

# 15 Variation

## 15.5 Direct Proportion

1. Use the data given to check whether or not it agrees with the statement given.

(a)	$x$	1	3	7	11	$y \propto x$
	$y$	4	12	28	44	

(b)	$q$	0	1	2	3	$p \propto q$
	$p$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	

(c)	$x$	1	5	10	$y \propto x$
	$y$	0.1	0.5	1	

(d)	$t$	0.1	1	2	$s \propto t$
	$s$	0.5	5	10	

2. Copy and complete each table using the statement given.

(a)	$y \propto x$	$x$	1	4	9
		$y$	5	?	?

(b)	$s \propto t$	$t$	$\frac{1}{2}$	1	3	5
		$s$	?	2	?	?

(c)	$q \propto p$	$p$	1	3	5	9
		$q$	?	9	?	?

(d)	$y \propto x$	$x$	$\frac{1}{2}$	1	5
		$y$	1	?	?

3. The yield  $Y$  of a tomato crop is directly proportional to the quantity of fertiliser  $F$  used. 5 kg of fertiliser produces 30 kg of tomatoes.

- (a) Find the relationship between  $Y$  and  $F$ .
- (b) What is the yield when 12 kg of fertiliser are used?
- (c) How much fertiliser was used to produce a yield of 42 kg?

4. A spring stretches when a mass is attached to one end. The extension  $x$  is directly proportional to the magnitude of the mass,  $m$ . When a mass of 50 g is attached, the extension is 1 cm.

- (a) Find the relationship between  $x$  and  $m$ .
- (b) When a mass of 120 g is attached, what is the extension?
- (c) What mass will produce an extension of 3.2 cm?

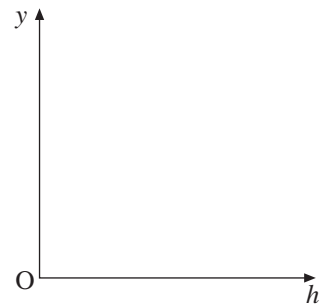
5. A launched rocket travels at constant acceleration.  
Its speed  $v \text{ ms}^{-1}$ , is proportional to the time  $t$ , in seconds, since launch.  
After 5 seconds its speed is  $120 \text{ ms}^{-1}$ .
- Find a relationship between  $v$  and  $t$
  - What is its speed when  $t$  equals
    - 1 second
    - 10 seconds
    - 60 seconds?
  - How long will it be before its speed is  $20\,000 \text{ ms}^{-1}$ ?

6. A ball is dropped to the floor from a height of  $h$  centimetres. It bounces to a height of  $y$  centimetres.  $y$  is directly proportional to  $h$ .

- Sketch a graph to show the relationship between  $y$  and  $h$ .

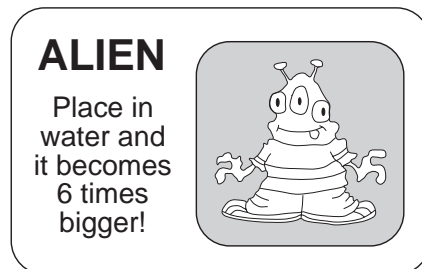
When  $h = 120$ ,  $y = 80$

- Find  $y$  when  $h = 150$ .



(LON)

7. (a) Explain why the volume of a cube increases by a factor of 8 when the side length is doubled.
- (b) June recently bought a small toy in the local shop.



It was originally 8 cm tall. After she placed it in water it grew to a similarly shaped alien. The height was then 14.5 cm.

Is the claim on the pack justified?

(AQA)

8. The distance,  $d$  km, it is possible to see on a clear day is proportional to the square root of the height,  $h$  m, above sea level.

Standing on a pier, 4 m above sea level, it is possible to see a distance of 10 km.

- Find a formula for  $d$  in terms of  $h$ .
- Standing on top of the cliffs I can see a distance of 35 km.  
How high are the cliffs?

(OCR)

9.  $y$  is directly proportional to the square of  $x$ . When  $y = 5$ ,  $x = 4$ .  
Find the value of  $y$  when  $x = 8$ .

(AQA)

# 15.6 Inverse Proportion

1. For each table of values below, determine whether they agree with the relationship stated.

(a)

$x$	1	2	4
$y$	12	6	3

 $y \propto \frac{1}{x}$ 

(b)

$q$	1	2	5
$p$	2	1	0.5

 $p \propto \frac{1}{q}$ 

(c)

$r$	$\frac{1}{2}$	1	2
$s$	2	1	$\frac{1}{2}$

 $s \propto \frac{1}{r}$ 

(d)

$x$	$\frac{1}{2}$	1	5
$y$	10	5	1

 $y \propto \frac{1}{x}$ 

2. Copy and complete each of these tables to match the stated relationship.

(a)  $y \propto \frac{1}{x}$

$x$	10	20	40
$y$	2	?	?

