

Understanding Rate of Change Using Motion Detectors: One Teacher's Voice, Perspective, and Growth

Abstract

This study discusses how learning experiences with computer-based motion detectors created through innovative professional development activities helped one teacher develop his own ideas about rate of change relative to velocity and position concepts. The teacher was interviewed before he taught a lesson on qualitative graphing using motion detectors and was observed in a group discussion after the lesson had been taught. The methods of data collection included audio-taped interviews and videotaped professional development meetings. The teacher's initial conception was that graphs of position versus time and velocity versus time were graphs of separate events. He refined his understanding and formulated that both graphs could represent different but related quantities of the same event (motion, for instance). The teacher also stated the importance of being able to simultaneously observe position versus time and velocity versus time graphs. These results illustrate how carefully crafted professional development actions and real-time graphing technology helped this teacher widen his knowledge about the quantities, velocity and position. This teacher was able to increase his initial understandings of rate of change concepts when he was given the opportunity to predict graphical shapes of given motions, to test these predictions, and to explain, reflect and analyze his thinking. We conclude that since teachers have often limited occasions to act as learners while preparing to teach, there exists a need for innovative professional development approaches that can help create both inquiry experiences and learning through reflection opportunities for in-service teachers.

Introduction

The use of rate of change is prevalent in secondary mathematics and includes a wide variety of subjects and applications. It is also the precursor to more complex problems that deal directly with differential and integral calculus (Wilhelm, Confrey, Castro-Filho, & Maloney, 1998). Current research advocates the introduction of rate of change in the curriculum at earlier grade levels (Kaput, 1994). For example, Confrey and Smith (1994) defend the use of rate of change as an entry point to think about functions.

Most studies on understanding rate of change have focused on students (Lobato and Thanheiser, 1999). Some of those studies have shown that by using a motion context of rate of change with computer-based motion detectors, the ideas of distance, speed, and acceleration can be experienced through modeling motion (Nemirovsky, Tierney, and Wright, 1998; Noble, Nemirovsky, Wright, and Tierney, 2001). Observing the results of one's own actions in a graphical environment can be a very powerful tool

for understanding. For example, one can compare how a position versus time graph or a velocity versus time graph changes when one increases or decreases his/her speed.

Confrey and Smith (1994) and Stroup (1995) argue that real-time graphing environments help learners develop knowledge about graphic patterns and rate of change concepts. The National Council of Teachers of Mathematics Principles and Standards (2000) state that, "Technology can help students learn mathematics...with calculators and computers students can examine more examples or representational forms than are feasible by hand, so they can make and explore conjectures easily. The graphic power of technological tools affords access to visual models that are powerful" (p. 25). Using computer-based motion detectors (that can plot a person's motion in real time), allows students to explore their conjectures and predictions regarding various motion situations. For example, students can predict the velocity versus time graph of a person walking back and forth at varying rates in front of the motion detector, and then check their predictions by carrying out the motion. Such real time graphing capabilities can assist with common misconceptions students have concerning various graphical circumstances. One common misconception that could be remedied involves the meaning of the negatively-sloped but positive valued region on a velocity versus time graph. Many students consider the negatively sloped region to mean a change in direction when in fact it simply means to move in the same direction, but at a slower rate.

Many research studies have been conducted examining mathematics pre-service and in-service teachers' understanding of general function concepts (Even, 1993; Lloyd and Wilson, 1998; Wilson, 1994), but few studies have investigated teachers' conceptual understanding of rate of change. Of the studies that have been conducted on this matter, it was found that mathematics teachers view rate of change mainly as a numeric quantity and have great difficulty discussing its use as a physical quantity (Simon and Blume, 1994; Thompson and Thompson, 1994; 1996). Such teacher limitations can, in turn, create serious learning obstacles for their students. Thompson and Thompson (1994, 1996) investigated the effort of a teacher to teach concepts of speed to a child. In their study, it was found that the teacher's instruction on speed involved an emphasis on numerical values instead of conceptual values. The teacher's "deep understandings of rate were often encapsulated within the language of numbers and operations" (Thompson and Thompson, 1996, p. 15). Bowers and Doerr (1998) explored pre-service teachers' understanding of rate of change while using motion detectors and a software called MathWorlds in a technology-based course. After doing a series of activities, teachers were required to create a three-lesson activity, implement the lesson, and then revise their plans based on students' learning. While Bowers and Doerr reported a development in teachers' understanding, they also found that some

teachers' conceptions constrained opportunities for students' learning. For example, one student's response was graded incorrectly because it didn't match the teacher's expectations of what she considered to be the correct velocity versus time graph. Later, the teacher realized that the student's response made sense for the given problem. Both studies support the idea that teachers' understanding and actions have a critical influence on student learning.

