

**ON PEDAGOGICAL KNOWLEDGE IN MATHEMATICS:  
HOW SECONDARY SCHOOL MATHEMATICS TEACHERS  
FACE THE CHALLENGE OF  
TEACHING A NEW CLASS\***

Catherine P. Vistro-Yu, Ed.D.  
Ateneo de Manila University  
Quezon City, Philippines  
Email: [cvistro-yu@ateneo.edu](mailto:cvistro-yu@ateneo.edu)

*The study investigated how six Filipino secondary school mathematics teachers prepared for the task of teaching a beginning college algebra class. The teachers were selected based on their performance in a mathematics test. They were interviewed, asked to teach a beginning college algebra course, and given a questionnaire to complete. Results revealed the limited ways with which these teachers prepare and plan their lessons. Results of the initial interview showed that the teachers were dissatisfied with their college preparatory program. The teaching episodes showed that they are able to teach in an organized manner, albeit lacking in depth in the content. Implications for teacher preparation programs and mathematics teacher educators are offered.*

**Introduction**

The old adage, “It is not easy to teach an old dog new tricks,” can very easily be applied to school mathematics teachers. For a teacher who has taught for 15 years, to be a student once again can be very disconcerting. Even for a young teacher who has experienced the pride of contributing to the cognitive development of about a thousand students over the last 3 years, to be back in the classroom learning advanced concepts in mathematics can also be an unwelcome proposition. But, the prospect of being able to improve themselves as a teacher and the expectation of being promoted to a higher rank in school can make graduate studies attractive even to teachers who have been in the field for more than 10 years. Besides, the hope that they will finally be able to improve their teaching skills and become much better teachers may soon become a reality.

How does one become a better mathematics teacher? How does one improve his or her teaching skills in mathematics? What kinds of knowledge are necessary for these? The general belief is that teachers must learn more mathematics. The higher the level of mathematics a teacher knows the better teacher he or she becomes. This, however, may not be necessarily true. Earlier studies on the relationship between teachers’ content knowledge in mathematics and their students’ success in mathematics showed the lack of correlation between the two (School Mathematics Study Group, 1972; Eisenberg, 1977; and General Accounting Office, 1984 as cited in Fennema and Franke, 1992). Many of these studies used standardized tests to measure teachers’ content knowledge.

Fennema & Franke (1992) showed, however, that something was also lacking in those studies. In these studies, they pointed out that “no attempt was made to measure the complexity of teacher knowledge or the relationship between the formal mathematics that

---

\* This study was supported by the Citibank, N.A Endowed Professorial Chair and High School Class '53 Faculty Development Fund, Ateneo de Manila University.

teachers knew and what they taught” (p. 149). There is more to taking plenty of higher-level mathematics courses for one to become a good mathematics teacher. Later studies have shown that what teachers do in the classroom, how they teach, how they make decisions all have as much influence in their effectiveness, as do their content knowledge. Certainly, the amount of mathematics content that teachers know affects their ability to respond to students’ dilemmas involving the *hows* and the *whys* of learning mathematics. However, pedagogical knowledge in mathematics is that kind of knowledge that a teacher uses to deal with the everyday task of teaching and relating to students in the classroom. It is that kind of knowledge that teachers hope to improve when they say they want to become better teachers because they realize that this is where they draw all the “tricks” that they can muster to make their students’ learning experiences valuable.

### **The Study**

The following study investigated how six (6) Filipino secondary school mathematics teachers prepared for the daunting task of teaching a beginning algebra class in college. Their preparation for such a task could reveal how they develop new pedagogical knowledge in mathematics particularly, in algebra. The teachers were strategically selected on the basis of their performance in a mathematics test given to 33 secondary public school mathematics teachers enrolled in a master’s program in mathematics education. The study hoped to see the different ways that these teachers coped with the task given them. The following questions guided the investigation:

1. How do teachers understand pedagogical knowledge?
2. What are their beliefs about pedagogical knowledge and other forms of knowledge that they know they should have?
3. How do teachers prepare for a new teaching task?
4. How do teachers evaluate their own performance in a new teaching task?
5. What do answers to the first four questions reveal about how teachers develop pedagogical knowledge in mathematics?

Answers to these questions could influence the ways in which teacher educators can help school mathematics teachers improve their pedagogical knowledge in mathematics.

