

# Mathematical Self-Efficacy: A Pilot Study Exploring Differences Between Student Groups

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## *Abstract*

This paper describes the results of a pilot study designed to investigate differences in mathematical self-efficacy for two groups of students taking a general mathematics unit as part of their year 1 computing and IT undergraduate studies. It further investigates two linear programming models to see whether mathematical self-efficacy scores can be used to indicate an appropriate choice of course for certain students on application to university. The results of the survey give some indication of differences between the groups and suggest that a larger study may yield benefits in the selection of students for courses and also the way mathematical material is taught.

**Key words:** Linear programming, mathematical efficacy, mathematics teaching.

## **Introduction**

The mathematical ability of students entering UK universities has been a matter of some concern and debate for a number of years (Henry, 2004) and this concern has been felt not only in terms of the mathematics required for general university entrance (usually GCSE mathematics at grade C or better) but also on courses for which mathematics is a primary requirement (Engineeringtalk, 2001).

At London South Bank University there are, in common with other similar institutions, concerns also about the variety of qualifications in mathematics which students exhibit on application (including for example Key Skills in Numeracy and a range of overseas qualifications) so that trying to match a student's abilities in mathematics with university entrance requirements is sometimes difficult. Within the computing and IT subject domain mathematics plays an important role and all students taking these subjects at London South Bank University are required to take a first year unit entitled 'Concepts of Mathematics and Statistics' the purpose of which is to provide students with a basic grounding in key mathematical ideas (number representation, number bases, basic algebra etc.) as well as provide an introduction to statistics and probability. The unit is taken by computing and IT students studying for honours degree (BSc) and also those taking Higher National Diploma (HND) courses. The entry requirements of these courses are different in terms of the advanced qualifications required but frequently tutors have commented on how difficult it is to

decide whether a candidate's mathematical background (in terms of formal qualifications) indicates acceptability for the three-year BSc programme, or whether a longer path to honours degree via the HND would be better for the student, even if the candidate is technically qualified for either. This is particularly true in the case of mature applicants who may have been away from education for a while and therefore not have studied mathematics for a long time or who may be offering knowledge gained through appropriate work experience in lieu of formal qualifications. Indeed, there are now courses (Foundation Degrees) which are specifically designed for students who have work experience but not necessarily many academic qualifications who wish to 'formalise' their knowledge through an appropriate qualification and for these courses applicants may well not have studied for many years and may have previously gained few qualifications. The consequences of a student joining an inappropriate course can be severe (in the event of failure) both personally and financially so it is important to use all possible means to match the student with the right level of course. Experience has also shown that mathematics is one area where many students have severe difficulties in their first year of university study.

This paper describes the results of a pilot study in which a questionnaire was used to try and elicit differences in computing and IT students' subjective views of their previous mathematical experiences (prior to joining the University) to see whether these differ between HND and BSc students and, if so, how they differ. Further it was intended to examine whether the data gathered could be used to discriminate between HND and BSc students. The questionnaire could then be used with course applicants who have unusual (or few) formal qualifications and could provide information suggesting which course might be most appropriate.

### **The Approach Taken**

In order to elicit students' subjective perceptions of their ability to cope with mathematical study the concept of mathematical self-efficacy was used. Self-efficacy is a type of personal cognition defined as "people's judgements of their capabilities to organise and execute courses of action required to attain designated types of performance" (Bandura, 1986). This concept has been applied within the field of educational research to a variety of subject domains (including mathematics) and at a variety of levels (Phan, 2000, Hall, 2005). An individual's self-efficacy beliefs are conjectured to be oriented around four core concepts: 'performance experiences', 'vicarious experiences', 'verbal feedback' and finally 'physiological and affective states'. Each of these contributes to the individual's ability to organise and execute effective learning and can be tailored to specific subject domains. A little more detail of these terms is given in Figure 1 below, where the descriptions are taken from Phan (2000).

<b>Source of Self-Efficacy</b>	<b>Description</b>
Performance Experience	An indicator of capability based on performance in past assessments, courses etc.
Vicarious Experiences	A source of evidence based on competencies and informative comparison with the attainment of others.
Verbal Persuasion	This refers to verbal feedback from teachers or adults.

Physiological and Affective States	Judgements of capability, strength and vulnerability to dysfunction.
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Figure 1: Self-Efficacy Sources

An individual's judgements regarding self-efficacy can be elicited by questionnaire (Betz, 1983) and in this case a sample of 48 first year computing and IT students (28 BSc and 20 HND) taking the first year mathematics unit were selected for the pilot shortly after the start of the new academic year. Each student was given a questionnaire consisting of 16 statements relating to mathematical self-efficacy and asked to indicate the extent to which they agreed with the statement on a 7-point Likert scale ranging from 1 (not true) through to 7 (very true). There were four questions relating to each of the four sources of self-efficacy and examples of these are shown in Figure 2.

