

# 12 Number Patterns

## 12.1 Simple Number Patterns

- Write down the next two numbers in each of the following sequences:
  - 6, 12, 18, 24, ...
  - 2, 4, 6, 8, ...
  - 50, 100, 150, 200, ...
  - 12, 24, 36, 48, ...
  - 13, 26, 39, 52, ...
  - 25, 50, 75, 100, ...
- Find the difference between successive terms for each of the following sequences. Hence, find the next two terms.
  - 1, 5, 9, 13, ...
  - 2, 7, 12, 17, ...
  - 2, 12, 22, 32, ...
  - 3, 10, 17, 24, ...
  - 3, 11, 19, 27, ...
  - 1, 12, 23, 34, ...
- In each case find (i), (ii) and (iii), using a calculator if necessary. Deduce the value of (iv) without using a calculator.
  - $3 \times 11 = ?$
    - $33 \times 11 = ?$
    - $333 \times 11 = ?$
    - $33333 \times 11 = ?$
  - $6 \times 9 = ?$
    - $6 \times 99 = ?$
    - $6 \times 999 = ?$
    - $6 \times 99999 = ?$
- A pattern of counting numbers is shown.

14, 15, 16, 17, 18, 19, 20, ...

  - Which of these numbers is a square number?
    - Which of these numbers is a multiple of nine?

The pattern is continued.

  - What is the next square number?
    - What is the next number that is a multiple of nine?
- Here is a sequence of numbers.

30    25    20    15    10    .....    .....

Write down the next two numbers in the sequence.

(SEG)

- Here is another sequence of numbers.

160    80    .....    20    10    .....    2.5

Write down the two missing numbers in this sequence.

(AQA)

