2 Formulae

2.4 Substitution into Formulae

1. The speed of a bike, v metres per second, is given by the formula

$$v = u + ft$$

when u is its initial speed (in m/s), f its acceleration (in m/s^2) and t, the time in seconds.

Determine v when

(a) $u = 0,\ f = 5$ and $t = 10$ (b) $u = 20,\ f = 2$ and $t = 5$

(c) $u = 20,\ f = 0$ and $t = 5$ (d) $u = 40,\ f = -5$ and $t = 5$

(e) $u = 40,\ f = -5$ and $t = 8$

In each case, briefly describe the motion of the bike.

2. The Fahrenheit scale, (F) and the Celsius scale (C) are related by the formula

$$\mathrm{F} = \frac{9}{5}\mathrm{C} + 32$$

(a) Give the following temperatures in Fahrenheit.

(i) Normal body temperature: 37 °C

(ii) Boiling point of water: 100 °C

(b) Give the following temperatures in degrees Celsius.

(i) Freezing point of water: 32 °F

(ii) Singapore's average daily temperature: 86 °F

3. If $x = 3,\ y = 4$ and $z = 7$, find the values of the following expressions:

(a) $5yz$ (b) xyz (c) $\dfrac{12}{xy}$

(d) $\dfrac{xy}{18}$ (e) $2x + 3y$ (f) $x - 5y + 2z$

(g) $xy + yz$ (h) $x^2 + y^2$ (i) $2z^2 + y$

(j) $y^2 + x^3$ (k) xy^2 (l) $4x^2y^2$

4. If $a = 3,\ b = 2$ and $c = -1$, find the value of each of the following.

(a) $a^3 + b^3 + c^3 - 2abc$ (b) $(2a + b - c)(4b - 3c)$

(c) $(a - b)^2 - (b - c)^2$ (d) $\dfrac{a}{b} + \dfrac{b}{c} - \dfrac{c}{a}$

(e) $\dfrac{a+1}{2} - \dfrac{b+c}{4} + \dfrac{c-a}{3}$ (f) $a^b - c^a + b^a$

(g) $2a - 3b^2 + 3abc^2$ (h) $a^2 + 3b^3 - 4c^5$

(i) $\dfrac{a+b}{c} - \dfrac{ab-c}{b}$ (j) $\dfrac{3a-b}{b-c} + \dfrac{a+c}{b-a}$

(k) $\dfrac{2c^2-3a}{bc-a} - \dfrac{4b}{3a}$ (l) $\dfrac{a^2-b^2}{c^2} - \dfrac{a^3-c}{(c-3b)}$

5. Find the value of $x^3 + 2xy^2 + y^3$ when $x = 2$ and $y = -1$.

6. Find the value of $\dfrac{x+1}{x-1} + \dfrac{2x-1}{2x+1}$ when $x = -2$.

7. Find the value of $2ab + 3bc^2$ when $a = 0,\ b = 5$ and $c = -2$.

8. The distance travelled, *s* metres, by a car is given by

$$s = ut + \frac{1}{2} ft^2$$

Here *u* is the car's initial speed (in m/s), *t* the time (in seconds) and *f* the acceleration (in m/s^2).

(a) Find *s* when

(i) $u = 0,\ t = 10,\ f = 5$ (ii) $u = 20,\ t = 5,\ f = 6$

(iii) $u = 50,\ t = 4,\ f = -5$ (iv) $u = 60,\ t = 10,\ f = -2$

(b) If the car travels 400 metres in 5 seconds with initial speed of 40 m/s, what is its acceleration?

9. (a) Work out the value of $3a + ac$ when $a = 4$ and $c = -5$.

(b) Work out the value of $3p^2 - 5$ when $p = 2$.

(Edexcel)

10. Use the formula

$$v = u + at$$

to find the value of v when $u = -10,\ a = 1.8$ and $t = 3.7$.

(AQA)

11. A table has four columns: A, B, C and D. Part of the table is shown.

A	B	C	D
1	6	94	36
2		93	49
4	9		
7			144

(a) This is the 2nd row of the table.

A	B	C	D
2		93	49

The number in column B = the number in column A + 5

Work out the number in column B in this row.

(b) This is the the 3rd row of the table.

A	B	C	D
4	9		

(i) The number in column C = 100 – the number in column B

Work out the number in column C in this row.

(ii) The number in column D = the number in column B squared

Work out the number in column D in this row.

