain *why* the method works.

Extension

Extend this method so that it works for *any* days in *any* century whilst we are using the Gregorian calendar.

ACTIVITY 14.3

Easter Day

The problem of finding the date of Easter Day is not an easy one to solve. It depends on both the *Gregorian* calendar (in which there is a leap year every four years, except for century years where only one in every four is a leap year) and the *Jewish* calendar (which follows a solar cycle for years, a lunar cycle for months and has a 19-year cycle in which the third, sixth, eighth, eleventh, seventeenth and nineteenth years have a thirteenth month).

In the Bible (St. Luke, Chapter 22), we have a description of how Jesus ate the Passover with his disciples, before going to Gethsemane. The Last Supper, the Agony in the Garden and the Crucifixion were at the time of the full moon following 21 March (Spring Equinox). The hurry to remove the body from the cross before the beginning of the Jewish Sabbath (Saturday) fixes the Crucifixion on a Friday.

The Church has sought to preserve this sequence. In 1961, T. O'Beirne published in the journal the *New Scientist*, an algorithm called 'Ten Divisions to Easter' to give the date of Easter Day for any year. The algorithm is shown below:

<i>Algorithm to find the date of Easter Day for any year</i>				
<i>Step</i>	<i>Number</i>	<i>Divide by</i>	<i>Answer</i>	<i>Remainder (if needed)</i>
1.	$x = \text{year}$	100	$b =$	$c =$
2.	$5b + c$	19	-	$a =$
3.	$3(b + 25)$	4	$r =$	$s =$
4.	$8(b + 11)$	25	$t =$	-
5.	$19a + r - t$	30	-	$h =$
6.	$a + 11h$	319	$g =$	-
7.	$60(5 - s) + c$	4	$j =$	$k =$
8.	$2j - k - h + g$	7	-	$m =$
9.	$h - g + m + 110$	30	$n =$	$q =$
10.	$q + 5 - n$	32	-	$p =$

Where a dash (-) is shown, it means that the number is not needed.

Easter Day is the p th day of the n th month in the year x .

Source: *Maths Focus*

1. Calculate the date of Easter Day this year. Check that you are correct by looking in a diary or on a calendar.
2. Calculate the date of Easter Day for each of the next 10 years – what are the *earliest* and the *latest* dates found?

Extension

Write a computer program to calculate the date of Easter Day for the next 100 (or 200) years. Analyse the results.

ACTIVITY 14.4

A Tour of Scotland

You are on holiday in Scotland and are staying in Glasgow. You want to visit Edinburgh and Aviemore on the same day. Use the bus timetables provided to plan a journey that allows you to spend as long as possible in the two places that you want to visit.

You should produce a plan for the journey that indicates how long you will spend travelling and how long you will have for each visit. You should give as much information as possible about each stage of the journey.

EDINBURGH	St Andrew Square Bus Stn	–	0920	–	–	–	1320	–	1615	–	1725	–
GLASGOW	Buchanan Bus Station	0715	–	0920	1120	1320	–	1520	–	1700	–	1900
Stirling	Goosecroft Bus Station	0755	–	–	–	–	–	–	–	1740	–	1940
Perth <i>arr</i>	Leonard St Bus Station	0838	1025	1035	–	–	1425	–	1725	1823	–	2023
Perth <i>dep</i>	Leonard St Bus Station	0845	1040	1040	–	–	1430	–	1730	1830	–	2030
Pitlochry	Aldour Bus Stop	0921	1116	1116	–	–	1506	–	1806	1906	–	2106
Pitlochry	Fishers Hotel	0926	1121	1121	–	–	1511	–	1811	1911	–	2111
Aviemore	Shopping Centre	1041	1236	1236	1410	1610	1626	1810	1926	2026	–	2231
Aviemore	Burnside Bus Stop	1043	1238	1238	1412	1612	1628	1812	1928	2028	–	2233
INVERNESS	Farraline Park Bus Stop	1121	1316	1316	1455	1655	1706	1855	2006	2106	2045	2311

