The Cultivation of Problem-solving and Reason in NCTM and Chinese National Standards

Xuehui Xie

School of Education
Nanjing Normal University.

Email: xuehui@runbox.com

Phone: 010-86560255

Abstract: This study will make a comparison on the cultivation of problem-solving and reason between national mathematics standards issued by National Council of Teachers of Mathematics (NCTM) in the United States and by Ministry of Education (MOE) of China. In order to make this comparison, the characteristics of problem-solving and reason in NCTM and MOE are discussed respectively. The ways of achieving problem-solving ability and reason will also be investigated. It will show striking differences on topic in the two types of standards.

The US national standards issued by National Council of Teachers of Mathematics (NCTM) have been regarded as the most influential guide in the United States K-12 mathematics education (Raimi & Braden, 1998; Council of Chief State School Officers, 1995). This study will focus on problem-solving and reason, disclosing its meaning, discussing its implications in mathematics. For this purpose, I will contrast against an alternative approach Chinese version of problem-solving and reason.

Both NCTM and MOE consider problem-solving ability as the main goal of mathematics education. Both of them believe that mathematical problem-solving ability should include both intellectual and non-intellectual aspects. The intellectual aspect includes the following contents: the ability to formulate, pose and investigate mathematics problems; the ability to collect, organize and analyze problems from mathematical perspective; the ability to seek proper strategies; the ability to apply learned knowledge and skills; and the ability to reflect and monitor mathematical thinking processes. The non-intellectual aspect includes the cultivation of positive dispositions, such as persistence, curiosity and confidence, the understanding of the role of mathematics in reality, and the tendency to explore new knowledge from mathematics perspective. Both NCTM and MOE view reasoning as a process of conjecture, explanation and justification. And both of them believe that mathematics education should foster students' inductive and deductive reasoning. [1]

Although both NCTM and MOE have similar ideas about the scope and roles of problem-solving and reasoning at a general level, a more detailed examination points to key differences between them. This study will examine these differences in three sections. Section I will examine the characteristics of problem-solving and reasoning in

[1] These contents are synthesized from NCTM and MOE documents.

NCTM. Section II will examine the characteristics of problem-solving and reasoning in MOE. Section III is a short summary.

Problem-Solving and Reasoning in NCTM

The major characteristics of problem-solving and reasoning in NCTM are discussed below.

Problem-Solving Is Not Only an End but Also an Approach in NCTM

NCTM specifies problem-solving as both an end and an approach:

> Problem solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings. Solving problems is not only a goal of learning mathematics but also a major means of doing so. (NCTM 2000, p. 51)

NCTM (1980, 1989 & 2000) places problem-solving ability as the overarching goal of mathematics education. NCTM proposes that problem-solving must be the focus of school mathematics and that mathematics should be organized around problem-solving, such as "a method of inquiry and application," using "problem-solving approaches to investigate and understand mathematical content" (NCTM 1989, p.76), and "building new mathematical knowledge through problem-solving" (NCTM 2000, p. 51). NCTM believes that centering mathematics learning on problem-solving helps students to learn key concepts and skills within motivating contexts:

> Instruction should be developed from problem situations. As long as the situations are familiar, conceptions are created from objects, events, and relationships in which operations and strategies are well understood ..Situations should be sufficiently simple to be manageable but sufficiently complex to provide for diversity in approach. They should be amenable to individual, small-group, or large-group instructions, involve a variety of mathematical domains, and be open and flexible as to the methods to be used. (NCTM 1989, p.11)

Students Learns on their own in Problem-Situations

The idea that students gain new mathematics knowledge through problem-solving leads NCTM to assign a very high priority to the use of **problem situations** inside and outside mathematics. It is argued that when students learn mathematics from problem situations, mathematical knowledge can be easily recalled by the students for many years afterwards by simply reflecting on the problem situations:

> Although a specific idea might be forgotten, the context in which it is learned can be remembered and the idea re-created. In this way, students develop a framework of support that can be drawn upon in the future, when rules may well have been forgotten but the structure of the situation remains embedded in the memory as a foundation for reconstruction. (NCTM 1989, p.11)