### **Methodology**

Six secondary school mathematics teachers were interviewed, were asked to teach a beginning college algebra course, and were also given a questionnaire to complete for this study. The six teachers were strategically selected from a pool of 33 secondary mathematics teachers using a mathematics test as a basis. The test consisted of 5 multiple-choice items and 5 free response items selected from the Singapore Secondary School Mathematical Olympiad in 1999 – 2000, Junior Section (Teo, To, and Wong, 2000). The purpose of using this kind of test was to determine how much teachers could extend their knowledge of mathematics to solve high-level type, i.e. Olympiad type, problems. However, no correlations or associations were inferred from the results of this test to the main focus of the study. The intent of the study is to describe how teachers face the challenge of teaching a new class, in this case, in fact, a beginning algebra class in college -- what they do to prepare, and how they actually carry out their lesson plan.

After the six teachers were selected, each was interviewed. The purpose of the interview was to determine their level of knowledge and understanding about teachers’ pedagogical

knowledge. The questions also probed into their past and present experiences in the hope of understanding how they developed the kinds of knowledge they needed for teaching mathematics.

Each of the six teachers was then given a lesson in a beginning college algebra course to teach to a class of 40 college freshmen in a city college. Due to time constraints, each teacher was given only 20 minutes to teach the assigned lesson. Part of the requirement by the researcher was for each teacher to submit a written lesson plan. The purpose of this was to determine how well teachers actually prepare for a lesson and to check for consistency and accuracy in the teachers' implementation of a plan. Two weeks after the teaching of the assigned lessons, the teachers were given a questionnaire that basically asked them to reflect on their teaching experience.

## Results

### *Demographics*

Four of the six teachers obtained low scores (0 – 3 out of 10) while the other two obtained mid-level scores (4 – 7 out of 10). For the record, the highest score among the 33 teachers was 5 while the lowest was 0. One of the 6 selected teachers obtained a score of 5. Table 1 shows some detailed demographics on the 6 teachers.

*Table 1. Some basic demographic data on the 6 teachers*

Teacher*	Gender	Test Score	Teaching Experience	Rank
Ali	Male	0 (Low)	6 yrs	Teacher 1
Mai-mai	Female	3 (Low)	12 yrs	Teacher 1
Gil	Male	4 (Mid)	5 yrs	Master Teacher 1
Bert	Male	5 (Mid)	14 yrs	Teacher 3
Lyn	Female	1 (Low)	3 yrs	Teacher 1
Susan	Female	2 (Low)	15 yrs	Teacher 1

\*Names are fictional

As shown, most of the teachers have the rank of Teacher 1, three of them have been teaching for more than 10 years, and one teacher who has taught for 5 years only has already reached the rank of Master Teacher 1. One can also notice that the two teachers in this sample who have the highest test scores are also the ones with the highest ranks – Master Teacher 1 and Teacher 3.

### *Results of Interviews*

When asked about their preparation courses in college, all six teachers expressed dissatisfaction in the way they were prepared for the actual teaching job. They acknowledged the importance of the coursework and the practicum; however, they still felt unprepared to teach. One teacher commented that the courses, especially, did not develop independent learning but found the practicum most helpful. Another said that although the courses were important, she felt the in-service training seminars that she attended after she began teaching were most helpful.

Most of the teachers had a good idea of what pedagogical knowledge refers to. They describe it as knowledge used for teaching, particularly, knowledge of teaching techniques, psychological principles, classroom management, and the teaching and

learning processes. It is generally “knowledge needed in teaching, other than content knowledge.” When asked to assess their pedagogical knowledge, most of them expressed that they felt lacking in this, although, some acknowledged that the practicum experience in college has contributed much.

When asked how they think they acquired or developed pedagogical knowledge, most of them stated that they learned by lectures, observations of peers or their very own teachers doing the teaching, by reading books, attending seminars, and sharing of lesson plans with peers during practicum. One teacher stressed the importance of modeling the right pedagogy by their own teachers or supervisors in college, which he did not experience in his own preparatory program.

Between content and pedagogical knowledge, two teachers believe that content knowledge is more important. Interestingly, one of these two is the teacher who obtained the highest score in the mathematics test. All six of them however, emphasized that content knowledge is useless without pedagogical knowledge. They believe that both types of knowledge should be developed together. Without pedagogical knowledge, teaching subject matter content is useless. Likewise, without content knowledge, pedagogical knowledge is irrelevant and shallow. To optimize learning, these teachers believe that both types of knowledge should be developed together. Furthermore, they believe it is the teacher’s responsibility to make connections between pedagogical and content knowledge. One teacher thinks that pedagogical knowledge can be developed without formal instruction while content knowledge is best developed through formal studies with a teacher and lots of self-study and practice exercises.

