

Language and Mathematics: A Mediatlional Approach to Bilingual Arabs

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Abstract

Students usually experience severe problems when the medium of instruction changes from their native language to another one. This phenomenon in the case of mathematics education brings dire consequences if unchecked by mathematics educators. This paper presents the outcome of an experiment that attempted to address the language barrier of preparatory year mathematics students, who are acquiring English as a new language of instruction at King Fahd University of Petroleum & Minerals, Saudi Arabia. The findings in the study indicate that the approach has minimized the language barrier problem, however, the issue merits further study.

1. Introduction

The role of language in the teaching and learning of mathematics has been noted in many curricula. For instance, communicating mathematical ideas is among the twelve components that are considered by the NCSM (National Council of Supervisors of Mathematics) as “essential” for any successful mathematics teaching and learning process (Ellerton & Clarkson, 1996). This position has also been endorsed by National Council of Teachers of Mathematics (NCTM) in the *Principles and Standards for School Mathematics* (2000). There, communication has been emphasized "as an essential part of mathematics and mathematics education (p.60)." The NCTM standards have also elaborated that all students in general, and second-language learners in particular, need to have opportunities as well as to be given encouragement and support for speaking, writing, reading and listening in mathematics classes. In particular, this practice will help second-language learners overcome barriers and thus facilitate communication in the

teaching and learning of mathematics. Similar endorsements and recognition of the importance of language factors in mathematics learning and teaching are found in many other curriculum documents, especially in countries that are multicultural like the USA, New Zealand, Australia and South Africa. However, it has been noted that the attention given to the centrality of the language factor by mathematics educators and researchers in both the research and practice domains is “little more than lip service” (Ellerton & Clarkson, 1996:1017).

One of the reasons why the language factor needs special attention these days is the fact that many students are currently learning mathematics in their second or third language (Austing & Howson, 1979; Ellerton & Clarkson, 1996). This phenomenon is gradually becoming the norm rather than the exception (Secada, 1991). The reason for this in the developed countries is largely due to the increase in immigration, while in the developing countries it is due to the legacy of colonialism, and the multiplicity of local languages. Another reason which can be described as a much stronger one is the fact that the language of science, technology and the internet is slowly but surely narrowing down to a few languages. Therefore, textbooks and other learning and teaching materials are increasingly likely to adopt these few languages. Although studies on the consequences of this bilingualism and multilingualism on student mathematics learning are inconclusive (Morrison & McIntyre 1972; Austing & Howson, 1979, Davidenko, 2000), some studies have shown that there is a relationship between the degree of bilingualism and logical reasoning (Secada, 1988; Ellerton & Clarkson, 1996; Brodie, 1989). Furthermore, several other studies have indicated that the language problem is one of the major factors contributing toward the poor performance of many students in mathematics; especially those who are bilingual and multilingual (see Secada, 1992; Barton & Barton, 2003). Studies have shown that students that are found to be very weak in the language of instruction have the tendency toward ill-comprehension as well as poor participation in classroom discourse (see Setati, 2003). Consequently, they cannot meet the desired objectives of their studies due to lack of communication skills, and also puts teachers in a big dilemma on how to correctly assess the sources of student difficulty: is it mathematics or language? (Secada & Cruz, 2000).

For students who are acquiring a language of instruction as well as learning mathematics in the new language, the language of mathematics is another source of difficulty and confusion. Mathematical terminology is often complex and the words used therein are endowed with meanings, which in most cases are completely different from their normal usage. For instance, the words: *root*, *similar*, *power*, *or* and *odd* have a different sense from the usual meanings when used in mathematics. Sometimes it may be difficult, “even for students who are not bilingual, to determine which meaning of ‘odd’ is intended in a problem (odd as in something peculiar or odd as in numbers that are not divisible by two)” (Raborn, 1995). Studies have shown that bilingual students, even at university level, are confusing the meanings of some of these mathematical terms (see Setati, 2003). The problem is greater for bilingual students who are acquiring a language of instruction such as the preparatory year students at King Fahd University of Petroleum & Minerals. This class of students has to cope with the difficulty of learning to understand the special terminology and syntax of mathematics (see Brodie, 1989; Durkin & Shire, 1991).