Source of Self-Efficacy	Example Statement
Performance Experience	"I am not good in mathematics" "I am generally please with my mathematics results"
Vicarious Experiences	"I don't have anyone to help me with mathematics" "I have a close friend who is good in mathematics"
Verbal Persuasion	"I like to get verbal feedback from my teacher" "When my teacher praises me I want to do well in mathematics"
Physiological and Affective States	"Mathematics is interesting" "I am always worried about mathematics"

Figure 2: Sample Statements from the Pilot Questionnaire

For each respondent, the four scores for each self-efficacy source were added to give a total score, and the total scores averaged over each of the four sources for the two courses. The four pairs of averages were then compared using a small sample t-test to see which of the sources of self-efficacy significantly differentiate between the two groups. The four sources were then to be used to investigate the possibility of establishing a mathematical function, using linear programming, that could suggest which course would be most appropriate based on a student's score on the mathematical self-efficacy questionnaire.

### Partitioning Data Using Linear Programming

In many other studies involving self-efficacy, researchers have used complex statistical methods such as linear discriminant analysis to try and partition data over several variables. There are difficulties with these procedures though since results are often difficult to interpret and the statistical assumptions implicit within the techniques are not necessarily valid for small samples of data. Other non-statistical approaches have been used quite successfully and one classical example is that of Linear Programming (LP). LP has been one of the most versatile operational research

techniques with applications covering a range of topics including resource allocation, product mix, transportation and location problems (Pidd, 1996). A well known application within the field of medicine was that of breast cancer diagnosis (Mangasarian, 1995) in which an LP was used to partition a set of data relating to breast tumours into benign and malignant sets based on the measured characteristics of individual cancer cells. The approach taken was to find a hyperplane that best partitioned the two sets of data by minimising the average distance of misclassified points from the hyperplane. Other approaches using ellipsoidal separation have also been used (Konno, 2002).

More formally, let us suppose that A and B represent our two groups to be classified and that  $a_i$  ( $i = 1, \dots, m$ ) and  $b_j$  ( $j = 1, \dots, n$ ) are the vectors of data (in our case this would be the self-efficacy data for each student) to be used for the classification process. Using the notation of Konno (2002) the method is to find a vector  $(c, c_0)$  which will partition the points exactly so that  $c^T a_i > c_0$  for all members of set A and  $c^T b_j < c_0$  for all members of set B and  $(c, c_0)$  defines the hyperplane that separates the sets. Now, of course, it is unlikely that complete separation will be possible so we introduce additional variables  $y_i$  and  $z_j$  (for sets A and B respectively) which measure the distance of misclassified points from the hyperplane. The objective function is to minimise the weighted average of these distances. The LP formulation is, after normalisation:

$$\text{Minimise:} \quad (1-\lambda) \frac{1}{m} \sum_{i=1}^m y_i + \lambda \frac{1}{n} \sum_{j=1}^n z_j$$

Subject to:

$$\begin{aligned} c^T a_i + y_i &\geq c_0 + 1 & i = 1, \dots, m \\ c^T b_j - z_j &\leq c_0 - 1 & j = 1, \dots, n \\ y_i &\geq 0 & i = 1, \dots, m \\ z_j &\geq 0 & j = 1, \dots, n \end{aligned}$$

In the above  $\lambda$  is just a weight used to express the relative importance of the two sets for classification purposes. This classical formulation has been very successful in partitioning the large data sets within the context its original medical application and it was felt to be an approach that could well be useful in partitioning the rather smaller data sets within this pilot study and any subsequent main study. However, given the smaller data sets generated here, a similar but alternative formulation was developed in which the objective was to find the hyperplane that correctly classifies the maximum number of data points and this entailed the formulation of a 0-1 integer linear programme (ILP). The ILP formulation is given below:

$$\text{Maximise:} \quad \sum_{i=1}^m v_i + \sum_{j=1}^n w_j$$

Subject to:

$$\begin{aligned} c^T a_i - c_0 &\geq v_i - (1 - v_i)M & i = 1, \dots, m \\ c^T b_j - c_0 &\leq -w_j + (1 - w_j)M & j = 1, \dots, n \\ v_i, w_j &\text{ are 0-1 variables for all } i \text{ and } j. \end{aligned}$$

In this formulation, the 0-1 variables take the value 1 if the data point is correctly classified and 0 otherwise. The constant,  $M$ , is just a number selected to be large enough to ensure feasibility of the ILP for data points incorrectly classified.