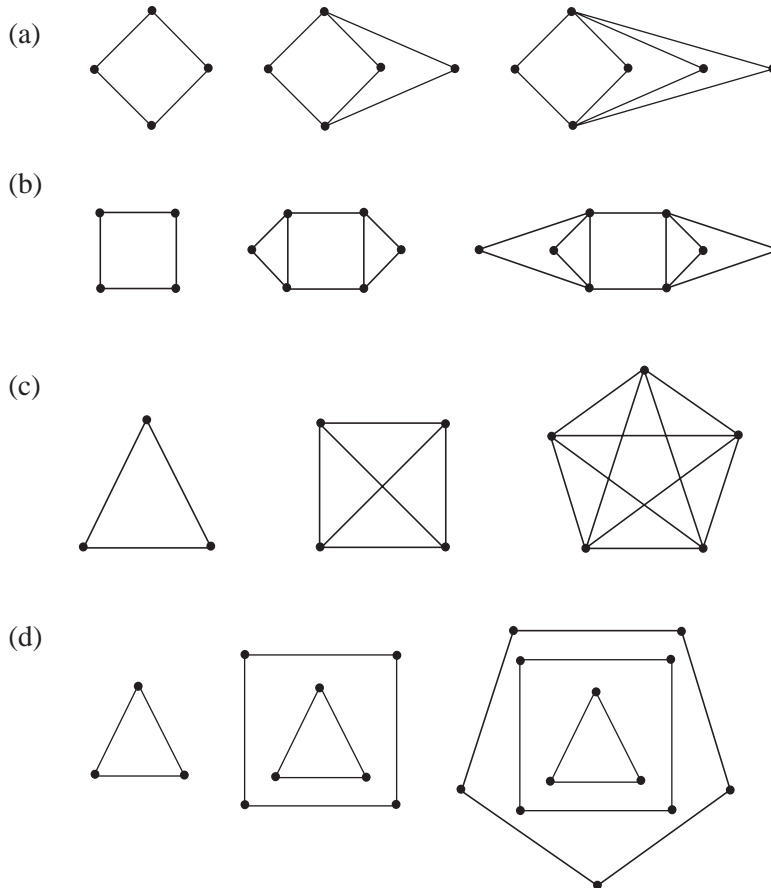
## 12.2 Recognising Number Patterns

1. Find the next two terms of each sequence below:

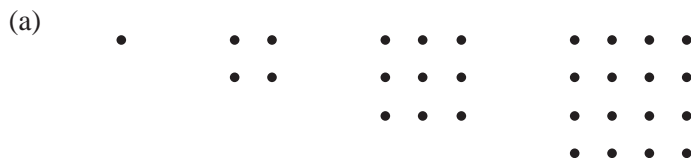
- |                             |                              |
|-----------------------------|------------------------------|
| (a) 3, 6, 10, 15, 21, ...   | (b) 1, 1, 2, 4, 7, ...       |
| (c) 6, 11, 17, 24, 32, ...  | (d) 0, 5, 15, 30, 50, ...    |
| (e) 5, 7, 11, 19, 35, ...   | (f) 10, 6, 1, -5, -12, ...   |
| (g) 0, -1, -3, -6, -10, ... | (h) 49, 42, 36, 31, 27, ...  |
| (i) 10, 14, 22, 34, 50, ... | (j) 100, 80, 62, 46, 32, ... |

2. Each sequence of shapes shown below is made up of lines which join two points. For each sequence:

- Write down the number of lines as a sequence;
- Predict, and check, the number of lines in the next shape.



3. Write down a sequence for the number of dots in each pattern, then explain how to get the next number. Check your prediction by actually drawing the next shape in the sequence.





4. (a) To generate a sequence of numbers, Paul multiplies the previous number in the sequence by 3, then subtracts 1.  
Here are the first four numbers of the sequence.

$$1, 2, 5, 14, \dots$$

Find the next two numbers in the sequence.

- (b) Here are the first four numbers of the sequence of cube numbers.

$$1, 8, 27, 64, \dots$$

Find the next two numbers in the sequence.

(MEG)

5. Neelan is investigating the number sequence

$$3, 4, 7, 12, 19, \dots$$

He draws up 2 difference tables and writes down his answers under the original sequence.

Sequence	3	4	7	12	19	<input type="text"/>
Differences		1	3	<input type="text"/>	7	
Differences of differences			<input type="text"/>	2	2	

Copy the sequences and fill in the missing numbers in all 3 boxes.

(LON)

6. The rule for a sequence of numbers is:

*'add the two previous numbers and then multiply by 2'.*

- (a) Write down the next two numbers in the sequence:

$$1, 1, 4, 10, \dots, \dots$$

The first eight numbers in a different sequence of numbers are:

$$3, 7, 4, 8, 5, 9, 6, 10, \dots$$

- (b) Write down the next two numbers in the sequence.

(LON)

7. Here are the first five terms of a sequence.

290    284    278    272    266

Write down the next two terms of the number sequence.

(Edexcel)

8. Patterns are made from shaded and unshaded squares.



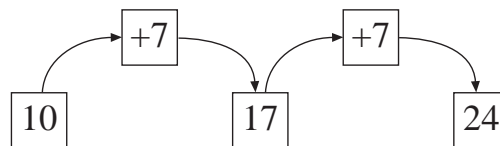
Pattern 1    Pattern 2    Pattern 3    Pattern 4    Pattern 5

- (a) Draw Pattern 6.  
 (b) How many shaded squares will there be in Pattern 20?  
 (c) How many unshaded squares will there be in Pattern 20?

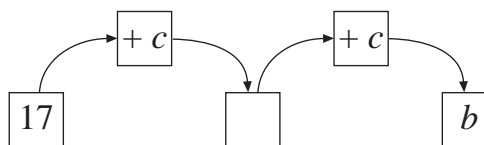
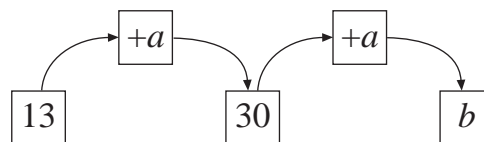
(AQA)

9. In all of the following diagrams, numbers are increased by two equal steps.

For example:



Find  $a$ ,  $b$  and  $c$ .