(b)  $p \propto \frac{1}{q}$

$q$	1	2	8
$p$	?	2	?

(c)  $s \propto \frac{1}{r}$

$r$	1	2	5
$s$	5	?	?

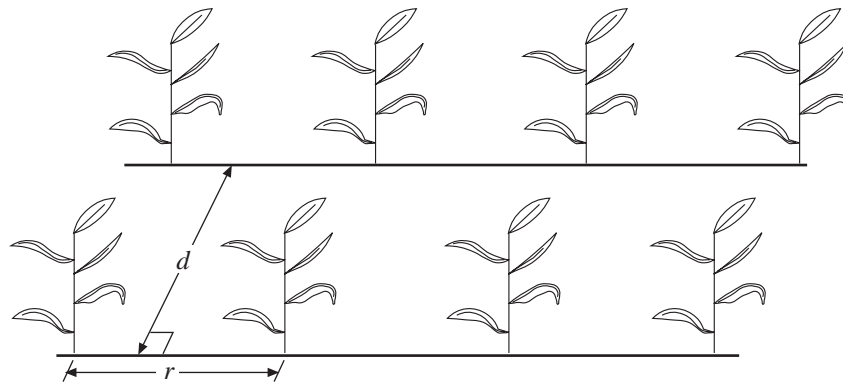
(d)  $v \propto \frac{1}{u}$

$u$	10	20	100
$v$	1	?	?

3. Two quantities,  $x$  and  $y$ , are such that  $y$  is inversely proportional to  $x$ . Also note that  $y = 4$  when  $x = 2$ .
- Find the relationship between  $x$  and  $y$ .
  - What is the value of  $y$  when  $x = 4$ ?
4. The value of a TV set is assumed to be inversely proportional to its age. When it is a year old it is sold for £400.
- What will its value be when it is 2 years old?

- (b) How many years old will it be when its value is first less than £100?
- (c) Is the assumption made here a reasonable one?
5. The value,  $v$ , of a train is assumed to be inversely proportional to its age,  $x$ . It was sold for £500 000 when it was 4 years old.
- (a) Find the relationship between  $v$  and  $x$ .
- (b) What is its value when it is 10 years old?
- (c) How many years old is it when its value is first less than £100 000?

6.



The diagram is taken from a book about growing maize.  
 The distance between the rows of plants is  $d$  metres.  
 The spacing between the plants in the rows is  $r$  metres.

The number,  $P$ , of plants per hectare is given by the formula  $P = \frac{10000}{dr}$ .

$d = 0.8$  and  $r = 0.45$ .

- (a) Calculate the value of  $P$ .  
 Give your answer to 2 significant figures.

The value of  $d$  is inversely proportional to the value of  $r$  and  $d = 0.9$  when  $r = 0.4$ .

- (b) Calculate the value of  $r$  when  $d = 1.2$ .

(LON)

7.  $y$  is inversely proportional to the square root of  $x$ . When  $x = 16$ ,  $y = 2$ .  
 What is the value of  $y$  when  $x = 0.25$ ?

(AQA)

8.  $p$  is inversely proportional to  $r$ .  $p = 7$  when  $r = 12$ .

- (a) Work out the value of  $p$  when  $r = 3$ .  
 (b) Work out the value of  $r$  when  $p = 24$ .

(Edexcel)

# 15.7 Functional and Graphical Representation

1. Write down the mathematical relationship between each pair of variables, using the information given.

- (a)  $y$  is proportional to  $x$ , and when  $x = 2$ ,  $y = 5$ .  
 (b)  $T$  is proportional to the square of  $x$ , and  $T = 4$  when  $x = 2$ .  
 (c)  $R$  is inversely proportional to the square of  $S$ , and  $R = 2$  when  $S = 1$ .  
 (d)  $q$  is proportional to the cube of  $p$ , and  $q = 4$  when  $p = 2$ .

2. Express each of the following in words

(a)  $y \propto \frac{1}{x^2}$                       (b)  $y \propto x^3$                       (c)  $y \propto \frac{1}{x^4}$

3. Copy and complete each of the tables below according to the given relationship.

(a)  $y \propto x^2$

$x$	1	2	3	4
$y$	?	?	27	?

(b)  $y \propto x^3$

$x$	1	3	6
$y$	?	9	?

(c)  $y \propto \frac{1}{x^2}$

$x$	1	2	4
$y$	?	1	?

(d)  $y \propto \frac{1}{x^3}$

$x$	1	2	4
$y$	?	1	?