We illustrate the two approaches to data classification by considering a simple 2-dimensional example. Figure 3 below shows two data sets containing 7 points for set A (represented by black squares) and 7 points for set B (represented by black diamonds) which cannot be uniquely partitioned by a straight line. In both this example and the application to real data described later a value of 0.5 was used for  $\lambda$  since there was no reason to treat either group as more highly weighted for classification purposes.

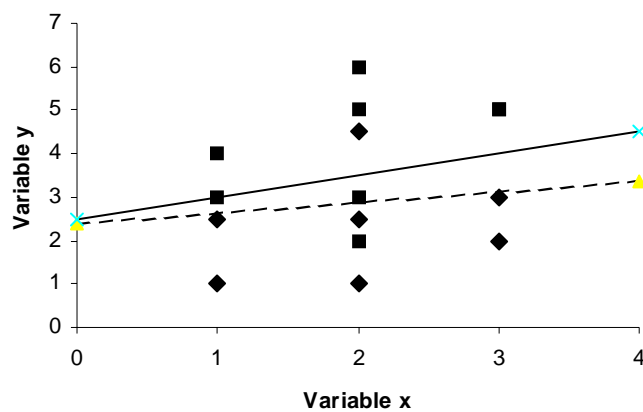


Figure 3: Sample Data Set with Partitioning Lines

The results of running both the LP and ILP formulations are shown, with the solid line representing the partitioning line given by the LP and the dashed line that given by the ILP. The LP misclassified 2 items from set A and 1 from set B whilst the ILP only misclassified 1 item from each set.

It was decided to use both the LP and the ILP formulations on the data set to see whether there was any indication that one would give an improved performance over the other. It must be remembered that with data sets larger those being used here an ILP can be difficult to solve (one 0-1 variable would be required for each of the data points, i.e. there would be  $i + j$  of them in total) since a search process such as the branch and bound method would need to be employed rather than the more efficient simplex method that would be used for a standard LP.

The student sample was randomly divided into two sets so that the first set of responses (18 BSc students and 10 HND students) could be used for investigation and for ‘training’ the classification hyperplane, and the second set (10 BSc students and 10 HND students) used for testing the classification hyperplane.

### Results of the Pilot Study

Table 1 below shows the average score achieved by students within each group for each of the four mathematical self-efficacy sources together with the result of the two sample t-test for the difference of means.

Source	Group	Mean	Standard Deviation	Significance Level	Combined Group Mean
Performance Experience	BSc Group	16.06	5.56	p = 0.044	14.46
	HND Group	11.6	5.1		
Vicarious Experiences	BSc Group	20.06	3.7	p = 0.017	18.18
	HND Group	14.8	5.45		
Verbal Persuasion	BSc Group	15.33	4.52	p = 0.257	16.12
	HND Group	17.50	4.79		
Physiological and Affective States	BSc Group	19.44	4.75	p = 0.836	19.25
	HND Group	18.90	7.35		

Table 1: Questionnaire Results – Descriptive Statistics by Group

We can see that there are only two of the sources that produce significantly different results between BSc and HND students: ‘previous accomplishment’ and ‘vicarious experience’. The result for ‘previous accomplishment’ would seem to indicate that the BSc students have had significantly better experiences in terms of their previous success in mathematical studies when compared to the HND students – a not unsurprising result given that BSc students will generally have a larger number of advanced qualifications which might indicate a more successful record of academic achievement. Interestingly, the mean for the combined group was lowest for this source possibly indicating that, generally, the students did not feel themselves to be good in mathematics as judged by their previous accomplishments in mathematics. The other significant difference between the groups was in terms of ‘vicarious experience’ and this leads us to the conclusion that the HND students have not had the same mathematical study support network (in terms of friends or relatives) which the BSc students have had or that working within any network that they did have was not a positive experience for the student. This would have implications for the type of support that needs to be offered to HND students within the university domain – support that might have been non-existent before and which might be a source of positive experiences for the students.

It is interesting that the two remaining factors showed no significant difference between the two groups. For ‘verbal persuasion’, the mean for the HND group was higher than that for the BSc group indicating that the HND group felt a stronger interaction with feedback from tutors (both in positive and negative terms) which, again may have implications for the way mathematics is taught and supported on the personal level with HND students. The final source, ‘physiological and affective states’, again showed no significant difference of means between the two groups but

this source scored highest in terms of combined average. This suggests that the students were by no means indifferent towards mathematics i.e. that there was a motivation and incentive in many students to seek ways of improving their mathematical abilities.