(AQA)

## 12.3 Extending Number Patterns

1. Find the 10th and 20th terms of each of the following sequences:

- |                              |                           |
|------------------------------|---------------------------|
| (a) 3, 6, 9, 12, 15, ...     | (b) 4, 8, 12, 16, 20, ... |
| (c) 100, 98, 96, 94, 92, ... | (d) 12, 9, 6, 3, 0, ...   |
| (e) 2, 3, 5, 8, 12, ...      | (f) 3, 4, 7, 12, 19, ...  |
| (g) 1, 4, 9, 16, 25, ...     | (h) 2, 4, 8, 16, 32, ...  |
| (i) 10, 12, 16, 24, 40, ...  |                           |

2. Look at this number pattern:

$$\text{Line 1} \quad 1 \times 1 = 1$$

$$\text{Line 2} \quad 11 \times 11 = 121$$

$$\text{Line 3} \quad 111 \times 111 = 12321$$

$$\text{Line 4} \quad 1111 \times 1111 = 1234321$$

$$\text{Line 5} \quad 11111 \times 11111 = 123454321$$

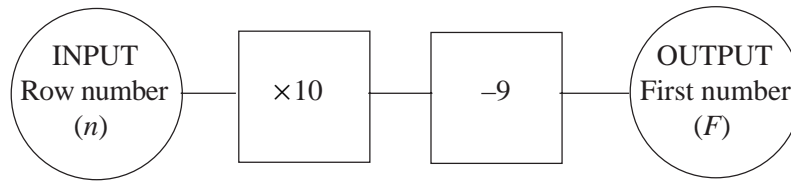
- (a) Write down the complete Line 6 of this pattern.
- (b) Use the pattern to help you find the value of  $111111111 \times 111111111$ .  
(MEG)
3. Ranjit is doing an investigation into powers. He begins to make a table as follows.

	<i>Column 1</i>	<i>Column 2</i>	<i>Column 3</i>	<i>Column 4</i>	..	..
<i>Row 1</i>	1	1	1	1	..	..
<i>Row 2</i>	2	4	8	16	..	..
<i>Row 3</i>	3	9	27	..	..	..
<i>Row 4</i>	4	16	64	..	..	..
<i>Row 5</i>	..	..	..	..	..	..
..	..	..	..	..	..	..
..	..	..	..	..	..	..

- (a) What is the 6th number in Row 3?
- (b) What is the 10th number in Column 2?
- (c) The number 49 appears in Column 2. In which row is it?
- (d) The number 6561 appears in Row 3. In which column is it?
- (e) The number 576 appears in Column 2. In which row is it?
- (f) What is the 20th number in Row 2?  
(SEG)
4. The odd numbers are arranged in rows of five, as follows:

<i>Row number (n)</i>	<i>First number in row (F)</i>					<i>Last number in row (L)</i>
1	1	3	5	7	9	
2	11	13	15	17	19	
3	21	23	25	27	29	
	and so on.					

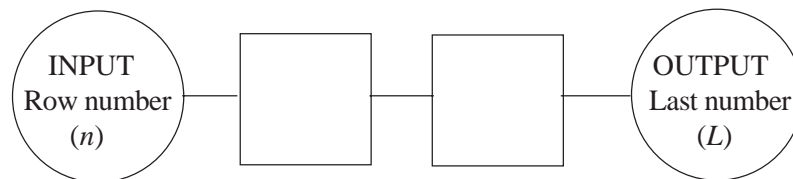
(a)



This number machine can be used to find the first number ( $F$ ) in row  $n$ .

The machine uses the rule  $F = 10n - 9$ .

- (i) Copy and complete the following number machine to give the last number ( $L$ ) in row number  $n$ .



- (ii) Write down the rule connecting  $L$  and  $n$ .

- (b) The numbers in Row 1 add up to 25.

- (i) Copy and complete the following table.

<i>Row</i>	1	2	3	4	5
<i>Sum of numbers in the row</i>	25				

- (ii) Work out which row has a sum of 875.

(MEG)

5. Here is a sequence of patterns.

**Pattern 1**

x  
o o  
x

**Pattern 2**

x x  
o o o  
x x

**Pattern 3**

x x x  
o o o o o  
x x x

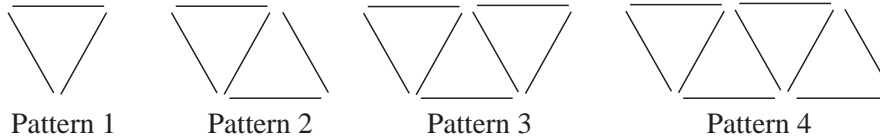
- (a) Draw Pattern 4.  
(b) Copy and complete this table.