4. The intensity of illumination,  $I$ , at a point varies inversely with the square of the distance,  $x$ , of the point from the light.

Express this in mathematical terms, and hence determine the ratio of the intensity of illumination produced by a light 8 m from the point, to the same light, 2 m from the point.

(SEG)

5. When a stone is thrown upwards with an initial speed  $s$  metres per second, it reaches a maximum height,  $h$  metres.

Given that  $h$  varies directly as the square of  $s$  and that  $h = 5$  when  $s = 10$ ,

- (a) work out the formula connecting  $h$  and  $s$ ,  
 (b) calculate the value of  $s$  when  $h = 20$ .

2 stones are thrown up. The ratio of their initial speeds is 3 : 1.

- (c) Work out the ratio of the maximum heights achieved.

(LON)

# 15.8 Further Functional Representation

1. For each table of values below, determine they agree with the stated relationship.

(a)	$x$	1	4	16	$y \propto x^{\frac{1}{2}}$
	$y$	$\frac{1}{4}$	$\frac{1}{2}$	1	

(b)	$x$	1	4	9	$y \propto \frac{1}{x^2}$
	$y$	1	$\frac{1}{2}$	$\frac{1}{3}$	

(c)	$x$	1	25	100	$y \propto x^{\frac{3}{2}}$
	$y$	0.2	25	400	

2. Copy and complete each table according to the given relationship.

(a)	$y \propto x^{\frac{1}{2}}$	$x$	1	4	9
		$y$	2	?	?

(b)	$y \propto \frac{1}{x^2}$	$x$	1	4	9
		$y$	?	2	?

(c)	$y \propto x^{\frac{3}{2}}$	$x$	1	4	9	16
		$y$	?	?	?	32

3. The increase in speed,  $V$  metres per second, at the lowest point of a 'Big Dipper' ride is proportional to the square root of the vertical height,  $h$  metres, dropped.

- (a) Write this relationship in mathematical terms.  
 (b) Use this relationship to find the ratio of the speeds obtained from heights 100 metres and 25 metres.

(SEG)

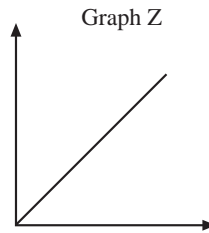
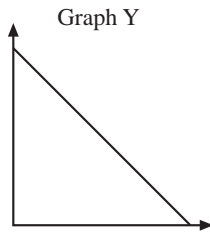
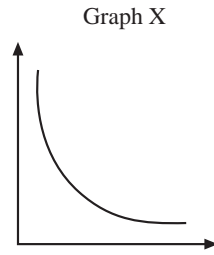
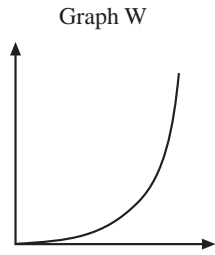
4.  $T$  is directly proportional to the positive square root of  $M$ .  
 $T = 32$  when  $M = 16$ .

- (a) Calculate  $T$  when  $M$  is 100.  
 (b) Calculate  $M$  when  $T$  is 9.6.

(NEAB)

5. Decide which graph matches each relationship.

**Graphs**



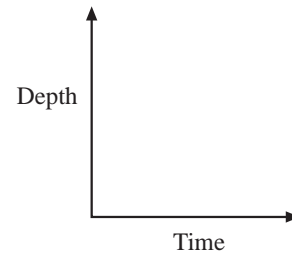
**Relationships**

- A: The area of a circle plotted against its radius.
- B: The circumference of a circle plotted against its radius.
- C: The length of a rectangle of area 24 cm<sup>2</sup> plotted against its width.

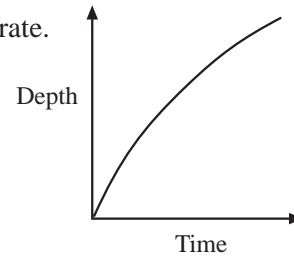
- (a) Which graph matches relationship A?
- (b) Which graph matches relationship B?
- (c) Which graph matches relationship C?

(SEG)

6. (a) Water flows into a cylinder at a constant rate.  
Sketch the graph of the depth of water against time.



- (b) Water flows into another container at a constant rate.  
Sketch the cross-section of the container that generated this graph.



- (c) The values of depth,  $d$ , against time,  $t$ , for a different container are shown in the table.

<b>Time</b> $t$ (secs)	1	2	3	4	5	6	7	8	9	10
<b>Depth</b> $d$ (cm)	0.1	0.4	0.9	1.6	2.5	3.6	4.9	6.4	8.1	10

Find the equation connecting  $t$  and  $d$

(SEG)