The use of problem situations in the classroom also provides students opportunities to "experience the power and usefulness of mathematics in the world around them [and provides] a consistent context for learning and applying mathematics. Problem situations can establish a 'need to know' and foster the motivation for the development of concepts" (NCTM 1989, p. 78). Therefore, students should be placed into classroom problem-solving situations from the very earliest stages of mathematics learning. For example, NCTM suggests that students learn proportion by making food in classroom based on a recipe. NCTM (1989) believes that this method is a tool for conceptual understanding and mathematics application:

> Knowledge often should emerge from experience with problems. In this way students may recognize the need to apply a particular concept or procedure and have a strong conceptual basis for reconstructing their knowledge at a later time. (NCTM1989, p.10)

It is also believed that when students are allowed to learn at their own pace, they reap the benefit of developing their own learning process, incorporating their individual personalities, peculiarities and their own language:

> The **idiosyncratic representations** constructed by students as they solve problems and investigate mathematical ideas can play an important role in helping students

> understand and solve problems and providing meaningful ways to record a solution method and to describe the method to others … It is important that students have opportunities not only to learn conventional forms of representation but also to construct, refine, and use **their own representations** as tools to support learning and doing mathematics. (NCTM 2000, p. 67) [Bold added]

Thus, **problem-solving is a major method for mathematics knowledge acquisition rather than merely applying the new learned mathematics knowledge to solve problems.** NCTM advocates that learning is led by the search to answer questions: first at an intuitive, empirical level, then by generalizing, and finally by justifying (proving) (NCTM 1989, p. 10).

Open-Ended Problems

Presenting students with open-ended problems is a very important characteristic of NCTM problem-solving. NCTM (1989) defines "open-ended problems" as:

> Situations that allow students to experience problems with "messy" numbers or too much or not enough information or that have multiple solutions, each with different consequences. (NCTM 1989, p. 76)

> Instruction should be developed from problem situations. As long as the situations are familiar, conceptions are created from objects, events, and relationships in which operations and strategies are well understood ..Situations should be sufficiently simple to be manageable but sufficiently complex to provide for diversity in approach. They should be amenable to individual, small-group, or large-group instructions, involve a variety of mathematical domains, and be open and flexible as to the methods to be used. (NCTM 1989, p.11)

It is also argued that such problems will better prepare [the students] to solve problems they are likely to encounter in their daily lives. That the problem situations should be close to students' daily lives is another important characteristic of NCTM problem-solving:

> School mathematics experiences at all levels should include opportunities to learn about mathematics by working on problems arising in contexts outside of mathematics. These connections can be to other subject areas and disciplines as well as to students' daily lives. Prekindergarten through grade 2 students can learn about mathematics primarily through connections with the real world. Students in grades 3–5 should learn to apply

> important mathematical ideas in other subject areas. This set of ideas expands in grades 6–8, and in grades 9–12 students should be confidently using mathematics to explain complex applications in the outside world. (NCTM 2000, p. 65)

Applying Multiple Strategies

NCTM (1989 & 2000) emphasizes the use of multiple strategies for problem-solving, and recommends that teachers encourage students to apply these strategies. The strategies include using manipulative materials, trial-and-error, trying special values or cases, guessing and checking, listing all possibilities, gathering and organizing data in tables, looking for a pattern from the tables, drawing a diagram, and working backward (NCTM 1989, p. 76 & NCTM 2000, p. 53). From this list, we can see that prior knowledge does not play a significant role in problem-solving but rather the trial-and-error method, although NCTM does advocate using prior knowledge. It seems that prior knowledge could be very important for this type of teaching but not prior knowledge in the sense of having certain **specific** problem solving processes, rather in the sense of progressively developing more “tools” for trial-and-error and so on, plus past experiences with general situations and how to develop solutions within them. The meanings of prior knowledge in NCTM and MOE are different. In NCTM prior knowledge is more focused on students’ previous experiences rather than specific mathematics knowledge as in MOE. The method of gathering and organizing data in tables and looking for patterns is related to statistics, which is another major method of learning mathematics in NCTM.