In order to fully consider how teachers' actions affect students' learning, it is crucial to investigate the depth of conceptual understanding in the teachers themselves. Given that most school curricula do not emphasize the study of rate of change and accumulation in an applied manner, it is not surprising that teachers may display difficulties while teaching these concepts for the first time. However, our analysis will not focus on teachers' difficulties but on the potential for learning that can happen as a result of implementing innovative technologically-based curriculum lessons with the support of a professional development team. In this paper, we explore the development of a teacher's understanding of rate of change while implementing lessons on qualitative graphing using computerized motion detectors and participating in a professional development program. We also highlight how the teacher's experiences made him begin to change his pedagogical content knowledge.

Methods

Context

The study was conducted at Tree High School, an urban school in central Texas with a majority population of Hispanic students. The study was part of a large collaborative effort between teachers of an urban high school and a research group from The University of Texas at Austin. The first phase of the collaboration between teachers and researchers was a summer workshop conducted with all thirteen mathematics teachers working at Tree High School. The workshop addressed issues about a reformed algebra curriculum based on national standards. Activities included content exploration (linear and quadratic functions), paralleling the introduction of technological tools that could be used in a reformed algebra program. Teachers used motion detectors to collect data on a ball's speed, time and distance traveled in different situations, such as bouncing and rolling up and down a ramp. The information was collected, then analyzed using computer software like Function Probe (Confrey, 1992a).

During the following Fall semester, a full day meeting was held with the teachers to discuss some possibilities of continuing their professional development and the improvement of the high school curriculum. Teachers were interested in using some of the activities from the workshop in their classroom. One of the agreed-upon ideas was the use of a replacement unit, a unit designed to be used in the place of the regular curriculum. Teachers

and researchers would create activities and materials centered around a specific content area during a short period of time. Eight mathematics teachers participated in a professional development program, which included experiencing (as learners) replacement unit content activities within an Algebra I curriculum. The unit spanned from qualitative graphing to linear function and simultaneous equations with an extensive use of technology. The teachers participated in weekly discussions about implementing the replacement unit within their own mathematics classrooms.¹

Participant

This paper focuses on one of the eight teachers, called Felipe. the criteria for choosing him were: 1) He was teaching the replacement unit; 2) He was teaching only math classes; 3) He was involved in the discussions about the replacement unit and 4) He had taught Algebra I before. From the eight participant teachers, four met that criteria and constituted the core group of the main research. For this particular paper, we choose to focus on Felipe because although he had shown conceptual difficulties about rate of change during the summer two-week workshop, participating in the unit implementation became an opportunity for him to develop his knowledge about the subject.

Although Felipe was a veteran math teacher who had been teaching at this school for eleven years, he had not used technology in his own classroom except for the occasional use of graphing calculators. He was an advocate of bilingual education and was more engaged in this issue than in discussions about content knowledge. For that reason, Felipe was usually considered an outsider since most of other teachers claimed that students should be taught only in English. Felipe had not used motion detectors for teaching before implementing the replacement unit. He had experienced the motion detector activities on two separate occasions, one at the summer professional development workshop and the other in a replacement unit-planning meeting prior to his use within his own classroom. Similar activities were used in each case. Analysis of both occasions revealed that he did not have difficulty analyzing graphs of position versus time. He was able to look at a position versus time graph, tell whether it was showing a person walking slow or fast, and relate velocity with the steepness of the graph. However, he showed some difficulties with velocity versus time graphs, in particular with the distinction between velocity and speed. During the summer workshop, most teachers, including Felipe, did not seem to know the distinction between velocity and speed. The excerpt below was drawn from the first time teachers and researchers discussed this issue.

Third author: Real quick. What s the technical difference between velocity and speed?

¹ For a full discussion about the research results, consult Castro-Filho (2000).