(c) This is the 4th row of the table.

A	B	C	D
7			144

Work out the number in column C in this row.

(AQA)

12. To cook roast lamb in a moderate oven, my recipe book gives these times.

78 minutes per kilogram, plus 35 minutes

(a) How long should I cook a 1.6 kg joint of lamb?
Give your answer in minutes to the nearest minute.

(b) Write your answer to part (a) in hours and minutes.

(c) I cooked one joint of lamb for 230 minutes. What was the weight of this joint?

(OCR)

2.5 More Complex Formulae

1. It is given that $v^2 = u^2 + 2as$. Find the values of u when $v = 0.8$, $a = 0.05$ and $s = 2.8$.

2. It is given that $y = \dfrac{18 - 5x}{2y}$. Find:

(a) the values of y if $x = -6.4$ (b) x if $y = 2\dfrac{1}{2}$

3. If $S = \frac{n}{2}\left[2a + (n-1)d\right]$, find

   (a) the value of S when $n = 10$, $a = -2$ and $d = \frac{1}{2}$

   (b) a when $S = 440$, $n = 10$ and $d = 5$.

4. The arithmetic mean, A, geometric mean, G, and harmonic mean, H, of three numbers, are given by the formulae

$$A = \frac{a+b+c}{3}, \quad G = (abc)^{\frac{1}{3}}, \quad H = \frac{3}{\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)}$$

   Find A, G and H for the following sets of numbers.

   (a) $a = 2$, $b = 3$, $c = 4$ (b) $a = 1$, $b = 3$, $c = 5$

   (c) $a = b = c = 3$ (d) $a = 2.5$, b = 3, $c = 3.5$

   What do you notice about the values of A, G and H?

5. Find z, given by each of the following formulae, for the given values of x and y.

   (a) $\frac{1}{z} = \sqrt{\frac{1}{x^2} + \frac{1}{y^2}}$, $x = 2$, $y = 3$

   (b) $z^2 = x^2 + y^2$, $x = 3$, $y = -4$

   (c) $\frac{1}{z} = \frac{x+y}{x^2+y^2}$, $x = -2$, $y = 4$

   (d) $\frac{x}{z} = \frac{x}{y} + 1$, $x = 5$, $y = 2$

   (e) $\frac{1}{z^2} = \frac{1}{x^2} + \frac{1}{y}$, $x = 4$, $y = 3$

6. The formula $v^2 = u^2 + 2fs$ connects the initial, (u), and final, (v), speeds of a car, with its acceleration, (f), and distance travelled, (s).

   Find v (in m/s) when

   (a) $u = 0$, $f = 10$ m/s^2, $s = 100$ m

   (b) $u = 20$ m/s, $f = 5$ m/s^2, $s = 50$ m

   (c) $u = 75$ m/s, $f = -10$ m/s^2, $s = 25$ m

7. A painting was valued at £600 on 1 January 2004. The value of the painting is predicted to increase at a rate of R% per annum.

   The predicted value, £V, of the painting after n years is given by the formula

$$V = 600 \times (1.055)^n.$$

   (a) Write down the value of R.

   (b) Use your calculator to find the predicted value of the painting after 15 years.

(Edexcel)

8. This rule can be used to work out the cost, in pounds, of buying time on a satellite link.

> Add 3 to the number of hours of time bought.
> Multiply your answer by 1000.

(a) Work out the cost of buying 4 hours of satellite time.

Julian bought some satellite time. The cost was £12 000.

(b) Work out the number of hours of satellite time that Julian bought.

The cost of buying n hours of satellite time is C pounds.

(c) Write down a formula for C in terms of n.

(Edexcel)

2.6 Changing the Subject

1. Make s the subject of each of the following:

(a) $2s - 8p = 14$ (b) $28 = 4s + r - s$ (c) $10 - 2s = 12r + 2s$

2. In each of the following, make y the subject:

(a) $y + x = 6$ (b) $m + y = 2 - n$ (c) $\frac{k}{5} = \frac{y}{3}$

(d) $3 + m = d + y$ (e) $5 = y - 3m$ (f) $2y + 6 = 48 + 2x$

3. (a) Given that $4a + b = c - a$, express a in terms of b and c.

(b) Given that $x - y = 3z$, express y in terms of x and z.

(c) Given that $pq = r$, express q in terms of p and r.

(d) Given that $a + b = 8c + 7$, express c in terms of a and b.