<i>Pattern</i>	1	2	3	4	5
<i>Number of circles</i>	2	3			
<i>Number of crosses</i>	2	4			

- (c) What pattern do you notice in the 'Number of crosses' row in the table?  
(d) How many circles are there in Pattern 20?

(OCR)

6. Patterns are made of sticks.



- (a) Copy and complete the table.

<i>Pattern</i>	1	2	3	4
<i>Number of sticks</i>	3	5		

- (b) How many sticks are there in Pattern 5?  
 (c) Here is a rule for working out the number of sticks.

Multiply the pattern number by 2 and add 1

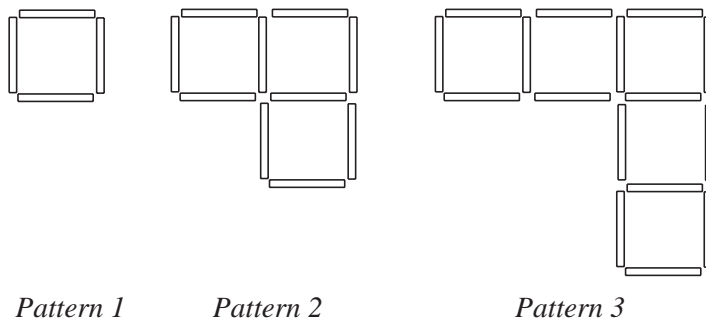
How many sticks are there in Pattern 10?

(AQA)

## 12.4 Formulae and Number Patterns

1. Use the formulae below to find the first 6 terms of each sequence.
- (a)  $u_n = 1 + 2n$       (b)  $u_n = 5n - 2$       (c)  $u_n = 2n^2 - 1$   
 (d)  $u_n = 3^n - 1$       (e)  $u_n = (n + 1)^2$       (f)  $u_n = (n + 2)(n - 3)$
2. Find (i) the 10th term and (ii) the 20th term of each sequence below:
- (a)  $u_n = 5n$       (b)  $u_n = 3 + 4n$       (c)  $u_n = 20 - 2n$   
 (d)  $u_n = 5 + n^2$       (e)  $u_n = n^2 + 4n + 4$       (f)  $u_n = \frac{1}{(n + 1)}$
3. Find the formula for  $u_n$ , the  $n$ th term, for each of the sequences given below:
- (a) 4, 7, 10, 13, 16, ...      (b) 2, 6, 10, 14, 18, ...  
 (c) 50, 43, 36, 29, 22, ...      (d) 5, 2, -1, -4, -7, ...  
 (e)  $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots$       (f) 7, 15, 23, 31, 39, ...
4. The  $n$ th term of each of the sequences below can be written in the form  $u_n = an + b$ . For each sequence, find the constants  $a$  and  $b$ .
- (a) 3, 5, 7, 9, 11, ...      (b) 4, 3, 2, 1, 0, ...  
 (c) 4, 11, 18, 25, 32, ...      (d) 100, 95, 90, 85, 80, ...

5. Write down the sequence (i)  $u_n = n^2$  and (ii)  $u_n = n^3$ . Use them to find the formula for the  $n$ th term of the following sequences.
- (a) 2, 5, 10, 17, 26, ...      (b) 2, 8, 18, 32, ...
- (c) 2, 9, 28, 65, 126, ...      (d) -1, -7, -17, -31, ...
- (e) 2, 16, 54, 128, 250, ...      (f) 0, 4, 18, 48, 100, ...
6. The three patterns below are made out of matchsticks.



- (a) Draw the next pattern in the sequence.
- (b) Copy and complete this table to show the number of matchsticks used for each pattern.