INVERNESS	Farraline Park Bus Stop	0635	–	0845	1045	1215	1315	1445	1445	1715	1715	1830
Aviemore	Burnside Bus Stop	0713	–	0923	1123	1253	1353	1523	1523	1753	1753	1908
Aviemore	Police Station Layby	0718	–	0928	1128	1258	1358	1528	1528	1800	1758	1913
Pitlochry	opp Fishers Hotel	0842	–	1042	1242	–	–	1642	1642	1914	1914	2027
Pitlochry	Aldour Bus Stop	0844	–	1044	1244	–	–	1644	1644	1916	1916	2029
Perth <i>arr</i>	Leonard St Bus Station	0920	–	1120	1320	–	–	1720	1720	1952	1952	2105
Perth <i>dep</i>	Leonard St Bus Station	0922	0930	1122	1330	–	–	1722	1730	2000	2000	2107
Stirling	Goosecroft Bus Station	1005	–	1205	–	–	–	1805	–	–	–	2150
GLASGOW	Buchanan Bus Station	1045	–	1245	–	–	1650	1845	–	2115	–	2230
EDINBURGH	St Andrew Square Bus Stn	–	1035	–	1435	1550	–	–	1840	–	2105	–

GLASGOW <i>dep</i>	Buchanan Bus Station	0630	0730	0800	0830	0900	0930	1000	1030	1100	1130	1200	1230
EDINBURGH	St Andrew Square Bus Station	0740	0840	0910	0940	1010	1040	1110	1140	1210	0124	1310	1340
GLASGOW <i>dep</i>	Buchanan Bus Station	1300	1330	1400	1430	1500	1530	1600	1630	1700	1730	1800	1830
EDINBURGH	St Andrew Square Bus Station	1410	1440	1510	1540	1610	1640	1710	1740	1810	1840	1910	1940
GLASGOW <i>dep</i>	Buchanan Bus Station	1900	1930	2000	2030	2100	2130	2200	2230	2300	0000	1900	
EDINBURGH	St Andrew Square Bus Station	2010	2040	2110	2140	2210	2240	2310	2340	0010	0110	2010	

EDINBURGH	St Andrew Square Bus Station	0730	0800	0830	0900	0930	1000	1030	1100	1130	1200	1230	1300
GLASGOW <i>arr</i>	Buchanan Bus Station	0840	0840	0940	1010	1040	1110	1140	1210	1240	0310	1340	1410
EDINBURGH	St Andrew Square Bus Station	1330	1400	1430	1500	1530	1600	1630	1700	1730	1800	1830	1900
GLASGOW <i>arr</i>	Buchanan Bus Station	1440	1510	1540	1610	1640	1710	1740	1810	1840	1910	1940	2010
EDINBURGH	St Andrew Square Bus Station	1930	2000	2030	2100	2130	2200	2230	2300	2330	0000	0030	
GLASGOW <i>arr</i>	Buchanan Bus Station	2040	2110	2140	2210	2240	2310	2340	0010	0040	0110	0140	

ACTIVITIES 14.1 - 14.2

Notes and Solutions

Notes and solutions are given only where appropriate.

14.1 1. 0.0078 day

2. 7.8 days

3. The seasons, in the northern or southern hemispheres will be getting out-of-step with the calendar.

4. 1600, 2000 - Yes (leap years)

1700, 1800, 1900 - No (not leap years)

(In fact, Britain did not immediately follow the decree from Rome, and had a leap year in 1700, eventually coming into line with Europe with 2 September 1752 followed by 14 September – with riots in the streets by those who thought that the state was stealing some of their days!)

Extension

There are 97 leap years in 400 years, so the average length is now

$$365 \frac{97}{400} \approx 365.2425 \text{ days}$$

giving an error of 0.0003 days, i.e. 3 days in 10 000 years.

14.2 2. (a) Friday (b) Saturday (c) Tuesday

3. There are 365 days in a year, which is 52 weeks and 1 day. So each day progresses by one the next year, except after a leap year, when it progresses by 2 days.

This method works as S is a counter, and it increases by 1 for each new year (Y increases by 1 and $\left[\frac{Y-1}{4} \right]$ goes up by one every four years after a leap year).

ACTIVITIES 14.3 - 14.4

Notes and Solutions

14.3 1. 4 April 1999

- 2. 2000 23 April
- 2001 15 April
- 2002 31 March
- 2003 20 April
- 2004 11 April
- 2005 27 March
- 2006 16 April
- 2007 8 April
- 2008 23 March
- 2009 12 April
- 2010 4 April
- 2011 24 April
- 2012 8 April

Earliest and latest dates will vary according to which years are relevant.

14.4 Variety of answers possible.