4. Make a the subject of the following formulae:

(a) $a + x = b$ (b) $a + h = k$ (c) $a - m = n$

(d) $a - k = h$ (e) $a - b = c + d$ (f) $a + c = d + e$

(g) $y + a = x$ (h) $z - a = 2k$ (i) $p = a - q$

(j) $5k = p - a$ (k) $7k = p + a$ (l) $a - b - c = k^2$

(m) $b - a + k = h^3$ (n) $m + n + a = k$ (o) $m - n - a = h$

(p) $7k - h - a = 2a$ (q) $5pq - a = p^2 - q$ (r) $3xy + a = x^2y$

(s) $5a = 15$ (t) $ax = 3y$ (u) $xay = 3k$

(v) $2xy = 3ak$ (w) $ak = p - q + k$ (x) $ax^2 = 5y - 4$

5. Make a the subject of the given formula.

(a) $ax = y$ (b) $a(p-4) = q$ (c) $ax + by = c$

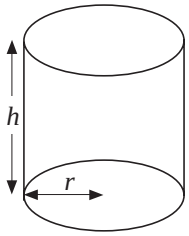
(d) $p(a+b) = c$ (e) $2a - 3m = 4a - 7$ (f) $5b - 2a = 3c$

(g) $\dfrac{a}{m} + b = c$ (h) $x = \dfrac{2a}{3} + 5z$ (i) $\dfrac{p+a}{5} = 3p$

(j) $R = m(a+g)$ (k) $2b = ax + a$ (l) $2m = 65 - 4a$

6. (a) The volume of a cylinder is given by

$$V = \pi r^2 h$$



(i) Make h the subject of this equation.

(ii) Find h when $r = 3$ cm and $V = 350$ cm^3.

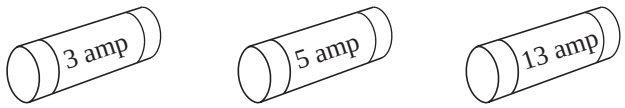
(b) The total surface area is given by

$$s = 2\pi r^2 + 2\pi rh$$

(i) Make h the subject of this equation.

(ii) Find h when $r = 3$ cm and $s = 300$ cm^2.

7. Electrical fuses are available as shown.



The correct fuse to use for an electrical appliance can be calculated using this formula,

$$F = \frac{P}{240}$$

where F = Fuse rating in amps,
P = Power rating in watts.

(a) Which fuse should be fitted for a toaster with power rating 1100 watts?

(b) An electric heater needs a 13 amp fuse. What is the largest power rating the heater could have?

(SEG)

8. The length of a man's forearm (f cm) and his height (h cm) are approximately related by the formula

$$h = 3f + 90$$

(a) Part of the skeleton of a man is found and the forearm is 20 cm long. Use the formula to estimate the man's height.

(b) A man's height is 162 cm. Use the formula to estimate the length of his forearm.

(c) George is 1 year old and he is 70 cm tall.
Find the value the formula gives for the length of his forearm and state why this value is impossible.

(d) Use the formula to find an expression for f in terms of h.

(MEG)

9. Make t the subject of the formula

$$D = 5t + \pi t + 5w$$

(Edexcel)

10. The cost, in pounds, to hire a conference centre is calculated by using this formula.

Cost = 4 × number of people attending + 250

(a) Find the cost of hiring the conference centre when 200 people attend.

(b) A company pays £650 to hire the conference centre.
How many people attend the conference?

(AQA)

11. This is the payment plan for Donal's mobile phone. He receives a bill every month.

(a) In January, Donal did not make any calls. How much was his bill?

(b) In February, Donal made 100 minutes of calls. How much was his bill?

(c) In March, Donal's bill was £7.50. How many minutes of calls did he make?

(AQA)

2.7 Further Change of Subject

1. The volume of a cylinder is given by

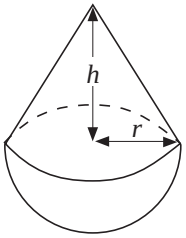
$$V = \pi r^2 h$$

where r is the base radius and h the height.

(a) Make r the subject of the formula.

(b) Find r when $V = 300 \text{ cm}^3$ and $h = 5 \text{ cm}$

2. The volume of a toy, consisting of a base hemisphere and cone top, is given by



$$V = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

Make h the subject of this equation and find h when $V = 300 \text{ cm}^3$ and $r = 3 \text{ cm}$.