Second author²: One is a vector, the other one is a scalar.

Helga: Yeah.

Third author: or, in easy language?

Second author: One has magnitude and direction, the other has just magnitude.

Third author: So, if you were doing a speed thing, you might think of whether I am going this way or this way, I am going the same speed. If you are gonna do this as velocity, you are gonna call one of them a positive velocity and the other one a negative velocity.

Felipe: You said velocity, positive and negative velocity?

Third author: Velocity includes both magnitude and direction.

Speed is, usually if I am just talking informally, I would say I went to your house at 35 mph and came back 35 mph. If I am using velocity, I would say, I went 35 mph and came back negative 35 mph.

This excerpt shows how Felipe did not recognize the distinction between velocity and speed. The same pattern was observed in the discussion about velocity and speed that took place during the replacement unit planning workshop. Teachers seemed to lack experience in relating velocity versus time to position versus time graphs. This lack of experience is not surprising given that mathematics courses and textbooks usually show only position versus time graphs, using them as applications of functions but without much reference to ideas such as rates of change and accumulation.

Felipe's ideas developed as a result of engaging in teaching and reflection with the technology and activities. This paper will emphasize two pivotal occasions documenting Felipe's understanding of rate of change and his evolution of thought. The first pivotal occasion occurred at an interview meeting (with author one and author two), and the second occasion was at a professional development meeting with all the mathematics teachers and researchers after most lessons using motion detectors had been enacted.

Materials and technology

Motion detectors are devices that can detect motion by emitting sound waves and processing the returning echo from these sound waves when they reflect off objects. The distance of the object from the motion detector is computed by multiplying the speed of ultrasound in air by the time it takes for the ultrasound to go from the motion detector to the object and back. Motion detectors can be attached to computers or calculators to plot position versus time, velocity versus time, and acceleration versus time graphs. In our work, we used computer linked motion detectors and MacMotion software (Thornton, Beardslee, Travers, Nolan, & Budworth, 1994). Motion

² 2nd author was a teacher during the workshop. She joined the research group one year later as a graduate student.

detectors were used to introduce the idea of slope as a constant rate of change.

When the motion detector was introduced to the teachers, a handout was provided consisting of a series of qualitative graphing activities designed to investigate situations involving motion. The handout requested for participants to imagine a person walking in different directions (back and forth in front of the detector) with a variety of different walking rates. Participant learners were then asked to predict what a position versus time and a velocity versus time graph might look like for such motions.

Methodology

This study was of a qualitative design and was a smaller part of a larger study concerning the entire mathematics department's use and implementation of the Algebra I replacement unit materials. We used the process of triangulation, which involved ethnographic methods. Data collected included detailed observation field notes, audiotaped interviews³ of the focus teacher, Felipe, videotaped professional development meetings, and videotaped classroom lessons. All audiotaped and videotaped data were transcribed. The first interview aimed at assessing Felipe's understanding of the lesson and his plans about how to teach the lesson. At first, the handout proposed for the lesson was discussed. The teacher was then asked to explain what the main mathematical ideas present in the lesson were and how they related to the unit. Another individual interview was conducted after the lesson was concluded. This interview focused on the teacher's reflections about the lesson. In general, we discussed how he evaluated the lesson in relation to what he had planned. During the pre and post-lesson interviews, we discussed, in depth, conceptual aspects about the task. Most of these interactions were not planned but were used as opportunities for the teacher to reflect on his understanding. During these interviews, my stance was to let Felipe explore his own conjectures about the knowledge. Felipe used the computer as a tool to check his predictions. Then he explained or refined his initial conjectures.

During the observations, the first author took observation field notes about the development of the lesson. Special focus was on how the teacher was using the technology. Other details were recorded concerning any difficulties or innovative ideas presented by teachers and/or students.

Teachers' responses to the interviews, and videotape transcripts of classrooms and meetings were analyzed using two methods: the constant comparison method (Strauss and Corbin, 1998) and the voice and perspective dialect (Confrey, 1995)

The constant comparison method involves three levels of coding: open coding, axial coding and selective coding. In this research, only the first two levels of data analysis, open and axial coding were conducted, because of

³ The interview was conducted by the first author.

the limited time (about eight weeks) of data generation. All data in the form of text were entered in a software database. Then, the data were coded according to the lesson or the event in which they occurred. The next type of coding consisted of identifying concepts within the data. These concepts were later grouped into categories and sub-categories.