In order to see whether the mathematical self-efficacy data could be used to indicate an appropriate course of study for a student the data was used as training data to determine a hyperplane capable of partitioning the two groups of students. The original LP formulation by Mangasarian was applied using data for all four sources of mathematical self-efficacy and the LP model solved using the Solver facility in Microsoft Excel. It was able to correctly partition 21 out of the 28 cases (75%). Table 2 below shows the training data results by course with the percentage of correctly and incorrectly classified cases indicated in brackets.

		Predicted Course	
		HND	BSc
Actual Course	HND	5 (50.0%)	5 (50.0%)
	BSc	2 (11.1%)	16 (88.9%)

Table 2: Training Data Results for Mangasarian LP Model

From Table 2 we can see that the LP is able to correctly partition 16 of 18 BSc students but is much less successful with HND students. When we apply the fitted hyperplane to the testing data then we get the results shown in Table 3:

		Predicted Course	
		HND	BSc
Actual Course	HND	8 (80.0%)	2 (20.0%)
	BSc	4 (40.0%)	6 (60.0%)

Table 3: Test data Results for Mangasarian LP Model

This gives an overall success rate of 14 out of 20 (70%) correctly classified which is slightly less than with the training data but the number of cases used is small in this pilot study.

Turning to the ILP formulation we get training data results as shown in Table 4 below:

		Predicted Course	
		HND	BSc
Actual Course	HND	6 (60.0%)	4 (40.0%)
	BSc	0 (0.0%)	18 (100.0%)

Table 4: Training Data Results for the ILP Model

This model gives a slightly higher level of correct classifications (85.7%) and the same hyperplane applied to the testing data gives the results as shown below:

		Predicted Course	
		HND	BSc
Actual Course	HND	8 (80.0%)	2 (20.0%)
	BSc	2 (20.0%)	8 (80.0%)

Table 5: Test Data Results for the ILP Model

This shows that in the test data 16 out of 20 (80%) of cases were correctly classified.

### Conclusions

This paper has presented some results from a pilot study which sought to investigate differences between HND and BSc students in terms of their mathematical self-efficacy and whether mathematical self-efficacy scores can assist tutors in deciding whether new students would be best suited to HND or BSc level study. Naturally, this is not the only determinant of academic course selection, but is an important contributing factor.

We have found that there are significant differences between HND and BSc students in terms of two sources that contribute to mathematical self-efficacy and that these lead to some preliminary conclusions.

Firstly, students who are admitted to the BSc course have, on average, better previous experiences of mathematics in terms of their achievements in mathematics than HND students. Note that this does not just imply recorded marks or qualifications gained, but is a personal view of their performance and progress. This would support the author's personal observations that HND students often lack confidence and also some basic skills that BSc students seem to have.

Secondly, HND students seem to have had significantly worse vicarious experiences which relate to the degree to which they have (or have not had) a network of friends who they can work with, and how they view their mathematical abilities in relation to their friends and classmates. One could argue that this is important in that if such students are mixed with BSc students for purposes of mathematics teaching (if such a module is common to both groups) then this could further weaken self-efficacy in HND students as they establish their mathematical abilities in relation to other (BSc) students in the class. Furthermore, in teaching HND groups, it would seem to suggest that these support groups need to be encouraged among similarly skilled students through group work and practical projects so that vicarious experience can be seen as a positive one by students.

Thirdly, neither group of students seem to 'not care' about mathematics. Some students may love the subject and others be worried about it but students do have motivation to improve their mathematics (if only to relieve the stress and worry that some students have about learning mathematics) and tutors need to get to know students to see which type of support and encouragement would be best for individual students.

Turning to the use of LP and ILP methods, we have demonstrated the mathematical self-efficacy scores can be used to separate the two groups of students with a reasonably high degree of success. This would imply that for cases of, say, mature

students with few formal qualifications, collection of this type of data at interview might well provide useful further information on which admissions tutors may base course recommendations. Of the two methods investigated, the ILP gave a higher rates of correct classification but there are two caveats to be applied. One is that these pilot data sets are small and the methods would need to be tried on larger groups and, secondly, the Mangasarian LP method has been further developed by re-applying the method to cases where the classification is in doubt so as to further refine the process. This was not possible in our case as the data sets were too small.

The author has learned much from this pilot study and will now be proceeding to a much larger study to be undertaken with the next intake of new students to these courses.

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