3. The surface area of a sphere is given by

$$S = 4\pi r^2$$

(a) Make r the subject of this equation.

(b) Find r when (i) $S = 100 \text{ cm}^2$ (ii) $S = 200 \text{ cm}^2$

By what factor does the radius change when the surface area is doubled?

4. Make x the subject of

(a) $y = 4x + 2$ (b) $y = 1 - 3x$ (c) $y = mx + c$

(d) $y = \dfrac{1}{x+1}$ (e) $y = 1 + \sqrt{x}$ (f) $y = \dfrac{1}{1+\sqrt{x}}$

(g) $y = \sqrt{\dfrac{5x}{a}}$ (h) $y = \sqrt{x+1}$ (i) $\dfrac{1}{y} = \dfrac{1}{x} + 1$

(j) $\dfrac{1}{y} = \dfrac{2}{3} - \dfrac{1}{x}$ (k) $y = \dfrac{1}{4} + \dfrac{1}{x}$ (l) $y = \dfrac{4}{\sqrt{2+x}}$

5. If $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$, make u the subject of this formula. Find u when

(a) $f = 5$ and $v = 1$ (b) $f = 3$ and $v = -2$

6. The percentage profit, p, on the sale of an item is given by the formula

$$p = \frac{100(s-c)}{c}$$

where s is the selling price and c is the cost price. Express c in terms of s and p.

(MEG)

7. Students conduct an experiment to find g, the acceleration due to gravity.

They measure the time, T seconds, for one complete swing of a pendulum of length L centimetres.

The formula for g is

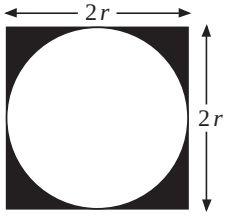
$$g = \frac{4\pi^2 L}{T^2}$$

(a) Find g when $L = 39.24$ and $T = 1.26$.
Take $\pi = 3.142$ or use the π button on your calculator.

(b) Rearrange the formula to express T in terms of L, π and g.

(SEG)

8. A star shape is made by cutting quadrants of a circle from a square of side $2r$.



(a) Show that the shaded area is given by the formula

$$A = 4r^2 - \pi r^2$$

(b) Rearrange the formula to make r the subject.

(AQA)

2.8 Expansion of Brackets

1. Copy and complete the following multiplication tables. Some have been done for you.

(a)

x	$y-2$	$3y$	$\frac{6}{y}$	$4-3y$
-1		$-3y$		
$\frac{1}{4}$				$1-\frac{3}{4}y$
2				
$-\frac{1}{2}$			$-\frac{3}{y}$	

(b)

x	$-k$	$\frac{2k}{3}$	$\frac{2}{3k}$	$2-2k$
6	$-6k$			
-4			$-\frac{8}{3k}$	
3				
$-\frac{1}{2}$				

2. Remove the brackets in each of the following algebraic expressions.

(a) $2(u-3)$ (b) $8(v+7)$

(c) $4(2x+3y)$ (d) $6(5a-b)$

(e) $-2(p-q)$ (f) $-5(a+b)$

(g) $-3(-2u-3v)$ (h) $8(-2u-3v)$

(i) $\frac{1}{2}(10p-6q)$ (j) $\frac{1}{5}(20x-15)$

(k) $-(b+c)$ (l) $-(p-q)$

(m) $-x(p+q)$ (n) $-y(-x+y)$

(o) $-(-p-q)$ (p) $-(-t+r)$

(q) $\frac{1}{2}\left(\frac{2}{3}a-\frac{4}{5}b\right)$ (r) $6a\left(\frac{1}{3}b-\frac{5}{6}c\right)$