Mathematical Dispositions

NCTM (1989 & 2000) considers mathematics dispositions as a very important objective of mathematics education. Mathematics education should not only focus on high scholastic achievement but also positive dispositions toward mathematics. NCTM (1989) lists the characteristics of the ideal mathematics dispositions below:

- confidence in using mathematics to solve problems, to communicate ideas, and to reason;
- flexibility in exploring mathematical ideas and trying alternative methods in solving problems;
- willingness to persevere in mathematical tasks;
- interest, curiosity, and inventiveness in doing mathematics;
- inclination to monitor and reflect on their own thinking and performance;
- valuing of the application of mathematics to situations arising in other disciplines and everyday experiences;
- appreciation of the role of mathematics in our culture and its value as a tool and as a language. (NCTM 1989, p. 233)

Clearly, NCTM assigns mathematics dispositions a vital role in mathematics education and assumes that such dispositions are best acquired through lots of problem solving activities and lots of individual explorations.

Reasoning in NCTM

NCTM (1980, 1989 & 2000) specifies the following goals to develop reasoning in students. First, "students should recognize that reasoning is based on specific assumptions and rules." Second, students should be encouraged to make and investigate conjectures. NCTM assigns conjecture a great role in reasoning. NCTM envisions a progression of reasoning skills, beginning with trial-and-error strategies (which are then examined and analyzed) to conjecture strategies. These conjecture exercises are essentially formulating hypotheses about problem solutions and testing them:

> Informed guessing ―is a major pathway to discovery. ..Young children will express their conjectures and describe their thinking in their own words and often explore them using concrete materials and examples. Students at all grade levels should learn to investigate their conjectures using concrete materials, calculators and other tools, and increasingly through the grades, mathematical representations and symbols. They also need to learn to work with other students to formulate and explore their conjectures and to listen to and understand conjectures and explanations offered by classmates. (NCTM 2000, p.56)

This excerpt shows the following characteristics of investigating conjectures. First, the recommendation that students should formulate and solve conjectures from their own exploration indicates that NCTM prefers students to learn mathematics from their personal experiences and from something like an innate problem solving ability believed to be present in human beings and capable of developing. Second, NCTM also recommends that students learn from each other by communicating and by working together. Third, NCTM assumes that students naturally progress to systematic reasoning at their own pace with the increase of their grade levels.

Unsystematic reasoning and induction are the major focus in grade pre-k-8, although NCTM also mentions deductive reasoning. NCTM encourages students to select and use various types of reasoning and methods, especially the trial-and-error method, in order to justify their conjectures and solutions. For example, NCTM recommends that young children use an unsystematic trial-and-error method. Here NCTM equates "unsystematic trying of many cases" to the trial-and-error method:

> Early efforts at justification by young children will involve trial-and-error strategies or **the unsystematic trying of many cases**. With guidance and many opportunities to explore, students can learn by the upper elementary grades how to be systematic in their explorations, to know that they have tried all cases, and to create arguments using cases. …At all levels, students will reason inductively from patterns and specific cases. Increasingly over the grades, they should also learn to make effective deductive arguments based on the mathematical truths they are establishing in class. (NCTM 2000, p. 58)

However, NCTM does not explain its use of the term "unsystematic" and does not explain what "systematic" would be. Therefore it is hard to tell how much they assume children should use random efforts to solve problems and how much children begin to develop systematic efforts over time. A question of learning theory as well as of cognitive development is left unresolved in their statements. From the methods recommended by NCTM, we can also see that reasoning in NCTM refers mainly to informal induction. Based on the meaning of "unsystematic," One would argue that “unsystematic trying of many cases” usually occurs when students have no or little idea about solutions. In that situation, they try many cases as they occur to the student. Students will have a basis for coming up with certain ideas to try, and then with lots of experience at this, past efforts will help to refine future ideas. This is still very different from MOE and from more traditional math education in the United States. Compared to the meaning of unsystematic trial-and-error method, in systematic trial-and-error, students may try some cases selectively rather than randomly. This may occur when students at least know something of the relationship between the means and the end of a solution.

