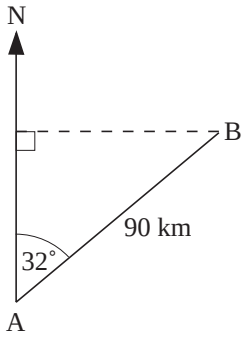
UNITS 4 – 6

Miscellaneous Exercises

1. A ship sails on a two stage journey from A to B to C.

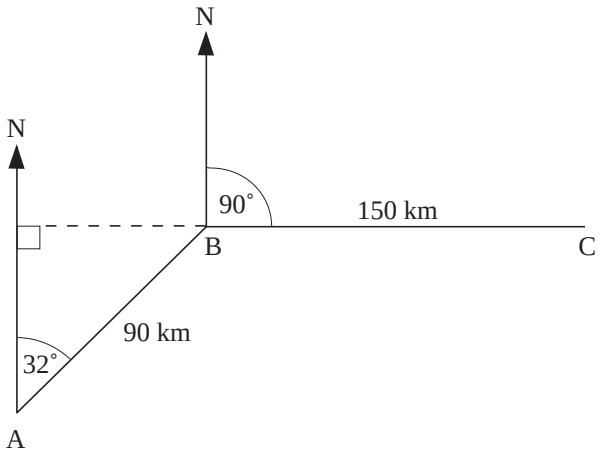


The first stage of the journey from A to B is shown.

A to B is a journey of 90 km on a bearing of $032°$.

(a) Calculate the distance travelled east during the first stage of this journey.

The second stage of the journey from B to C is a distance 150 km on a bearing of $090°$.



(b) Find the total distance travelled east on the journey from A to C.

Hence calculate the bearing of C from A.

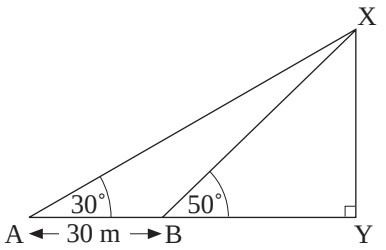
(SEG)

2. In the diagram, XY represents a vertical tower on level ground. A and B are points due west of Y. The distance AB is 30 metres.

The angle of elevation of X from A is $30°$.
The angle of elevation of X from B is $50°$.

Calculate the height, in metres, of the tower XY.

Give your answer correct to 2 decimal places.



(LON)

3. A TV mast stands on level ground. The Bearing of the foot of the TV mast from a point A is $040°$ and from a point B is $330°$. A is due west of B and the distance AB is 200 m.

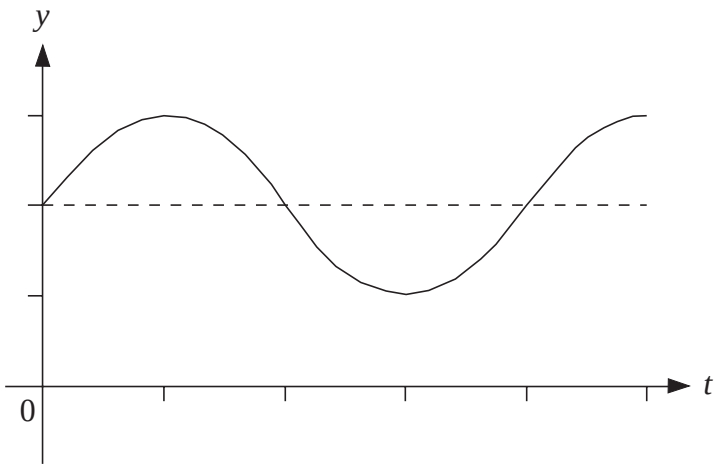
Find the perpendicular distance of the foot of the TV mast from the line joining A and B.

(SEG)

4. The depth of water in a harbour varies according to the formula

$$y = 10 + 5\sin(30t)°$$

{y is the depth of the water in feet; t is the time in hours.}
Here is a sketch of the graph formula.



(a) Complete the labelling on the t and y axes for this graph.

A ship wishes to leave the harbour, but needs a depth of water of 13 feet to do so safely.
When the time is 1200 hours the value of t *is* zero.

(b) At what time can the ship first leave the harbour safely?

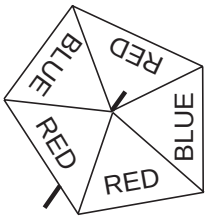
(LON)

5. Find two different values of x between 0 and 180 for which

$$\sin(2x)° = \sin 30°$$

(LON)

6. Alistair has a fair spinner with five equal sectors.
The spinner has three red sectors and two blue sectors.



(a) He spins it once.
What is the probability that it lands on red?