3. Simplify each of the following algebraic expressions.

(a) $(3x-2y)+(4x-y)$ (b) $(p-m)+(m-2p)$

(c) $5(x-2)+3(4-x)$ (d) $(3a+2b)-(a-b)$

(e) $2(3m+n)-3(m-3n)$ (f) $(x-y)-(y-z)-(z-x)$

(g) $3a(b-c)+(3b-2)a$ (h) $m(m-n)-n(n-m)$

(i) $x(y-z)+y(z-x)+z(x-y)$ (j) $3(2y+5z)-4(2y-x)$

4. Multiply out and simplify each of the following expressions.

(a) $6(3x + y)$ (b) $5z(z - 2y)$

(c) $\frac{1}{2}(2xy - 4yz)$ (d) $q(p + 2r - 3s)$

(e) $(p + q)(r + s)$ (f) $(x + y)(z + 2w)$

(g) $(3a + b)(a + c)$ (h) $(m + 2n)(2p + 3q)$

(i) $(a - b)(c + d)$ (j) $(2e - f)(2g - h)$

(k) $(3p - 4q)(s + 2t)$ (l) $(a + 7)(2b + 5)$

(m) $(x + 3)(x + 4)$ (n) $(a + 5)(a - 3)$

(o) $(x - 7)(x - 6)$ (p) $(3 + c)(6 - c)$

(q) $(1 - 3x)(4 + 3x)$ (r) $(2p + 3)(p + 5)$

(s) $(4x + 5y)(2x + 3y)$ (t) $(d - 7)(d - 5)$

(u) $(a + 5)^2$ (v) $(x - 3)^2$

(w) $(b + 2)^2$ (x) $(e - 4)^2$

(y) $(2x + 1)^2$ (z) $(3x - 2)^2$

5. (a) Expand $p(p^2 - 3p)$. (b) Factorise $y^2 + 5y$.

(c) Factorise completely $2x^2 + 6xy$.

(Edexcel)

2.9 Factorisation

1. Factorise the following:

(a) $2x + 4$ (b) $9 - 3x$

(c) $2 + 10x$ (d) $-5 - 15x$

(e) $x^2 + 2x$ (f) $x - 3x^2$

(g) $4x + 2x^2$ (h) $3x^2 - 9x$

(i) $10x - 5x^2$ (j) $7x^2 + 21$

(k) $3x^2 - x^3$ (l) $2x + 8x^3$

(m) $2x^3 + 10x^2$ (n) $4x^2 - 4$

2. The following expressions have been partly factorised.

   Complete the factorisation.

   (a) $2x^2 - 4x = 2\left(x^2 - 2x\right) = \ ?$ (b) $10x - 5x^2 = x\left(10 - 5x\right) = \ ?$

   (c) $4x^3 + 8x = 4\left(x^3 + 2x\right) = \ ?$ (d) $8xy + 16x^2 = x\left(8y + 16x\right) = \ ?$

   (e) $5xy + 10x^2y^2 = 5\left(xy + 2x^2y^2\right) = \ ?$

3. Factorise the following:

   (a) $10a - 15b$ (b) $50py - 120p$

   (c) $24abc - 8ab$ (d) $6abc + 12bcd$

   (e) $16m^2 + 12n^2$ (f) $p^2y + p^2y^2$

   (g) $18s^2t - 12st^2$ (h) $10a + 15a^2$

   (i) $c - c^2$ (j) $2a^2b^2 - 8a^2b$

   (k) $m^2n - mnl$ (l) $6xy - 3y + 9x$

   (m) $pqr + p^2 + pr$ (n) $abc + a^2b + bc$

   (o) $8abc + 6ab^2c + 4abc^2$ (p) $5s^2t - 3st - 4st^2$

2.10 Algebraic Manipulation

1. Make a the subject of each of the following formulae.

   (a) $\dfrac{k(m + a)}{m} = \dfrac{4}{x}$ (b) $5(a - b) = 7$

   (c) $v = m(a + c)$ (d) $y = \dfrac{7ab + k}{7 - 4a}$

   (e) $z = \dfrac{5 - 2a}{3 - a}$ (f) $x = \dfrac{7 + 3a}{a - 4}$

   (g) $y = \dfrac{x^2a - b}{a - 4}$ (h) $z = \dfrac{4ab + 5c + 2}{2ax + 5y}$

2. Make the letters in brackets the subject of the following formulae.

   (a) $(x + p)a = q(2x - q)$ (x) (b) $\dfrac{an - 5x}{3a - 4x} = \dfrac{1}{3}$ (a)

   (c) $a = \dfrac{2b + c}{b}$ (b) (d) $\dfrac{y - 2x}{3y} = 2x - 7$ (x)

   (e) $T = \dfrac{4pr}{p + 4s}$ (p) (f) $\dfrac{1}{v} + \dfrac{2}{u} = \dfrac{3}{f}$ (u)

   (g) $x = \dfrac{y}{2 - y}$ (y) (h) $x = \dfrac{x + y - 2}{x - y + 1}$ (y)