Another method of analysis used was the heuristic called "voice and perspective dialect," proposed by Confrey (1994). Within this heuristic, the researcher first examines the content presented by the learner as a viable model. This is accomplished by articulating "a model that we suspect may be operating for the student [learner]" (p. 8). Confrey (1995) refers to this articulation as student voice. In the second part of the heuristic, the experience of voice leads to a deep reexamination of the content by the researcher. His/her perspective about the content knowledge is changed during the process. This analysis was developed to investigate students' thinking. We expanded it here to investigate the teacher's thinking in the context of his practice. This process is cyclical instead of linear, and it starts during an interview or observation when the researcher tries to articulate the model used by the learner (teacher or student). In the case of interviews, the researcher allows the learner to test and refine his/her own conjectures. We used this heuristic for classroom and collegial discussions analysis as well.

Results

The results will be presented by the context in which they happened. At first, we present our analyses of the interview conducted with Felipe followed by his implementation of the lesson. The analyses focus on how Felipe reflected on his own knowledge about position versus time and velocity versus time graphs and refined his own understanding about these ideas. Later on this paper, we present an excerpt from a meeting with the teachers in which Felipe refined the distinction between velocity and speed. In the discussion we summarize Felipe's understanding of rates of change and the reasons for his conceptual development.

Distinguishing position versus time and velocity versus time graphs

This interview was conducted before motion detectors were used within Felipe's own classroom. The interview began with a request for Felipe's thoughts and responses to the posed questions on a motion activity handout (the same handout would be used with Felipe's class). The first question asked to "describe how one would walk to produce the following position versus time graph," (see Figure 1).

Felipe accurately described how he would walk and then demonstrated using the motion detector, which was to walk away from the detector at a constant rate. However, he became confused on the next handout question, which asked to predict the corresponding velocity versus time graph. He explained that a velocity versus time graph meant a graph produced by a

motion, and that a graph of position versus time meant a graph produced by a series of positions:

That's actually the velocity versus time graph [referring to the graph shown in Figure 1]. And the other one, the position again, is the one where we get the students, more, get more of those and just do that.

[Insert Figure 1 here]

The interviewer, unclear about what Felipe was thinking, probed him on this matter and asked Felipe to produce what he was calling the position versus time graph using the motion detector. In response, Felipe lined up a series of mouse pads in front of the motion detector and dropped each one at different intervals of time, creating a step graph⁴. He seemed to think that the position versus time graph was only a series of discrete points created by lining up multiple people or objects in front of the motion detector with no motion occurring (except for the motion of each object dropping out of the way so that the detector could detect the next object immediately behind the first). He also seemed to believe that the velocity versus time graph was this same position versus time graph made smooth and continuous, and that only the velocity versus time graph was produced when back and forth motion occurred in front of the detector. This interpretation was consistent with Felipe's idea that the motion detector worked by calculating discrete distances, but was displayed as a continuous graph. He used this idea many times during the interview.

What is interesting about this discussion is that if the interviewer had asked Felipe only about the first position versus time graph, it would appear that Felipe understood the graphing situation. However, when the follow-up question concerning velocity was introduced, the contextual circumstance became complex which allowed the interviewer to document different aspects of Felipe's understanding.

To better understand Felipe's thinking on this matter, the interviewer suggested that Felipe walk in front of the detector at a constant pace while both position versus time and velocity versus time graphs were visible on the computer screen (Figure 2). After the real-time graphs were produced, Felipe examined what the graphs were showing.

I can see why the distance [versus time graph] is gonna [going to] keep on, the further I get, the higher is gonna go...And this [the velocity versus time graph] I understand too because it is a constant, I was going at a constant velocity so it's gonna be the same.

⁴ In the workshop, Felipe had participated in producing a step graph through lining up people or objects at different positions in front of the motion detector.

[I insert Figure 2 here]

However, Felipe was confused with the idea that one motion could produce two separate graphs: " See, I need to think why is that we are gonna do the same walk, one to measure position versus time, then do the same walk to measure velocity versus time." He further described how velocity could be obtained from the position versus time graph (shown in Figure 2), by finding its slope. In fact, he thought that this made more sense than to have a separate velocity versus time graph.