<i>Pattern number</i>	1	2	3	4	5	6
<i>Number of matchsticks</i>	4	10	16			

- (c) How many matchsticks would be needed for the 20th pattern?  
Show clearly how you worked out your answer.
- (d) Write down an expression for the number of matchsticks in the  $n$ th pattern.  
(NEAB)

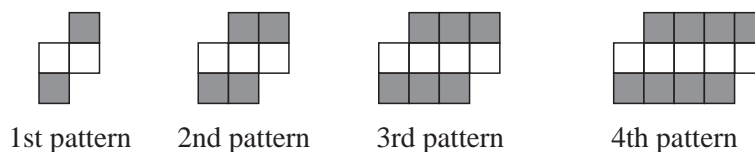
7. A sequence of numbers is shown.

5      9      13      17      21

- (a) Find an expression for the  $n$ th term of the sequence.
- (b) Explain why 83 will not be a term in this sequence.

(AQA)

8. Patterns are made from shaded and unshaded squares.



- (a) How many shaded squares are there in the  $n$ th pattern?
- (b) How many unshaded squares are there in the  $n$ th pattern?

(AQA)

9. (a) Find the eighth term of the sequence whose  $n$ th term is  $4n - 1$ .  
 (b) The first three patterns in a sequence are shown below.



Write down, in terms of  $n$ , the number of ticks when there are  $n$  crosses.

- (c) Here are the first four terms of a sequence.

5      8      13      20

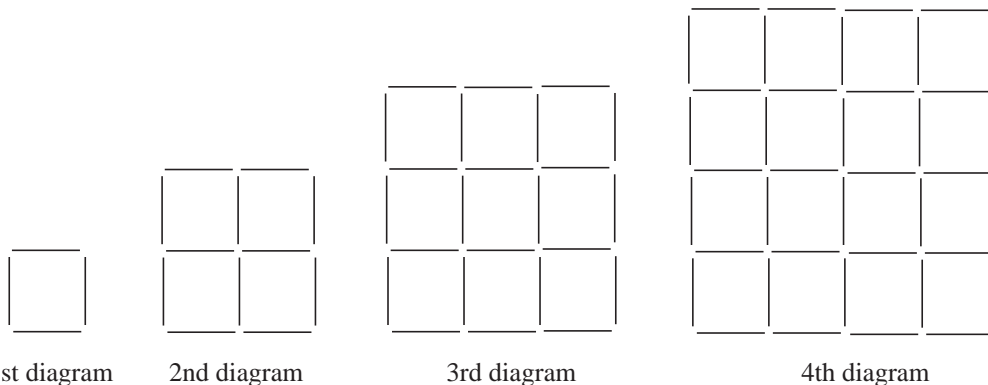
Find the  $n$ th term of this sequence.

(OCR)

## 12.6 Quadratic Formulae

1. Show that each of the following sequences has a constant second difference, and use this to find the next 2 terms of the sequence.
- |   |  |
|---|--|
| (a) 2, 6, 11, 17, ...<br>(c) 15, 13, 10, 6, ... | (b) 1, 1, 2, 4, 7, 10, ...<br>(d) -3, -10, -24, -45, ... |
|---|--|
2. The third, fourth and fifth terms of a quadratic sequence are 16, 26 and 38. Find the first, second and sixth terms of the sequence.
3. Find a quadratic formula which describes each of the following sequences:
- |  |  |
|--|--|
| (a) 2, 5, 10, 17, 26, ...<br>(b) 2, 6, 12, 20, 30, ...<br>(c) 7, 11, 16, 22, 29, ... |  |
|--|--|
4. The 9th, 10th and 11th terms of a quadratic sequence are given by 167, 205 and 249. Find the formula for the  $n$ th term.

5.



Each diagram consists of squares made from rods. The diagrams form part of a sequence.

- (a) Copy and complete the table.

<i>Number of diagram</i>	1	2	3	4	5
<i>Number of rods used to make that diagram</i>	4	12	24		

- (b) Write down the answers to these multiplications of consecutive numbers.

$$1 \times 2 = \dots\dots\dots \quad 2 \times 3 = \dots\dots\dots \quad 3 \times 4 = \dots\dots\dots$$

- (c) How many rods are used to make the 12th diagram in the sequence?

- (d) How many rods are used to make the  $n$  th diagram in the sequence?

(SEG)

6. (a) A number pattern begins 4, 8, 12, 16, 20, 24, ...