Furthermore, NCTM does not provide a definition of systematic reasoning but a list of informal methods in fostering reasoning at different grade levels. For example:

> Systematic reasoning is a defining feature of mathematics. It is found in all content areas and, with different requirements of rigor, at all grade levels. For example, first graders can note that even and odd numbers alternate; third graders can conjecture and justify—informally, perhaps, by paper folding—that the diagonals of a square are perpendicular. Middle-grades students can determine the likelihood of an even or odd product when two number cubes are rolled and the numbers that come up are multiplied. And high school students could be asked to consider what happens to a correlation coefficient under linear transformation of the variables (NCTM 2000, p. 56).

Implicit in this except is a theory of cognitive-development suggesting that, largely left to their own and put into the right environments students will undergo a movement from unsystematic to systematic problem solving. However, the movement through cognitive stages implied here could take place with more structured input from teachers and perhaps could take place more rapidly and successfully with such input. Studies would have to be done to determine this.

Problem-Solving and Reasoning in MOE

Thinking Ability as the Major Focus of MOE

Although MOE (1992, 2000 & 2001) regards fostering problem-solving ability as a main goal of mathematics education, MOE stresses that developing students' **thinking ability is the core of mathematical ability**. Thus, though this section focuses on problem-solving and reasoning, thinking ability must be discussed in order to elaborate on its ideas on problem-solving and reasoning. MOE defines thinking ability as:

> Being able to observe, experiment, compare, conjecture, analyze, synthesize, abstract and generalize; being able to reason by using induction, deduction and analogy; being able to express their own thoughts and opinions logically and appropriately; being able to apply mathematical concepts, principles, thought and methodologies to differentiate mathematical relations; and being able to form high-order thinking skills. (MOE, 2000)

This excerpt illustrates two differences between the MOE and NCTM views. First, MOE seeks to guide students to such thinking processes as analysis and synthesis, and abstraction and generalization. On the other hand, NCTM stresses the development of other types of thinking process, such as conjecture, reflection, etc. For MOE, the process of mathematics learning is also a process whereby the abilities of abstraction and synthesis are developed. Students are expected to identify general characteristics from their observation of particular things, and then apply them in new situations. Second, "being able to apply mathematical concepts, principles, thought and methodologies to differentiate mathematical relations" indicates that MOE encourages students to apply what they have learned to solve problems rather than placing students in pilot situations to develop their problem-solving ability. This is very different from NCTM, where problem-solving is not only an end but an approach to achieve problem-solving ability.

Students Learn through Teachers' Instruction

MOE (2000&2001) defines problem-solving and reasoning as:

> Problem-solving: being able to solve mathematical problems occurring in daily life, workplace and in other subject-matters; **being able to use mathematical language to express, communicate and form mathematical thinking.** (MOE 2000) [Bold added]

> Reasoning: being able to develop mathematics conjectures through observation, experimentation, induction and analogy, and being able to seek evidence or prove their conjectures; being able to clearly and systematically express their thinking process; and **being able to use mathematical language to discuss and question the reasonableness during the communication with others**. (MOE, 2001) [Bold added]

These definitions express similar goals as those of NCTM. However, unlike NCTM, again MOE does not mention that students learn on their own but rather that they should apply the learned mathematics language to think or communicate mathematically. This can also be seen in the goal of middle school mathematics education as specified by MOE:

> **Enable** each student to learn fundamental algebraic and geometric knowledge and skills, which are essential for each person to adjust to daily life, to succeed in workplace, and for advanced study; further **educate** computation skills; develop thinking ability and spatial concepts; **enable** them to **apply** the knowledge they learned to solve simple realistic problems; gradually **form** mathematics creativity; **foster** positive mathematics disposition; and **form** preliminary dialectical materialist point of view. (MOE 2000, *National Mathematics Guidelines*) [Bold added]

It clearly shows that teachers and instruction are to play a significant role in students' learning.