Why then, does velocity fit the idea that it is only a change of positions, and we measure that in a slope, that is not zero slope. And then looking at the other side, well because the velocity isn't gonna change, that's got to be zero, only changes right here at the beginning.

Felipe was referring to the velocity versus time graph having a zero slope since it is a horizontal line. He then began to articulate that there are two different quantities (position and velocity), which are represented by two different graphs for the same motion.

So, it really comes down to the idea that you are graphing, ah, relating two different situations...You are walking to get the position. And you are walking to get the velocity...But then of course, what you are gonna graph is something that is change in position, this is the position [pointing to the position versus time graph], this is the change in position versus time [pointing to the velocity versus time graph].

Felipe's interpretation was not completely accurate for the velocity versus time graph, since he was not saying *change in position over change in time* versus *time*. However, he was correct about the complexity of the situation when he explained, the graphs of position versus time and velocity versus time to be graphs of two different comparisons. Felipe made further sense of the two separate graphs, when he explained how one could find *velocity* in a position versus time graph.

For every meter, you go one second. Or other meter, you go another second. And then, when you put a line to it, that's [it]. This is gonna be that graph. And you are saying distance to time, which is speed⁵...And that's why I expected, when they say, well, graph the speed. Well, that one right there [the distance versus time graph] is a speed graph...The bottom [the velocity versus time graph], just show you, you are comparing the velocity from one second to the next. Cause it stays constant... And I understand that's what it tells. But when it asks

⁵ Felipe at this time is still making no distinction between speed and velocity, or distance and position.

where is the velocity versus time, I might be inclined to say, oh, where is the velocity graph? That's it right there [pointing to the distance versus time graph]. And I guess you have to say, where is the velocity versus time graph. That's this one [the velocity versus time graph].

From Felipe's perspective, he illustrated the importance of a position versus time graph since it not only displayed a position of an object at any time, t , but also displayed the physical rate of change quantity, velocity, in its slope (which was what seemed to create his confusion during the first part of the interview). The interview ended with a discussion concerning why it would be important for his students to experience this same motion detector activity. He mentioned that students needed to understand ratio, slope, and "the different comparisons," meaning position to time and velocity to time.

Implications for teaching

Felipe's investigations during the interview were reflected in his classroom practice. When teaching the lesson on qualitative graphing, he discussed with students both graphs of position versus time and velocity versus time. He initially had students explore, in groups, graphs of position versus time using motion detectors. He then had students walk while the motion detector graphed simultaneously position versus time and velocity versus time. The following excerpt shows Felipe's discussion with one of the groups.

Felipe: ... OK. Now...you are not comparing distance to time by itself anymore. Now you want to compare speed or velocity to time. What happens if I walk like you were all doing a while ago, just a constant. I am just walking the same way, so what do you have here? [Pointing to the graph on the computer screen] I go at a certain speed, but I keep it. I go let's say 5 miles an hour.

B4: Stays flat.

Felipe: Ok, very good, is that why this thing is flat like that (pointing to a velocity versus time graph)? So what do you think is going to happen, how am I gonna make it go up. Let's say goes up a little bit and still be flat.

B5: You walk faster.

B4: Go faster.

Felipe: Ok, so he is saying go faster but keep the same pace. Just drive faster. If you are doing 5 miles per hour here, just do it 7 maybe 8 miles per hour here and stay the same (constant).

Felipe also analyzed with his class graphs that displayed two separate motions of a student walking at a "slow" rate and at a "fast" rate. For the position versus time graph, Felipe explained how the distance was always

going up. For the velocity versus time graph, he discussed with students how the graph was showing two horizontal lines, one for each constant velocity. In the post interview, Felipe commented that he found it important to use both graphs with students.

Felipe: I liked the idea that we all did the distance versus time, and velocity versus time, [that] students [were] seeing both of the graphs simultaneously. Because they did a comparison... And when you see [the two graphs], it makes a difference and of course, what I want to point out, very distinctly is [in one] you are comparing distance to time. And [on] the other one, you are comparing velocity to time.