Describe this number pattern.

- (b) Another number pattern begins 1, 4, 9, 16, 25, 36, ...

(i) Describe this number pattern.

(ii) What is the next number in this pattern?

Each number in this pattern is changed to make a new number pattern.

The new number pattern begins -1, 2, 7, 14, 23, 34, ...

(iii) What is the next number in the new pattern?

Explain how you found your answer.

(SEG)

7. (a) (i) Write down the multiples of 5, from 5 to 40.

(ii) Describe the pattern of the units digits.

- (b) SEQUENCE P is 3, 6, 9, 12, 15, 18, 21, ...

Explain how SEQUENCE P is produced.

- (c) Copy the table below, and fill in the blanks.


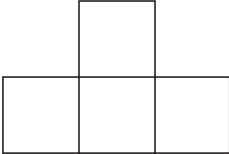
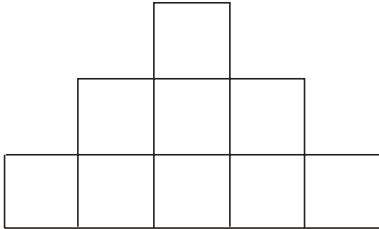
<u>SEQUENCE P</u>	→	<u>Add 1 and then multiply by 2</u>	→	<u>SEQUENCE Q</u>
3	→	$3 + 1 = 4, \quad 4 \times 2 = 8$	→	8
6	→	$6 + 1 = 7, \quad 7 \times 2 = 14$	→	14
9	→	.....	→	.....
12	→	.....	→	.....
15	→	.....	→	.....
18	→	.....	→	.....

- (d) (i) Find the next two terms in the sequence 1, 4, 10, 19, 31, 46, 64, ...

(ii) Explain how you obtained your answer to part (d) (i).

(MEG)

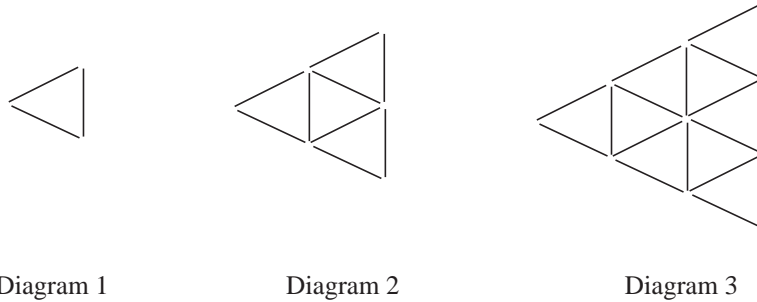
8. The diagram shows the first 3 members of a sequence of patterns of cm squares.

<i>Pattern Number</i>		<i>Number of cm squares</i>
1		1
2		4
3		9

- (a) Draw pattern number 4.
- (b) Write down the total number of cm squares in pattern number 8.
- (c) Express in symbols the number of cm squares in pattern number  $n$ .
- (d) Express in symbols the number of cm squares in the bottom row of pattern number  $n$ .

(LON)

9. The following diagrams form a sequence.



The sequence is continued.

- (a) How many lines are needed for diagram 4?

Lines drawn like this  are called *vertical*.

- (b) How many non-vertical lines are needed for diagram 5?
- (c) How many vertical lines are needed for diagram 6?

(d) Copy and complete the following table.

<i>Diagram</i>	<i>Number of non-vertical lines</i>	<i>Number of vertical lines</i>	<i>Total number of lines</i>
1	2	1	3
2	6	3	9
3	12	6	18
4			
5			
6			

(e) For the  $n$ th diagram, write in terms of  $n$ ,

- (i) the number of non-vertical lines,
- (ii) the number of vertical lines,
- (iii) the total number of lines.

(SEG)

10. The triangle number sequence is

1, 3, 6, 10, 15, 21, ...

The  $n$ th term of this sequence is given by

$$\frac{1}{2}n(n+1)$$

- (a) Write down an algebraic expression for the  $(n-1)$ th term of the sequence.
- (b) Prove, algebraically, that the sum of any two consecutive triangle numbers is a square number.

(AQA)