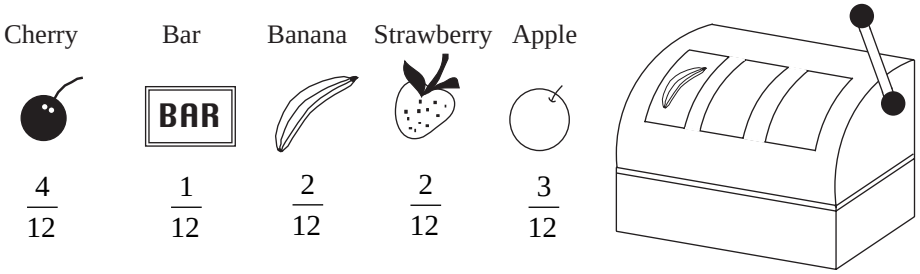
(b) He spins it twice.
List all the possible outcomes.

(c) The probability that both spins land on red is 0.36.
What is the probability that both spins do *not* land on red?

(SEG)

7. A game in an amusement arcade can show the following pictures.

The fraction under each picture shows the probability of the picture being shown at the first window.



Calculate the probability of the game

(a) *not* showing a Bar at the first window,

(b) showing a cherry or an apple at the first window.

(LON)

8. Peter and Asif are both taking their driving test for a motor cycle for the first time.

The table below gives the probabilities that they will pass the test at the first attempt, or if they fail the first time, the probability that they will pass at the next attempt.

	Probability of passing at first attempt	Probability of passing at next attempt if they fail the first attempt
Peter	0.6	0.8
Asif	0.7	0.7

On a particular day 1000 people will take the test for the first time.

For each person the probability that they will pass the test at the first attempt is the same as the probability that Asif will pass the test at the first attempt.

(a) Work out an estimate for how many of these 1000 people are likely to pass the test at the first attempt.

(b) Calculate the probability that both Peter and Asif will pass the test at the first attempt.

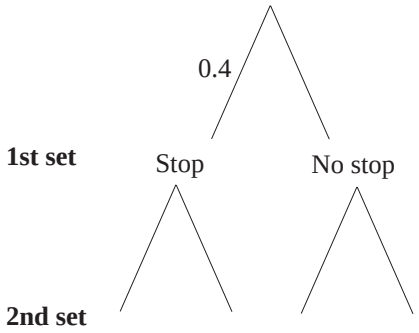
(c) Calculate the probability that Peter will pass the test at the first attempt and Asif will fail the test at the first attempt.

(d) Calculate the probability that Asif will pass the test within the first two attempts.

(LON)

9. Sarah cycles to college each day. She passes through three sets of traffic lights.

The probability of her stopping at the first set is 0.4 and at the second set is 0.3.



(a) By copying and completing the tree diagram above,

(i) find the probability of her stopping at *both* of the first two sets of lights;

(ii) find the probability of her stopping at only *one* of the first two sets of lights.

The probability of stopping at the third set depends on whether she has stopped at the second set of lights. If she stops at the second set of lights, the probability of her stopping at the third set is 0.4. If she does not stop at the second set of lights, the probability of her stopping at the third set is 0.2.

(b) (i) Find the probability that Sarah does *not* stop at any sets of the lights.

(ii) Find the probability that Sarah stops at two or three of these sets of lights.

(SEG)

10. A school is running a lottery in which people buy tickets for £1 each. For each ticket, you choose two *different* numbers from

1, 2, 3, 4, 5 and 6.

When the tickets have been sold, the winning pair of numbers is announced. The order of the numbers on the ticket does not matter.

You win £5 if your ticket has both winning numbers on it, and you win £2 if it has just one winning number on it.

You have bought *one* ticket.

(a) (i) Find the probability of winning £5.

(ii) Find the probability of winning £2.

(b) Your expected winnings are

£5 × (probability of getting jackpot) + £2 × (probability of one correct) – £1.

Evaluate this expression.

(c) Suggest suitable changes to the value of the prizes so that the lottery is more profitable for the school.

(SEG)

11. In this question you *must* use your calculator and you *may* write down any stage in your calculation.

Evaluate $\dfrac{(23.4 + 35.6) \times 5.7}{200.3 \times (16.2 - 8.15)}$

(LON)

12. The diameter of an atom is 0.000 000 03 m.

(a) Write 0.000 000 03 in standard form.