2. Theoretical Background on the Subject

A significant amount of research has been carried out in the recent past and is still being done on the concept of understanding in general. A part of it is directed towards the understanding of mathematics in particular. Ernest (1987) traced the origin of this research to what he called the “monumental research work of Piaget and his co-workers” (p.10). According to Ernest (1987), Piaget’s work is “concerned with the elaboration of a theory of the growth of understanding in children” (p.10). However, some other researchers such as Bruner, Hart, Mellin-Olsen, and Skemp looked at “understanding” from a different point of view (see Ernest, 1987). In all these studies, language is seen as a necessary condition for understanding. However, the point of departure among the researchers is whether thought precedes language or vice versa. The former would imply that the language is merely a means of expressing our thoughts while the latter would imply that the language determines and is a prerequisite for thoughts (Brodie, 1989). Silby (2000) summarized the two schools of thought as follows:

Language of thought theories fall primarily into two views. The first view sees the language of thought as an innate language known as mentalese, which is hypothesized to operate at a level below conscious awareness while at the same time operating at a higher level than the neural events in the brain. The second view supposes that the language of thought is not innate. Rather, the language of thought is natural language. So, as an English speaker, my language of thought would be English (Silby, 2000).

On one hand, the Sapir-Whorf hypothesis theorized that the language habits of our community predispose certain choices of interpretation (Durkin & Shire, 1991). This simply means, according to Durkin & Shire (1991), that people think and perceive things in a way made possible by “the vocabulary and phraseology of their language” (p.12). Hence, “concepts not encoded in their language will not be accessible to them, or at least will prove very difficult” (p.12). On the other hand, Einstein, in his response to Hadamard’s (1945) informal survey, was quoted as having said that “the word or the language, as they are written or spoken, do not seem to play any role in my mechanism of thought” (Davis & Hersh, 1980:308). It is worth noting that, although the Sapir-Whorf hypothesis has not been generally accepted, especially in the domain of mathematics education (see Zepp, 1989), there is evidence that shows the language we speak has an influence on our thought patterns (Brodie, 1989; Durkin & Shire, 1991; Silby, 2000). On the other hand, studies have shown that the cognitive style of most mathematicians coincides with Einstein’s statement (see Hadamard, 1945). Nevertheless, it is necessary to make a distinction between the process of creating mathematics by professional mathematicians and that of communicating mathematics to students in a classroom. While the former is dominated by thought, the later is mediated by language. Therefore, in whatever school of thought one might be, language has a crucial role to play in communicating and developing mathematics education.

Many frameworks have been developed from both a sociolinguistic as well as a psycholinguistic point of view in an attempt to link the various elements of language and mathematics. Among these frameworks is the one developed by Gawned (1990). According to Ellerton & Clarkson (1996), Gawned’s framework is based on a “sociolinguistic premise” (p.990). The framework acknowledges that the language of the classroom has a “formative effect on the learners’ understanding of mathematics” (p.990). According to the framework, as far as the mathematics

learner is concerned, “mathematical concepts only have meaning within the linguistic and social context from which they were derived” (p.994). As noted earlier, studies are inconclusive on the effect of bilingualism and multilingualism on student mathematical learning. However, some studies have shown that student proficiency in his or her first or second language plays a role in his or her cognitive activities. (Qi, 1989; Secada, 1992; Silby, 2000). Cummins’s ‘threshold hypotheses’ states that for learners who speak two or more languages, the interplay in the learning process between the language codes may either assist or detract them from learning. On one hand, if a bilingual or multilingual student has reached a “threshold” of competence in the two or more languages, then the learner may have a cognitive advantage. On the other hand, those bilingual or multilingual students who are not really fluent in either of the two or more languages tend to experience difficulty in mathematics (Ellerton & Clarkson, 1996).