(i) $w = \dfrac{a-b}{ac-1}$ (a) (j) $y = \dfrac{ax+b}{cx+d}$ (x)

(k) $\dfrac{1}{v} = \dfrac{u}{f} - 1$ (f) (l) $xy - 1 = 5(2x+3)$ (x)

(m) $\dfrac{F+40}{9} = \dfrac{c+40}{5}$ (c) (n) $P = \dfrac{ER}{k+R}$ (k)

(o) $k = \dfrac{2x-1}{x+4}$ (x) (p) $3h = k\left(\dfrac{x}{2} - y\right)$ (x)

(q) $P - mg = \dfrac{mv^2}{r}$ (m) (r) $c = \dfrac{nE}{k+na}$ (n)

(s) $\dfrac{3}{5} = \dfrac{y-4a}{y+7b}$ (y) (t) $\dfrac{a}{k} + h = \dfrac{b}{k}$ (k)

(u) $\dfrac{1}{a} + \dfrac{2}{b} = \dfrac{3}{c} + \dfrac{4}{d}$ (b)

3. Make a the subject of the following formulae.

(a) $\sqrt{a} = b$ (b) $\sqrt{2a} = b$ (c) $\sqrt{m+a} = b$

(d) $e = \sqrt{5a-8}$ (e) $\sqrt{\dfrac{a}{2}} = b$ (f) $l = \sqrt{\dfrac{k}{ma}}$

(g) $x = \sqrt{\dfrac{2a}{5c}}$ (h) $\sqrt{3a-2} = \sqrt{\dfrac{a}{b}}$ (i) $\sqrt{3a-2k} = z$

(j) $2a^2 = b - 3$ (k) $3a^2 - 2 = 3c$ (l) $k = ba^2 + z$

(m) $b = \sqrt{\dfrac{a^2}{5c}}$ (n) $m = n + \dfrac{na^2}{b}$ (o) $A = 4\pi a^2$

(p) $\sqrt[3]{a-b} = c$

4. Make the letter in brackets the subject of the formula.

(a) $a = \sqrt{a+2b}$ (b) (b) $a^2 + b^2 = c^2$ (b)

(c) $(x+y)^2 = x$ (y) (d) $e = \sqrt{3c-7a}$ (c)

(e) $x = 2w^2 + b$ (w) (f) $\sqrt[3]{y-1} = z$ (y)

(g) $\dfrac{a^2}{x^2} + \dfrac{b^2}{y^2} = 1$ (b) (h) $\sqrt[3]{2x^2-7} = \dfrac{y}{z}$ (x)

(i) $t = \sqrt{\dfrac{4x^2}{m-3}}$ (x) (j) $t^2 = \sqrt{\dfrac{m+2}{m-5}}$ (m)

(k) $\dfrac{1}{a} - \dfrac{1}{b} = \dfrac{1}{c-2}$ (c) (l) $y = \dfrac{nx}{a(4x-3)}$ (x)

5. Find the value of x by making x the subject of each of the following.

(a) $\dfrac{2}{5x} = \dfrac{4}{(x-1)}$ (b) $\dfrac{5}{x} + \dfrac{1}{4} = \dfrac{3}{7}$

(c) $\dfrac{2x}{2x+3} = 2$ (d) $\dfrac{x+2}{3} = \dfrac{2x-1}{14}$

(e) $\dfrac{3}{(x+1)} + \dfrac{1}{(2x+1)} = 0$ (f) $\dfrac{3x}{8} - \dfrac{x}{4} = \dfrac{1}{2}$

2.11 Algebraic Fractions

1. Simplify each expression into a single fraction.

(a) $\dfrac{x}{4} + \dfrac{x}{8}$ (b) $\dfrac{x}{6} - \dfrac{x}{12}$ (c) $\dfrac{x}{5} + \dfrac{x}{10}$

(d) $\dfrac{2x}{3} - \dfrac{x}{6}$ (e) $\dfrac{x}{2} - \dfrac{x}{8}$ (f) $\dfrac{4x}{7} - \dfrac{x}{9}$

(g) $\dfrac{5}{a} + \dfrac{10}{b}$ (h) $\dfrac{1}{2a} - \dfrac{1}{3b}$ (i) $\dfrac{3}{a} + \dfrac{2}{3b}$