Using the most powerful microscope, the smallest objects which can be seen have diameters which are *one hundredth* of the diameter of an atom.

(b) Calculate the diameter, in metres, of the smallest objects which can be seen using this microscope.

Give your answer in standard form.

(LON)

13. (a) Calculate the value of 2×5^9.

(b) (i) Calculate $\dfrac{28.3 + \sqrt{0.512}}{(18.9 - 2.75)^2}$

(ii) Paul gives his answer to (i) correct to 5 significant figures.

Give one reason why this is *not* an appropriate degree of accuracy.

(SEG)

14. (a) Use the formula $v = \sqrt{u^2 + 2as}$ to find the value of v when $u = 24$, $a = -9.8$ and $s = 10\dfrac{1}{4}$.

(b) *Without using a calculator*, use approximation to check that your answer to (a) is of the correct order of magnitude.

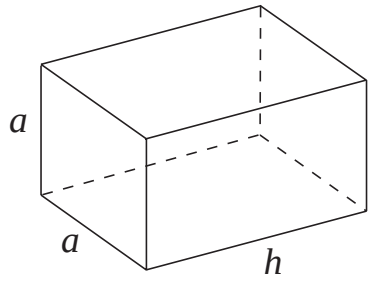
You *must* show all your working.

(SEG)

15. The winner of a 100 m running race is timed at 11.2 seconds by a hand-held stopwatch and 11.20 seconds by an electronic timing device. Explain, with appropriate working, why these timings are different.

(SEG)

16. The volume of a square-based block is given by $V = a^2 h$.



(a) In an experiment, the length a, and height, h, are measured as 4 cm and 11 cm respectively, each measured to the nearest cm.

What are the maximum and minimum possible vales of V in cm^3?

(b) In another experiment, the volume of the block is found to be 350 cm^3, measured to the nearest 50 cm^3, and its height is measured as 13.5 cm, to the nearest 0.5 cm.

(i) What are the maximum and minimum possible values of the length, a, in centimetres?

(ii) How many significant figures should be used to give a reliable answer for the value of a?

(SEG)

17. On the scales in Ali's book shop the weight of a book correct to 2 decimal places is 0.62 kg.

(a) Write down

(i) the lower bound of the weight of the book,

(ii) the upper bound of the weight of the book.

Ali needs to work out the weight of 50 copies of the book. He used his value for the weight of one book.

(b) Calculate

(i) the lower bound of the weight of 50 books,

(ii) the upper bound to the weight of 50 books.

(c) Calculate the greatest possible error that could occur in calculating the weight of 50 copies of the book.

(d) Write down the greatest possible error that could occur in calculating the weight of 500 copies of the book.

(LON)

18. Kim is doing an experiment using a pendulum. She uses the formula

$$g = \frac{40L}{T^2}$$

where g is the constant acceleration, L the length of the pendulum, and T is the time for one swing of the pendulum.

In Kim's experiment the length L is 1 metre, correct to the nearest centimetre.

She measured the value of T to be 2 seconds, correct to the nearest 0.2 of a second.

Calculate the upper bound and the lower bound of Kim's values for g.

Give your answers in metres per second correct to two decimal places.

(LON)

19. (a) Write down a number which is greater than 17 and less than 18 that has a rational square root.

(b) Give an example of two different irrational numbers c and d such that $c \times d$ is a rational number.

(LON)

20. (a) Write the following rational number as a fraction in its simplest form.

$$3.272\dot{7}\dot{2}\dot{7}$$

(b) Complete the table below for the sum, $x + y$, where *x and y* are any two numbers.

Use a tick (✔) if TRUE and a cross (✘) if FALSE.

$x + y$	**Always irrational**	**Always rational**	**Sometimes rational and sometimes irrational**
x rational, y rational	.....	.....	.....
x irrational, y rational	.....	.....	.....
x irrational, y irrational	.....	.....	.....

(SEG)

21. (a) A number x is always irrational.

Decide whether the following are

ALWAYS IRRATIONAL, ALWAYS RATIONAL, COULD BE RATIONAL OR IRRATIONAL

$$2x \quad , \quad \frac{1}{x}, \quad x^2, \quad \frac{10}{x} + x$$

(b) IRRATIONAL $\times$ IRRATIONAL = IRRATIONAL

(i) Give an example for which this statement is true.

(ii) Give an example for which this statement is *not* true.

(SEG)