The teachers were asked to describe their idea of what good teaching is. Most of them focused on the teacher’s attitudes and values when describing what good teaching is. Words and phrases such as **committed, modeling good attitudes, has the dedication, and able to relate well with students** dominated their descriptions. One teacher added that one has to know the subject matter and is successful in teaching students to use mathematics while another mentioned using the performance of one’s students as a basis of good teaching.

The teachers were asked to describe how they typically prepare for a mathematics class. Most of them admitted that they do prepare for class daily but because they have taught for sometime and have accumulated enough resources and materials, they do not prepare much. They rewrite old lesson plans and recycle the activities and materials that they have used before. When it is a new class, one teacher said that he does prepare a little bit more.

### *Results of Actual Teaching*

Each of the six teachers was given at least a week to prepare for a teaching task. Each teacher was asked to teach a regular lesson to a beginning college algebra class of a university in a neighboring city. This class was held in the campus of a nearby private university and was composed of 40 students enrolled in the undergraduate Accounting Program. Due to time constraints (this was nearing the end of the semester), each of the teachers could only be given 20 minutes to teach the lesson. As requested, the teachers prepared an outline of the lesson that they were going to teach during their allotted time. The lesson plans were short, not very detailed but clear. There were no incorrect concepts or procedures.

The teaching methodology that all six teachers used was exposition. The flow of the lesson was standard – the teacher gave an introduction, defined terms, explained, presented practice exercises, and solved the exercises for the students. Overall, the six teachers complied and followed the lesson outline that they have prepared. One teacher deviated a bit, which was really not much of a problem because she got back to her “script” soon after. Besides, the regular teacher was there to continue what was not finished. Although the class was a college class where formal discussions of algebra were more expected, the researcher encouraged the teachers to infuse interesting activities into the lessons. All of them followed that advice, which made the lessons more interesting than the lessons with the regular teacher. (Although, perhaps, it could also be the novelty of having a new person teach them that made the students seem more interested.) The teachers did not make any mistake in their teaching of the concepts assigned to them.

Of the six teachers, two of them were assigned lessons that are not included in the regular high school mathematics curriculum. The lesson was on the Set of Complex Numbers: Introduction and Operations on Complex Numbers. These two teachers accepted the challenge willingly. One could see that they were uneasy at the beginning of the teaching task but once they began, both appeared to be more or less at ease but still not quite. Indeed, one could see that these two teachers lacked the experience in teaching complex numbers. The discussion lacked depth and one could see a very mechanical approach to solving the problems that they posed to the class. In general, however, all six teachers were well prepared, i.e., their lectures were well scripted and their examples were well planned. One teacher appeared more natural and comfortable than the rest. He was also successful in making a usual textbook type problem appear more realistic, especially since he brought some props to liven his lesson. The class was attentive and showed signs of understanding of the concept.

#### *Results of the Post-Teaching Questionnaire*

In spite of the seemingly well-run lessons, all six teachers did not feel good about the time given them to teach their lessons. They had wanted to do some form of a written assessment so they could determine how well the students understood the lesson. One teacher was also unhappy because she felt she was not able to follow the outline that she had prepared. In reality, she did skip some examples and switched some parts of her “script” but she was able to make up for it quickly. The regular teacher of the class had to finish off what she began, which was really not a problem since the concept of complex numbers was new to the students.

Another teacher felt very disappointed in her performance, because the topic assigned to her was one that she had never taught before. She was also apprehensive by the fact that the class was a college class and her 15-year experience was with a high school class. Two others mentioned their discomfort in teaching a college class, in spite of the researcher’s assurance that the level of discussion is still very similar to the high school level because of the lack of preparedness of these students to handle college level mathematics. Two teachers felt very positive about how they did in the class. They based their reactions on the students’ behavior and responses.

When asked how they prepared for this new teaching task, four of them mentioned consulting at least three more references in addition to the student text, while the two, who

claimed that the lesson assigned them was something they have taught before, revised what they did in the past.