Throughout the different versions of MOE standards, the overall emphasis discussed above has not changed. In its updated standards, MOE (2001) claims that Chinese education is in transition from the center of teachers to the center of students. MOE also claims that the new standards have paid more attention to students. It is should be noted that in any education the ultimate goal is to see whether **students** rather than the teacher master a given knowledge, or develop a given skill and ability. The difference lies in the approaches in achieving these goals. Clearly MOE realizes some problems of the teacher-centered education and wants to overcome them by paying more attention to students. Interestingly, this focus on students in the new MOE standards is still quite different from that of NCTM. It shows that teachers still play a central role in students' learning. For example:

> **Choose** materials that interest students or that may happen in students' lives, and develop mathematics problems from these materials; **provide** opportunity for students' active thinking and communication; **present** mathematical knowledge in multiple ways to meet students' needs and increase their motivation of learning mathematics; when introducing and developing knowledge, **create** situations to inspire students to explore and recreate the process of formation of knowledge; gradually permeate important mathematical concepts and thought; value the connection among different parts of mathematical knowledge; **provide** differentiated curriculum for students with different abilities; **introduce** mathematical historical background knowledge in order to **enable** students to understand that mathematics develops from the needs of human beings, and to understand the role of mathematics in human lives. This type of historical background knowledge may be provided in reading materials of the curriculum. (MOE, 2001) [Bold added]

Mastering Fundamental Knowledge and Skills as a Main Approach for Fostering Thinking Ability

MOE (1992, 2000 & 2001) emphasizes that students must master fundamental mathematics knowledge and skills. During this process, teachers are required to gradually foster students' thinking ability and finally their problem-solving ability. MOE (2000) also provides definitions of fundamental knowledge and skills:

> Fundamental knowledge: the concepts, rules, nature, formula, axioms, theorems and the mathematical thought and methodologies that are reflected within them.

> Fundamental skills: being able to compute, draw, and think logically according to given sequences and steps.

According to MOE, mathematics education must seek not only to teach students mathematics knowledge but also to demonstrate the thinking process of discovering and developing knowledge. The latter is considered as very important in cultivating students' abilities. MOE (1992, 2000 & 2001) emphasizes the importance of developing students' thinking ability through helping them to master mathematical thought and mathematical concepts, formulae, theorems and rules, and problem-solving strategies. The aim is to expand and promote students' thinking ability within these processes, thus develop their scientific spirit and creativity, enable them to form and develop new knowledge, and lead them to use the newly learned knowledge to solve problems.

The Relationships among Knowledge, Skill and Ability

MOE (1992, 1995, 2000 & 2001) holds that knowledge, skill, and ability coexist and influence each others' growth. Skill is considered as a lower level than ability because skill tends to be specific, whereas ability covers a broader and more general scope. For example, in *Guidelines of Elementary School Mathematics* (2000), MOE divides computation skills into three levels based on proficiency. The first level is the skill to compute be able to compute correctly according to given procedures. The second level is the skill to compute more proficiently through mathematical **training and practice**. At this level students are able to compute correctly and relatively quickly. The third level is the skill to compute correctly and quickly. At this level students are able to choose the simplest way to compute flexibly. MOE also indicates the goal of further developing students' abilities based on skills at this level.

Besides the above, the definition of computation ability also shows that MOE takes a different approach from NCTM:

> Computation ability: being able to compute according to rules and formulas, and understand how and why computation is done in certain ways; and being able to find the simplest way of computation based on the given conditions of the problems. (MOE, 2000)

MOE (2000 & 2001) requires students to find the **simplest solution** in order to sharpen their thinking and to foster their flexibility of thinking. This differs from NCTM, which only encourages students to use multiple strategies or methods but does not encourage students to find the simplest one. This definition shows that MOE also stresses the importance of conceptual understanding, since it requires students to "understand how and why computation is done in certain ways." For example, in an American classroom, the mathematics teacher gives a problem such as 24 + ? = 73. One student uses trial-and – error method to obtain the answer of 49: 24+46=70. 24+47=71; 24+48=72; 24+49=73. So the answer is 73. Another student obtain 49 by 73-24. The two students use different strategies to obtain the answer. However, the strategy of the second student is simpler and shows that the second student has advanced conceptual understanding. The teacher approves both students. If the same problem happens in the Chinese classroom, the teacher would point out that students use the second strategy .

The Different Levels of Ability and Skills

MOE spend considerable space on defining mathematical knowledge, skills and diverse abilities, as well as the different levels of abilities and skills. This information is elaborated in MOE *Elementary School and Junior High School Mathematics Guidelines* (1995& 2000) and the *National Mathematics Standards* (2001). For example, MOE (1995, 2000 & 2001) specifies the mathematical thinking ability at four levels. The first level is to **know**. At this level, students are able to recognize and identify, differentiate and classify what they learn at the perceptual level. The second level is to **understand**.