Felipe's position reflects how teachers are influenced by their own experiences. If he had not explored both graphs in the pre-interview, it would be unlikely that he would have discussed both in the lesson. Also, he would have experienced difficulties conducting the discussion because he was not making a distinction between both graphs before the pre-interview. This indicates how teachers need to experience the concepts as learners before using activities with students. This entire episode also illustrates how Felipe's experience influenced his classroom pedagogy. He believed students needed to engage with both position and velocity versus time graphs simultaneously in order to actively "see" how the two graphs represented the same motion. Felipe still had some problems with the lesson as he was not making distinctions between velocity and speed. During the meeting between teachers and the research group, Felipe, along with the other participant teachers continued to explore the conceptual aspects of motion detector.

The Meeting

Most professional development meetings began with video clips of different classroom lesson enactments that displayed the various ways the replacement unit was being implemented by the mathematics teachers. This particular meeting occurred after most teachers finished their lessons on motion graphing. One of the clips shown was Felipe's lesson using motion detectors (which was described earlier). Two other video clips were shown of this same lesson enacted by two other math teachers. Discussion ensued on the pedagogical aspect of whether position versus time and velocity versus time graphs should be taught simultaneously or separately. Felipe argued that both graphs should be taught at the same time. Some teachers agreed with Felipe that it was important to show both graphs with the stipulation that lessons began with the position versus time graph for at least a small period of time before introducing the velocity versus time graph. Other teachers felt there was no need to ever show the velocity versus time graph (and did not do so within their classrooms).

During the meeting, one of the researchers asked the teachers to predict how a person should walk in order to produce the velocity versus time graph shown in Figure 3.

[Insert Figure 3 here]

Felipe instantly stood up to interpret the graph. This bold act was out of character for Felipe⁶, but he seemed to feel empowered by his new understandings. He spoke as he acted out the "walk" communicating to his peers his actions.

That will be, when you get the detector you are already in motion, that will be constant motion [referring to segment A to B]. So you've been moving at the same time and then you slow down [referring to segment B to C], and then you start speeding up again [referring to segment C to D], oh, at a constant rate [referring to segment D to E]. One flat, the flat part would be when the detector hit you, you don't start from the detector cause then you would start from zero on the velocity. [referring to the origin (0,0) on the figure 3]

Felipe was precise on his interpretation of the initial speed. He knew that one needed to already be moving with a constant velocity by the time the motion detector picked up the motion. However, Felipe had problems interpreting the graph, particularly in regard to negative velocity. He also indicated that "you don't start from the detector cause then you would start from zero on the velocity," which was incorrect since a velocity versus time graph never shows the initial starting position.

Felipe: Ok, so you start up with a speed and then you are slowing down and then you start picking up again [Felipe was walking at a constant pace, then slowing down and then speeding back up] to a constant, a constant velocity that is slower than what you initially had. [Felipe never changed his direction.]

Two other teachers raised concerns about his interpretations and suggested the point where the graph change from positive to negative could mean a change in direction.

Helga: You turn and change directions at that point? At the zero point. Is that a turning point? Where you then, Felipe said, speed up again...you're speeding back up about the same speed but you are going the other direction. Is that what it is?...

Michelle: You have to be moving the other direction cause if I was gonna see it, I was gonna go like negative ten miles per hour so that would mean that you would be moving backwards

⁶ As we mentioned before, Felipe usually engaged in conversations about bilingual education but not on issues of content knowledge.

in my car. I mean you really can't literally go to negative ten but you can signify it. I mean, you are still going ten miles per hour but if you base your velocity on a point where you are trying to go.

This discussion revealed a couple of interesting points. Felipe did not distinguish speed from velocity. This created for him a difficulty in interpreting what happens when a graph of velocity becomes negative. Felipe used his ideas about graphs and changing speed to interpret the situation. He understood that a zero slope meant no change in speed, so he started with a constant speed. He realized he needed to already be moving in order to produce a graph with an initial velocity different than zero. Felipe also interpreted the negative slope in the velocity versus time graphs as if the speed were decreasing. Since he was not making any qualitative distinction between negative and positive speed, he assumed that a negative speed would just be smaller than a positive speed. Michelle and Helga admitted that they found the idea of negative speed strange but accurately interpreted the change from positive to negative as a change in direction. Teachers were struggling with the meaning of negative with speed, which they considered a quantity but not a potential deficit (such as a negative transaction or a withdrawal). Another teacher, Virginia, said she never thought there was a distinction between the two: "It just never occurred to me, this difference between velocity and speed." The dialogue continued with teachers and researchers discussing the distinction between velocity and speed.