### **Discussion**

The interviews and the actual teaching episodes by the 6 teachers show that their pedagogical knowledge in mathematics is quite limited. Aside from the fact that their college preparatory programs were limited, it seems that in their real teaching job they do not have enough room or flexibility to try out new methods. They seem to follow standard procedures and methods that eventually do not fit the classroom situation they find themselves in. When one teacher opined that pedagogical knowledge could be acquired through informal means, she may be telling more the reality but not necessarily what should be the case. In other words, her observation indicates that preservice teacher programs could be lacking in their instruction for teachers to help them prepare and develop this type of knowledge. It does not help, therefore, that teachers do not get enough help from their preparatory courses in dealing with questions of how to teach a lesson, how to make students understand, how to relate with students, how to evaluate, etc. Interestingly, however, preservice programs for mathematics, especially, have often been criticized to teach too many pedagogical courses but not enough content courses. Could it be therefore, that all these simply indicate that our preservice programs are no longer effective in helping future teachers of mathematics prepare for the real teaching job, whether it be in the area of content or of pedagogy? What then, can be done?

Certainly, there is a need to improve the level of content courses in the preservice programs. However, the focus of concern in this research is teachers' pedagogical knowledge. It appears that there is a need to re-conceptualize the pedagogical courses for preservice teachers. All 6 teachers acknowledged that the practicum was a big help. However, a more scientific approach to teach them pedagogy and other related matters dealing with students and teaching material is called for. A course on Didactics in Mathematics may certainly help future teachers in developing a more structured, objective, and permanent way of learning how to teach and how to get students to understand mathematics. Didactics is a required course in many teacher education institutions in Europe. These countries have a good record of producing teachers who are more capable in dealing with questions of pedagogy. There is a need to push for Didactics to help teachers develop more confidence in facing problems related to pedagogy.

When asked how the 6 teachers prepared for the new teaching task, most of them revealed that they consulted books. They meant textbooks. This is consistent with what was revealed in the TIMSS-R study in which more than 70% admitted that they consult students' texts more when deciding what to teach (Brawner, Golla, Ibe, de Guzman, Ogena, Talisayon, and Vistro-Yu, C., 2000). This shows that teachers have limited content knowledge as well as limited outlook on where to seek advice when assigned a new topic to teach. The textbooks as one's main resource provide inadequate ideas to teachers who should be armed with more knowledge on how to teach a concept. Surprisingly, none of these teachers thought of engaging a professor in a discussion to get ideas on how to teach a topic, nor did they mention discussing with their colleagues. One reason why they did not seek advice from other people could be that they wanted to tackle the challenge on their own. And to them, this meant working things out by themselves

without help from others. There was a hint of this perspective from some of them during the interview. A couple of them revealed that they value independent learning.

One striking revelation shows some incongruence between teachers' own concept of good teaching and expected standards of good teaching. As mentioned earlier, words and phrases such as "committed", "modeling good attitudes", "has the dedication", and "able to relate well with students" dominated their description of a good teacher or of good teaching. However, when teachers are evaluated, it is not their character but their actual performance that is being assessed. The expectations are totally different. Could this be an underlying reason why teachers feel inadequate in the classroom? They feel they have good character and possess the right values but somehow they are told that they are still not good enough. Teachers need to be enlightened on this matter so that they will feel much better about what they do in the classroom.

There are several challenges that face Filipino mathematics teacher educators in light of the findings in this study. One is the challenge to make preservice courses more responsive to mathematics teachers' needs to develop stronger pedagogical knowledge. The change in the quality of the courses and not in quantity is certainly something that must be examined. A longer and more focused practicum program could be the answer. An organized and well-meaning mentorship program might also help.

A second challenge is for Filipino mathematics teacher educators to encourage peer teaching as a way of providing the much-needed support that preservice teachers need as they prepare themselves for the real teaching job.

In today's world where new information and knowledge come to us at a rate faster than what we can manage to learn at one time, mathematics teachers need to be both creative learners and creative teachers. They should know how to adapt and learn new methods of teaching old and new material. They should learn to do this for both their own personal and their students' advancement.

### **Conclusion and Recommendations**

This study showed that Filipino mathematics teachers can take up the challenge of a new teaching task but may not be able to develop the needed pedagogical knowledge for such a task, which leads to their dissatisfaction and feelings of inadequacy. The study showed that teachers have limited ways of acquiring knowledge on how to teach a new or old lesson and on how to make use of novel approaches to teach a new set of students.