At the start, Cummins distinguished between what he called basic interpersonal communicative skills (BICS) and cognitive academic language proficiency (CALP). According to Cummins, while conversational fluency is often acquired to a functional level within about two years of initial exposure to the second language, it takes at least five years to catch up with native speakers in academic aspects of the second language. Cummins’s distinction between the conversational and academic has made a lot of impact on many educational policies and practices in both North America and the United Kingdom (see Cline & Frederickson, 1996). Similarly, many of the current empirical studies on the implications of bilingualism revolve around this distinction. The results all point to the fact that linguistic factors have a significant effect on student learning of mathematics. However, it has been observed that most of the research on bilingualism and multilingualism is carried out in developed countries. Therefore, the need for urgent research investigation, particularly in developing countries, to determine the extent and nature of the role of bilingualism has been called upon (Austing & Howson, 1979; Ellerton & Clarkson, 1996).

As part of our effort to minimize the language difficulties of our bilingual Arabs mathematics students, who are experiencing and acquiring English as a new language of instruction, an experiment was carried out. In the next section, a detailed report of the experiment is presented.

3. Our Experiment at KFUPM

In an attempt to address the language difficulties faced by our mathematics students in the Preparatory Year of KFUPM, an experiment was conducted. It may be noted that many other studies have been carried out and are currently being carried out to determine the level of difficulty that language has been causing students in their achievement and understanding of mathematics. These studies indicate that in a school where the students' first language is different from the predominant language of instruction, students tend to benefit more if mathematics is taught in their first language (see Clarkson, 1994; Ellerton & Clarkson, 1996; Setati, 2003). On the other hand, intensive research has been done on "code-switching" (see Adler & Setati, 2001) in a mathematics classroom. Adler & Setati (2001) explain that Code-Switching refers to bilingual or multilingual settings; which simply means to switch between the language of learning and teaching and the learners' first language. The approach enables learners to harness their local language as a learning resource as well as to increase their participation in classroom discourse. Adler & Setati (2001) referred to many studies that have been carried out on code-switching, and they concluded that most of these studies have demonstrated or argued for the use of the learner's first language as a "support" in the teaching and learning of mathematics. These calls make sense, especially for students who are being introduced to the new language of instruction, as it will make it possible for the learner to continue to develop proficiency in the new language and, at the same time, learn mathematics.

The Rationale behind the Experiment

Our students in the preparatory year program at King Fahd University of Petroleum & Minerals (KFUPM) consist of males whose average age is 18. They are mostly recent graduates from the Kingdom's high schools where Arabic, the first language of almost all of the students, is the medium of instruction. Consequently, they have only a limited command of English - the official medium of instruction at KFUPM. To prepare the students for this sudden change, almost all new students undergo a rigorous two-semester English program during the preparatory year. At the same time, they are required to take one mathematics course each semester. As per the university policy, these two mathematics courses are taught in English. Even though the contents

of these two courses are quite similar to what the students learnt in high school, they still face difficulties in following the classroom lectures and understanding the textbook. Some of them find it difficult to ask or respond to questions in the classroom due to lack of proficiency in the new language of instruction. Another problem faced by the students is writing in English from left –to-right, instead of right-to-left, as in Arabic. As a result of these problems, many students tend to get confused, lose confidence and question their intellectual capabilities. Most faculty acknowledges the fact that these students have serious communication problems due to the language barrier, especially in the mathematics classes. However, what has not been established in the KFUPM preparatory year program is the consequence of this sudden “language-switching” on student understanding and achievement in their mathematics courses. It is generally accepted that the performance of the students is below expectations, and their mathematics background seems to be shaky contrary to the fact that they undergo “rigorous” selection process before admission. However, if inferences can be derived from other experiences, then one can conclude that the language factor might be perhaps one of the causes of the low performance of our students (see, Secada, 1992, Setati, 2003, Adler, 1996, Barton & Barton 2003). This assumption makes sense if one looks into the weak English background of the students. An early study has indicated that proficiency in English is a reliable predictor of students’ success at this early level at KFUPM (see Aldoghan, 1985).