At this level, students should have theoretical knowledge, be able to use mathematical language to express the meaning, be able to know the function, and be able to know the connection among different parts of mathematical knowledge. The third level is to **master**. At this level, students should be able to analyze, compute and make decisions, and be able to explain and justify their decisions. At this level, students should also be able to apply the learned knowledge into new situations on the basis of understanding. The fourth level is to **apply flexibly**. At this level, students should be able to synthesize and integrate the newly learned mathematics knowledge and skills with other prior knowledge, and be able to flexibly and reasonably select and apply what they learn to solve practical problems. For example, in geometry study, MOE would ask first students to learn and to recognize a triangle, a rectangle, or other shapes. Second, students know the characteristics of these shapes as well as their relationships, such as a rectangle can be divided into two triangles; based on this relation, students obtain the formula to compute the area of a triangle. Third, students master the formula and be able to use it to compute area of a triangle. Forth, students will be use it flexibly, such as to solve problems using shapes that are composed by triangles, rectangles ..etc

MOE (2001) regards skills as having three levels **imitate, solve mathematics problem independently, and transfer**. By the transfer stage, students should be able to make connections with other knowledge, transform complex problems into simple ones, and grasp the "fundamental structure" that unifies mathematical concepts and knowledge. In MOE, this fundamental structure refers to interconnection among mathematics knowledge and mathematical thought and methodology as reflected within. In addition, the process from imitation to solving problems independently indicates that MOE advocates that students learn from teachers rather than from their own efforts alone as NCTM proposes. MOE (2001) also defines the process of learning mathematics. First, students **get experience from their participation** in learning mathematical subject-matter. Second, students **agree, accept, and appreciate** mathematics. Third, students **internalize** what they learn.

From the analysis above, MOE emphasizes internalization in learning mathematics. This reflects two kinds of learning process. In NCTM, the learning process is that learning by doing and by trial-and-error, which would be the NCTM version of internalization. However, what is internalized differs and how internalization occurs is regarded differently. It seems that MOE emphasizes a process by which ideas, methods, skills and so on are introduced from outside and practiced and used until fully internalized and understood. NCTM thinks that students will try out different ideas, concepts, and so on, developing skills along the way and end up by internalizing "what works" for them which is all the more internalized since it began with their own ideas. The way this is supposed to happen is less theorized by NCTM than by MOE. MOE makes many distinctions ranked in order of how they are acquired and build upon each other. NCTM is vague about how unsystematic trial-and-error will lead to systematic trial-and-error.

The Use of Relationships between Conditions and Conclusions in Fostering Ability

MOE (1992, 2000 & 2001) seeks to train students to know relationships between conditions and conclusions. For example, MOE says that students must "be able to find and design the reasonable and simplest solution based on the **given conditions of the problems**," and "be able to draw shapes according to **given conditions**, and be able to question the reasonableness toward the source of the data, the methods of dealing with the data, and the conclusions drawn from within" MOE clearly regards the distinction between conditions and conclusions as very important. The idea is that, first, one can only draw conclusions from given conditions. If conditions change, the conclusions will also change. Second, conditions help students to understand the role of confinement at a given level of mathematical knowledge. For example, the statement "two straight lines are either parallel or they intersect" is true only within the confinement that the two straight lines are on the same plane. Understanding the conditions from which a given conclusion is drawn helps students to transfer mathematical knowledge and skills into other situations but within the appropriate confinement. MOE regards this important distinction something for teachers to make apparent to students until students internalize it and automatically apply it on their own.