#### *How do teachers understand pedagogical knowledge?*

Teachers in this study view pedagogical knowledge as that other type of knowledge from where you draw ideas on how to teach a lesson, how to relate to students, how to manage a class, and how to prepare tests. They know it is different from content knowledge.

#### *What are their beliefs about pedagogical knowledge and other forms of knowledge that they know they should have?*

Teachers in this study believe that content knowledge can be developed through formal courses while pedagogical knowledge may be developed through other means. For example, they cited their practicum in college as one source of pedagogical knowledge as

well as the in-service seminars and workshops that they attended after they have begun their teaching jobs.

*How do teachers prepare for a new teaching task?*

Teachers in this study claimed that they prepare for a new teaching task by reading more and studying more. They consult 3 or 4 more textbooks. They practice working on the problems related to the lesson. When applicable they use old lesson plans and revise them as needed. Results of the interview suggest that they also consult their peers and their superiors or the more experienced teachers when preparing for new lessons. For this study, however, they did not seek help from other people.

*How do teachers evaluate their own performance in a new teaching task?*

Teachers in this study based their performance on how the students reacted and participated in the lesson. Several had wanted to give some kind of a quiz to determine for sure that the students actually understood but did not have the time to do so. They also used their lesson plan to determine their success. Doing what they had outlined for themselves was important to them. A couple of teachers also used their intuitive feel of how they actually did. They seemed to have their own internal criteria for evaluating their teaching performance. Surprisingly, none of them sought the researcher or the regular teacher's opinion on how they performed the task assigned to them. This shows that they were satisfied with seeing how the students participated.

*What do answers to the first four questions reveal about how teachers develop pedagogical knowledge in mathematics?*

Answers to the previous questions indicate that teachers rely more on experience when developing pedagogical knowledge. It appears that formal pedagogy courses did not have much impact on these teachers. This was supported by their constant mention of the practicum as that part of their preparatory course that helped them more in this aspect of the teaching job. It seems that for these teachers the formal way of developing pedagogical knowledge is by studying the content material more (studying 3 more textbooks). The most important part of the preparation is informal – consulting with more experienced teachers and their peers and participating in in-service seminars and workshops. Unfortunately, teachers do not always employ all methods when faced with a new teaching task as what happened with the six teachers in this study.

Based on the data, the study recommends that the quality of preservice courses for mathematics teachers in Philippine universities, particularly, those that supposedly help teachers acquire or develop pedagogical knowledge be improved. Didactics as a new course could be considered as part of the teacher preparation programs in all teacher training institutions and universities in the Philippines. Didactics would be able to provide a more structured and scientific approach to learning how to teach mathematics, something that Filipino teachers might also need to widen their views of how they can become better teachers. The study also recommends that school administrators and superiors enlighten teachers about what external evaluations consider as good teaching. This is to dispel teachers' notions that good character equals good teaching. Data from this study showed that teachers are on a different plane when they describe what a good teacher is.

Lastly, this study shows that teachers can extend their knowledge enabling them to accept new teaching tasks. However, they do need help and support in preparing for these since

their experiences and knowledge are quite limited, in spite of the many years of teaching experience. These are the challenges that face both Filipino mathematics teachers and mathematics teacher educators.

### References

- Brawner, F., Golla, E., Ibe, M., de Guzman, F., Ogena, E., Talisayon, V., & Vistro-Yu, C. (2000). *TIMSS-R Philippine Report Volume 2: Mathematics*. Manila: Department of Education, Culture, & Sports, Department of Science and Technology – Science Education Institute, & University of the Philippines National Institute of Science and Mathematics Education.
- Eisenberg, T. A. (1977). Begle revisited: Teacher knowledge and student achievement in algebra. *Journal for Research in Mathematics Education*, 8, 216 – 222.
- Fennema, E. & Franks, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*, 147 – 164. NY: Macmillan Publishing Company.
- General Accounting Office. (1984). *New directions for federal programs to aid math and science teaching* (GAO/PEMO-85-5). Washington, DC: Author.
- School Mathematics Study Group. (1972). Correlates of mathematics achievement: Teacher background and opinion variables. In J. W. Wilson and E. A. Begle (Eds.), *NLSMA Reports* (No. 23, Part A). Palo Alto, CA: Author.
- Teo, K. M., To, W. K., & Wong, Y. L. (2000). *Singapore Secondary School Mathematical Olympiads 1999 – 2000*. Singapore: Singapore Mathematical Society.