Nevertheless, the rationale of our studies is not to investigate to what degree the language is a factor of low performance and understanding of students in mathematics. Instead, it is to develop a form of mediated teaching approach that might reduce the language barrier of these new student, i.e., to provide a minimal language support that may cushion the process of students language transition as well as improve their understanding in mathematics and, at the same time, abide by the university regulations regarding English as a medium of instruction.

Teaching Design

Our proposed teaching approach was slightly different from traditional classroom teaching. In the experiment, overhead projector transparencies were regularly utilized. We confined our experiment to the first two chapters in the coursebook which cover more than $\frac{2}{3}$ of the technical

terms of the whole course. These chapters are basically the preliminaries, equations and inequities in the standard college algebra book, and contain more than seventy percent of the mathematics terminology and vocabulary of the entire course. Although English was strictly maintained as the medium of instruction, a handout of Arabic translations of the mathematics terminology of the entire course was given to the students. The insertion of the Arabic translation of the terms was to help students to read and recall their high school knowledge and make a connection with the preparatory course lesson (see Appendix I for a sample). It is our belief that when the students are able to progress smoothly with the first two chapters, they can easily carry on with the remaining chapters with little or no difficulty.

As stated earlier, most of the students have problems reading the recommended textbook despite the fact that the University provides them with the best and latest texts available on the subject. To address this aspect, we incorporated the examples from the text in the transparencies as part of the lecture material. Similarly, after an explanation of each concept, in-class exercises were given to the students.

Sample

The subjects of this study were the Math 001 (first math course) students of the second semester during the 2001/2002 academic session. All the students that registered for Math 001 in that semester participated in the experiment. Almost two-thirds of the students were repeating the course while the remainder was fresh from high school. The experiment involved eighteen sections of students with seven instructors. The average number of students per section was 20. The nature of the sample is summarized in table I.

Number of Registered Students	426
Number of Repeaters	295
Number of Sections involved	18
Section Size	14-29
Number of teachers	7

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Results

As stated earlier, all students of Math 001 in that particular semester were involved in the experiment. Therefore the setup of the experiment was not designed for a comparison with another group of students. Data was collected both qualitatively and quantitatively. The quantitative data was collected from the teacher survey and also from students achievement in their exams. On the other hand, qualitative data was collected from both students and teachers via informal interviews. Also, during the period of the experiment, the course instructors regularly met to monitor the progress of the experiment. In the meetings, teachers freely expressed their views and also those of their students about the experiment. The findings of this experiment are summarized as follows:

1. All the teachers found that the insertion of an Arabic translation of key words and concepts on the transparencies helped most of the students recall the concepts they had learnt at high school level.
2. Teachers regularly observed the "*Aha, now I've got it!*" expression on the faces of students after reading the Arabic translation.
3. The insertion of the Arabic translation was found to be very helpful in minimizing student language difficulty in learning mathematics.
4. Students found the Arabic insertion very helpful.
5. There was an increase in teacher-student interaction in the classroom.
6. There seemed to be an increase in the students learning process.
7. The presentation of the material on transparencies was found to be useful to the students.
8. The students seemed satisfied with the classroom presentation of the material on transparencies.
9. The pace of coverage was found to be faster with the help of transparencies than the usual pace of coverage of similar material in the normal lecture.
10. The students consumed a fair amount of classroom time in attempting the in-class exercises.
11. The use of examples and in-class exercises from the textbook was found helpful in motivating the students towards reading the textbook.

12. Contrary to past experience, the students did not run here and there to get their section changed as usually happens at the start of each semester. This point, to a certain extent, possibly reflected the confidence of the students in understanding the material with all instructors, and possibly also reflected the success of the uniformity of the teaching approach.

Student Achievements

A majority of the students, being second time repeaters of the course, were weak and less motivated at the start of the experiment. However, considering the level of the exams, we observed that the performance of these students was more encouraging during the experiment when compared to a similar group in past semesters. Below are the average scores of the students in the three major exams and classwork (continuous assessment):

Exam I	Exam II	Final Exam	Class work	Total
53.6	65.4	49.2	65	57.8

As can be observed in table II, there is an impressive progress in students' performance from Exam I to Exam II, which was basically the period of the experiment. A possible interpretation of the low performance in the final exam is the fact that the final exam is comprehensive, whereas most repeaters are generally found with the problem of low retention of course material. In general, the student performance was considered to be slightly above average. Table III gives the letter grade summary of the overall results.