Students' Creativity, Mathematics Dispositions, and Independent Thinking

These characteristics are reflected in the following excerpts:

> The creativity that students should cultivate: Have curiosity toward the phenomena that occur in society and natural world, constantly pursue new knowledge, think independently, know how to look for and raise problems from mathematical perspective; and use mathematical methodologies to discover and solve problems. (MOE, 2000)

> Applications of mathematics include: Being able to recognize the mathematical information in reality and the wide application of mathematics; being able to independently to solve problems based on prior mathematics knowledge and skills; and when meet new mathematics knowledge, being able to independently explore and understand its existence and the value of its application. (MOE, 2001)

> The dispositions of students should include: Have correct learning goals, and have interests, confidence, and persistence in learning mathematics; have scientific attitude of looking for the truth from practical problems, be willing to explore and create new knowledge, as well as to apply knowledge to reality. (MOE, 2000)

Thus, MOE does assign importance to creativity, independent thinking and mathematics dispositions of students. MOE argues that students' creativity, mathematics dispositions and independent thinking should build on the mathematics knowledge learned from teachers. This is in contrast to NCTM, where independent thinking is encouraged from the outset of mathematics study. In other words, MOE places independent thinking on the foundation of established textbook concepts and knowledge, while NCTM considers independent thinking to be the foundation of learning. While NCTM seeks to allow positive mathematics dispositions to develop naturally with minimum direct intervention by the teacher, MOE guides students to positive dispositions by seeking to directly cultivate a learning attitude.

Summary

In NCTM, the term "problem-solving" is used to refer both to an end and an approach; while in MOE, problem-solving mainly as a goal. NCTM believes that students naturally develop problem-solving skills through their own exploration of mathematics knowledge. In general, MOE follows the sequence of mastering mathematics knowledge and skills internalize these knowledge and skills, and form mathematical thinking ability form problem-solving ability. In this sequence, students learn mathematics under the guidance of teachers, so trial-and-error is not encouraged as a strategy to learn mathematics in MOE. On the other hand, NCTM makes the use of trial-and-error as a significant strategy because students are placed into pilot situations in the classroom from the outset. Apparently NCTM believes that mathematical ability originates from students, while MOE sees imitation and internalization as important foundations to develop students' mental process.

In line with the NCTM contention that students learn best by doing, it is recommended that students should use manipulative materials and should be allowed to measure, record and look for patterns in the data that they observe in problem-solving. These strategies are not used as major methods in MOE mathematics learning.

Since NCTM proposes developing students’ problem-solving ability by exposing them to problem situations, encouraging them to manipulate objects, to use trial-and-error, inductive reasoning is the major focus, although NCTM does mention deductive reasoning. On the other hand, MOE proposes to develop problem-solving and reasoning through instruction, where teachers demonstrate the developing thought process and help students to master underlying mathematical methods and methodologies.

NCTM advocates open-ended problems because they are regarded as close to problems that occur in real-life situations. In MOE, most problems only have one answer, although problems may have multiple approaches to reach the answer. Under this framework, problems are presented with strict conditions, from which students are expected to draw conclusions.

The primary learning process in NCTM and MOE are different. In NCTM, the process is conjecture, monitor, reflect and validate. In MOE, the process is comparison and contrast, analysis and synthesis, abstraction and generalization from particular to general and from empirical to theoretical.

Different theories of cognitive development seem to underlie these approaches. MOE has worked out stages of knowledge, each stage building on the previous one and the process beginning with teacher explanations and modeling while ending with full student internalization and mastery. NCTM has not worked out specific stages in the way MOE has. It has an underlying theory of self-initiated and directed movement from unsystematic to systematic problem-solving efforts but the complexities of this process are not explained.

References

Council of Chief State School Officers, Washington, DC (1995). *State curriculum frameworks in mathematics and science: How are they changing across the States?*

Ministry of Education of China (1992). *The elementary and junior high school mathematics curriculum plan of nine-year compulsory education.*
1992

Ministry of Education of China (1992). *The description about curricular plan of nine year compulsory elementary and junior high school mathematics*.
1992

Ministry of Education of China (2000). *The guidelines of junior high school mathematics education*.
2000

Ministry of Education of China (2000). *The guidelines of elementary school mathematics education*.
2000

Ministry of Education of China (2001). *The guidelines of fundamental education curriculum reform.*
2001

Ministry of Education of China (2001). *The national compulsory mathematics curriculum standards*.
2001

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*.

Raimi, R. & Braden, L. (1998). State mathematics standards: An appraisal of math standards in forty-six States, the district of Columbia, and Japan. Thomas B. Fordham Foundation, Washington, DC.