2. Express the following as fractions with a single denominator.

(a) $\dfrac{x}{2} + \dfrac{x-1}{4}$ (b) $\dfrac{3y}{4} - \dfrac{y-1}{2}$

(c) $\dfrac{z}{2} - \dfrac{x+2}{3}$ (d) $\dfrac{1}{3x} - \dfrac{1}{3y}$

(e) $\dfrac{4}{ac} + \dfrac{2}{ab}$ (f) $3 - \dfrac{n-p}{m}$

(g) $\dfrac{4x^3y^2}{8xy^2} - \dfrac{x^2}{4}$ (h) $a + \dfrac{b}{ca} - \dfrac{c}{ab}$

(i) $\dfrac{3ab}{5x} - \dfrac{ab}{2x} - \dfrac{ab}{10x}$ (j) $\dfrac{2y+1}{5} - \dfrac{3y-2}{10} + \dfrac{y}{2}$

(k) $\dfrac{b+4}{6} - \dfrac{b}{3} + \dfrac{b-3}{12}$ (l) $\dfrac{a}{2} + \dfrac{a}{3} - \dfrac{3a}{8}$

(m) $\dfrac{c-1}{5} - \dfrac{2c+3}{3}$ (n) $\dfrac{2y-3}{3} + \dfrac{y-2}{4}$

(o) $\dfrac{a+1}{5} - \dfrac{a+1}{10} - \dfrac{a}{15}$ (p) $\dfrac{e-4}{5} + 1$

(q) $\dfrac{2}{3}(x-y) - \dfrac{3}{5}(x+y)$ (r) $\dfrac{1}{3x} + \dfrac{1}{5x}$

3. Simplify the following algebraic fractions.

(a) $\frac{x}{2}+\frac{x-3}{3}-\frac{x-4}{4}$ (b) $\frac{2x-y}{2}+\frac{x-y}{3}$

(c) $\frac{x+y}{2}-\frac{x+5y}{4}+\frac{5x-4y}{8}$ (d) $\frac{2x-3y}{5}-\frac{x-6y}{10}+\frac{5x+6y}{15}$

4. Express the following as fractions with a single denominator.

(a) $\frac{1+x}{3}+\frac{x-5}{6}$ (b) $\frac{1}{3x}-\frac{2}{5y}$

(c) $\frac{x+1}{2}-\frac{x-3}{3}$ (d) $\frac{2(x+y)}{x}+\frac{3(x-3y)}{5x}$

(e) $\frac{5}{x-y}-\frac{7}{y-x}$ (f) $\frac{1}{a}-\frac{2}{a+b}$

(g) $\frac{x}{a-b}-\frac{1}{b-a}$ (h) $3-\frac{x-2}{3x}$

5. Simplify:

(a) $\frac{c}{3}-\frac{c}{6}$ (b) $\frac{x}{10}+\frac{2x}{5}$

(c) $\frac{y}{2}+\frac{3}{4}$ (d) $\frac{2x}{3}+\frac{x-1}{5}$

(e) $\frac{2x+3}{4}-\frac{3x-2}{12}$ (f) $\frac{u-2}{3}+\frac{2u+3}{9}$

(g) $v+\frac{1}{v}$ (h) $2y+\frac{1}{y}$

(i) $m-\frac{2}{3m}$ (j) $\frac{1}{6x}+\frac{1}{3x}$

(k) $\frac{1}{2x}+\frac{3}{4x}$ (l) $\frac{3}{4y}-\frac{2}{y}$

(m) $\frac{1}{2}-\frac{1}{x-2}$ (n) $\frac{1}{m}+\frac{5}{mn}$

(o) $\frac{1}{p+q}+\frac{2}{3p+3q}$ (p) $\frac{8}{x+y}-\frac{3}{4x+4y}$

(q) $\frac{7}{x-y}+\frac{2}{5x-5y}$ (r) $\frac{5}{r-t}+\frac{1}{3r-3t}$

(s) $\frac{3}{x+1}-\frac{2}{x-1}$ (t) $\frac{5}{x-2}+\frac{7}{x+3}$

(u) $\dfrac{1}{x+2}+\dfrac{4}{x-1}$

(v) $\dfrac{2}{x-2}+\dfrac{6}{x-3}$

(w) $\dfrac{x}{4}+\dfrac{3(x-2)}{5}$

(x) $\dfrac{2y+3}{3}-\dfrac{2(y-1)}{7}$

6. Simplify fully

(a) $\left(2x^3y\right)^5$

(b) $\dfrac{x^2-4x}{x^2-6x+8}$

(Edexcel)