No.	Letter Grade	Students Count (out of 381 Students)	Overall %
1	A+	35	9.18
2	A	15	3.93
3	B+	20	5.25
4	B	47	12.33
5	C+	51	13.38
6	C	69	18.11
7	D+	29	7.61
8	D	19	4.99
9	F	96	25.20
10	GPA (All Sections): 1.66/4.00		

Problems Faced During the Experiment

Although the mode of the experiment and the relevant preparations were thoroughly discussed in several meetings, the course instructors faced some problems during the period of the experiment, which they could not have been foreseen. Some of them are narrated below:

1. Two classrooms, each equipped with an overhead projector, were allocated exclusively for 18 sections of Math 001. As mentioned earlier, special transparencies were prepared to display the material to be covered during the class lecture. It was realized within the first week of the classes that the overhead projectors were not capable of carrying an operational load of 9 hours during the lecture days. The projectors broke down on several occasions as a result of overusage. Therefore, the class instructors had to seek technical assistance during class time. This drawback caused a disturbance in the smooth functioning of the classes and wasted classroom lecture time as well, especially at the beginning of the experiment.
2. The transparencies covering the lecture material were mostly prepared a week ahead of the relevant lecture. The experiment coordinator was responsible for the preparation and typing of the material. Other instructors participating in the experiment had to review it thoroughly before the final preparation of the transparencies. On some occasions, the work overwhelmed the experiment coordinator and the material prepared by him sometimes only came out on the day of the class lecture. This delay left little time for the proposed review and the instructors often had to present it in class without appropriate preparation.
3. The spring semester selected for the experiment usually involves several students who repeat the course as a result of their failure. These students are poorly motivated and do not pay due attention to class lectures. In our view, the outcome of the experiment would have been more encouraging if it had been conducted with the newly admitted students during the fall semester.

4. Summary and Conclusions:

In this paper, we have discussed the crucial role of language in the teaching and learning of mathematics. In addition, we have presented the details of a one-semester experiment that aimed to reduce the language constraints of mathematics students admitted to the preparatory year program at KFUPM. These students, after completion of their high school studies in Arabic, experience English as the main medium of instruction at KFUPM. During the preparatory year, the students are placed in an intensive program of English and also take preparatory mathematics courses. Because of the language deficiency, these students face severe problems mostly related to language while taking these preparatory mathematics courses. Consequently, we felt the need to develop a teaching approach for these courses, which addressed the language problem. In order to achieve this objective, we took two initiatives. As a first step, the translation of all the key terms of the entire course from English to Arabic was prepared in the form of a handout and delivered to the students for quick reference. Secondly, we designed transparencies for the class lectures in which an Arabic translation was inserted after each key technical term, during the experiment period. One of the findings of this experiment was that the insertion of an Arabic translation motivated the students to recall the equivalent concepts studied at high school level. Consequently, this enabled them to connect their previous knowledge with the current one, and also let them attempt the relevant text problems. In addition, we observed an increase of class interactivity in this approach. Considering the fact that the majority of the subjects in our experiment were academically weak (mostly repeaters), we concluded that an overall performance of the students in the exams was encouraging. The students and instructors both responded positively to the approach adopted in the experiment, and they agreed that this approach did minimize the language barrier in the classroom. Moreover, it encouraged the students to read the textbook as well. It is to be noted that English was strictly maintained as the language of instruction throughout the period of experiment.

Based on the findings of our study, we suggest that the teaching strategy as devised in our experiment be adopted, with suitable modification, for Arab bilingual students who are acquiring English as a new language of instruction. Nevertheless, we stress that the issue of a sudden language switch needs a deeper and longitudinal study to assess the implications of sudden

change of the language of instruction on student understanding and their achievement in mathematics. Consequently, a comprehensive teaching approach can be developed to cater for the difficulties of these students, especially during the transition period.