Teachers then talked about the distinctions between graphs of position and distance. This led to additional insight into how velocity and speed are used differently. In the case of velocity and position, it matters not only how much or how fast you walk but also in which direction along a one-dimensional line.

Throughout this velocity versus speed discussion, Lupe often interjected which helped to document his knowledge construction. At one point he indicated that he had actually done a "walk" of a speed versus time graph as opposed to a velocity versus time graph and had neglected to factor in the direction component of velocity. He further explained his thinking by comparing the motion activity with another replacement unit activity that had to do with bank accounts. The bank account activity asked students to analyze two different graphs, one was a graph of an account's balance versus time (an accumulation graph) and the other was a graph of daily transactions versus time (a rate of change graph). He explained how both replacement unit activities would help students to better understand slope. He and others further described that a "negative speed" (velocity, a vector quantity that has both magnitude and direction components) would be like a

“negative daily transaction” (or a withdraw from an account) where both situations indicated the direction component of the rate of change quantity.

Discussion

The development of Felipe’s understanding can be summarized as follows. He initially correctly explained how to produce a linear positively sloped position versus time graph when using a motion detector. However, after probing further within this same interview Felipe explained that a graph of position versus time could only be produced by lining up people or objects in front of the motion detector, but with no motion. He also thought that motion was only necessary to produce a graph of velocity versus time. After visualizing both graphs on the computer screen for one single motion, he realized that the graphs actually represented different comparisons. One graph was comparing position to time and the other graph velocity to time. However, he was still intrigued about why one would need two graphs when velocity is constant. In this linear case, he argued that only the position versus time graph would be necessary since the velocity could be directly seen through the slope of the position versus time graph.

The next pivotal step in Felipe’s development was realizing that velocity and speed are different concepts. Through the professional development meeting discussion with other teachers, Felipe realized that speed had only magnitude, and velocity had both magnitude and direction.

A superficial interpretation of Felipe’s ideas could be that he had misconceptions that needed to be corrected. An alternative interpretation is that his ideas were in development and needed more reorganization and refinement, rather than replacement (Smith III, diSessa, and Roschelle, 1993). To examine this alternative, we need to discuss the possible sources of Felipe’s ideas.

One possible source comes from Felipe’s awareness of the dynamics of motion detectors. Felipe described in a follow-up interview that the motion detector takes discrete data of distances away from the motion detector and then displayed them as a continuous graph. Even though his understanding on how a motion detector works created some difficulties for his interpretations, it suggests a deeper understanding of the dynamics involved in the use of motion detectors.

Another possible source may be the fact that the activities used the linear case, that is, walking with a constant velocity to produce the position versus time graph.⁷ Felipe’s questioning of the need to use two different graphs to represent the same action is especially valid in the case of linear graphs. Felipe was able to find the velocity from a position versus time graph; therefore his conception was not a limitation but a reflection of his deeper understanding of constant slope. Given that the slope of a linear graph is constant, one could argue that the position versus time graph also

⁷ In another interview after the meeting, Felipe pointed out that most of the walking done was with a constant speed.

plainly shows the velocity, and therefore a separate graph of velocity versus time is not necessary (Stroup, 1996). The same cannot be said for a velocity versus time graph with constant velocity. To find the position at any given time, one would need to know the initial position, which is not information displayed within a velocity versus time graph.

Although Felipe had explored motion detectors and the materials before teaching them⁸, his ideas began to change only after he had to teach the materials and were confronted by issues of students' learning. During previous workshops, for example, the discussions about velocity and speed were short and restricted to giving definitions of speed and velocity. In contrast, the discussion presented here showed Felipe bringing different examples to the conversations. Even though he still experienced difficulties, he engaged more in the discussion. It was when Felipe realized that his non-distinction was causing a conflict in his interpretation of the graph, that a need for distinguishing velocity and speed was created. During the meeting, he also realized that direction of motion could be represented graphically. After the meeting the researchers admitted they expected improvement, but were quite surprised by the depth of discussion and engagement among teachers, including Felipe.