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2.1 Linear Equation ()

Equation

“A Statement () with Equality sign Between Two Expressions”

Examples: $3+2 = 5$, $9x^2 + x + 3 = 0$

Note the Difference ()

- i. $9x^2 + x + 3$ is an Expression ()
- ii. $9x^2 + x + 3 = 0$ is an **Equation** ()
- iii. $3+2 = 5$ is an Equation without Variable
- iv. $x + 3 = 9$ is an Equation with Variable

[True and False Statements]

1. $x + 3 = 9$ is a True Statement for $x = 6$.

This means: The Number 6 Satisfies the Equation $x + 3 = 9$

2. $x + 3 = 9$ is a False Statement for $x \neq 6$.

• Solve an Equation

- **Roots** () or **Solutions of an Equation**

“All values of the Variable that Satisfy () the Equation”

Example: $x = 6$ is a **Root or Solution of the Equation** $x + 3 = 9$

Equivalent () Equations

Equations having Exactly () Same Solutions

Exercise 1:

i. Is $x = 3$ a Root of the Equation:
 $x^2 - x + 3 = 6$?

ii. Does $x = 4$ Satisfy the equation:
 $x^2 - x + 3 = 15$?

Exercise 2: Determine which are the Equivalent Systems?

1. $\frac{x-9}{2} = 5$ and $9 = x - 10$

2. $\frac{6}{x-3} = \frac{x+3}{x-3}$ and $18 = x + 15$

“**Procedure** () to Solve an Equation in Variable x ”

Write a Sequence of Equivalent Equations by “Allowed” Operations till we produce an Equation or Equations of the Form
“ $x = a$ Constant”

Allowed () Operations on An Equation

1. Combining Like Terms on any side of an Equation

2. Using Different Laws like Commutative, Associative, Distributive.

3. On Both Sides of the Equation:

- Adding or Subtracting the same Quantity
- Multiplying or Dividing by the same Nonzero Quantity

Exercise 3: Identify () the Property used in Solving the Following Equation:

$\frac{x-1}{2} + \frac{x-1}{2} = \dots$ i. $\frac{x-1}{2} = \dots$

ii. $5x - 15 = 10$ (?)

iii. $5x - 15 + 15 = 10 + 15$ (?)

iv. $5x = 25$ (?) v. $x = 5$ (?)

Linear Equation in Single Variable x

$ax + b = 0$, where a and b are Constants with $(a \neq 0)$

Important

Once you solve an Equation, Verify () its Solution by Substituting () the Answer in the Equation

Exercise 4. Solve

$(x - 2)(2x + 3) = 2x(x - 1)$ [Ans. $x = 6$]

Also, state the Property with Each Step of your Solution and Check the Solution

Types of Equations (Related to Solution)

Contradiction

()

An Equation with no Solution

$2x = 2x + 5$

Conditional () Equation

An Equation that is True for Some Values of the Variable but not True for Other Values of the Variable

$2x + 1 = 5$

Identity ()

An Equation that is True for Every Real Number for which it is Defined

$(x - 1)^2 + 3 = x^2 - 2x + 4$

Exercise 6. Match the Equation with its Type

Equation	Type	Equation	Type
1. $3x+6 = x-6+2x$	a. Identity	3. $3x+6 = x+6+2x$	c. Conditional
2. $3x+6 = x-6$	b. Contradiction	4. $3x+6 = 2x$	d. None
[Answers: 1 b 2 c 3 a 4 c]			

When we Multiply Both sides of an Equation with $(x-a)$, we always Assume that $x \neq a$.

[Reason: Multiplying Both Sides of an Equation by Zero is Not Allowed]

$x \neq a$ is called a Restriction on the Equation.

Exercise 6. Solve Each Equation and Specify the Restriction.

a. $\frac{4x}{x+3} = 3 - \frac{4}{x+3}$

b. $\frac{x}{x-5} = 1 - \frac{5}{x-5}$

[Ans: a. $x=5$; b. No Solution]

(Note: When we Multiply Both Sides of Equation (a) by $(x+3)$, we must Assume that $x \neq -3$)

Question: What Restriction do you suggest for Equation (b)?