A crucial element within Felipe's conceptual development was the use of technology. Motion detector and computer use allowed Felipe to test his conjectures and reflect on the inconsistency of his predictions. After he refined his conjecture, he could test it and see how it was holding up for different conditions. The real time graphing environment allowed Felipe to observe the results of his actions and to test his conjectures. Seeing both graphs of position versus time and velocity versus time were so powerful to him that he argued for using both graphs together within the same lesson. Another factor in his development was his interaction with other teachers and researchers. The interview and collegial discussions were opportunities for Felipe to reflect and refine his understanding. Discussing his own ideas and listening to different perspectives helped surface and clarify his confusion. Teachers do not usually have such an opportunity. They are usually expected to have learned about content knowledge away from their practice.

The implication of our study highlights, not only, the need of developing teachers' content knowledge and pedagogical content knowledge within their own practice, but also potential solutions that involve innovative professional development approaches supplemented with technology. This professional development design included support (from researchers and other teachers) for teachers who were trying to implement new curricula. Integrated within this action plan were numerous experiences and learning opportunities for in-service teachers who normally have little or no time to

⁸ As we mentioned before, Felipe undertook the activities two times, in the 1997 summer workshop, and again in the 1998 workshop while preparing to teach the replacement unit.

be a “ learner.” This design can serve as a future model to improve teaching practices. We also discussed how apparently limited ideas can reflect a deeper understanding of other issues, and how those ideas can be developed. We explored the concept of rate of change as a particular example, but we argue that similar results could be found in other content areas.

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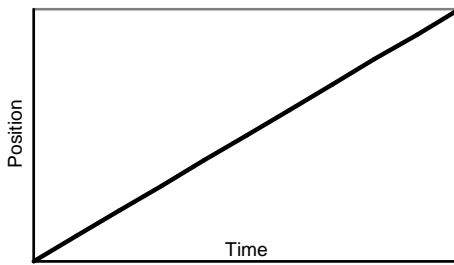


Figure 1 – Position versus time graph discussed with Felipe.

Understanding Rate of Change Using Motion Detector

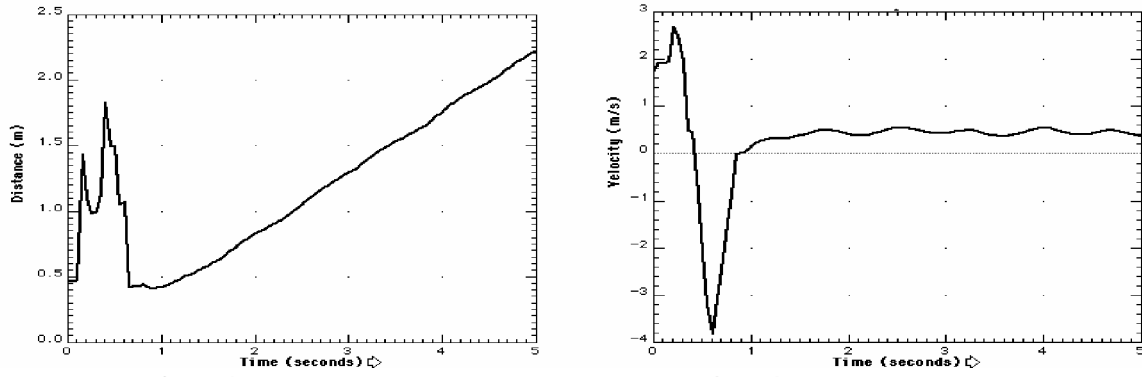


Figure 2 – Graphs of Position versus Time and Velocity versus Time produced by the motion detector for Felipe's walk at a constant pace.

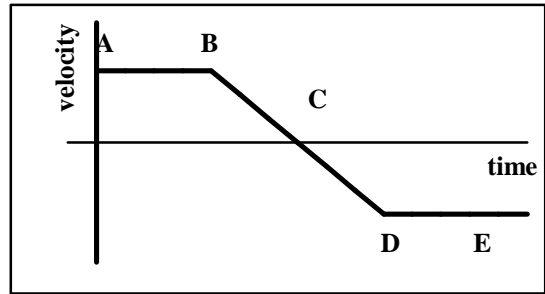


Figure 3 – Graph of Velocity versus Time discussed in the meeting.