ROLES OF SEMANTIC STRUCTURE OF ARITHMETIC WORD PROBLEMS ON PUPILS' ABILITY TO IDENTIFY THE CORRECT OPERATION

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Abstract

This paper draws on findings from a study conducted in seven primary schools in Seychelles about pupils' proficiency in one-step arithmetic word problems to discuss the roles of semantic structures of the problems on the pupils' ability to identify the operation required to solve them. A sample of 530 pupils drawn from Primary 4, 5 and 6 responded to a 52 multiple choice item test in which they were to read a problem and select the operation that is required to solve it. The results showed some consistencies with other international studies in which pupil related better to additive than to multiplicative problems. Moreover, they could solve better those problems that do not involve relational statements. The general picture indicated that there is an overall increase in performance with an increase in grade level yet, when individual schools are considered this increase in performance in significant in only one school. In the other results there were many cases of homogenous pairs of means. Analysis of distractors conducted on the pupils' responses to the seventeen most difficult items reveal that in most cases when an item required the operation indicating an additive or multiplicative structure, the most popular wrong answer would be one of the same semantic structure. Otherwise there is a strong tendency for the pupils to select the operation of addition when they could not identify the correct operation for a particular problem. Several other research questions emerged as a result of this present study.

Introduction

The question of which operation to use has become a rather instantaneous reaction when primary school pupils are given arithmetic word problems to solve. Many teachers in the primary schools in Seychelles have expressed the view that pupils in their classes find difficulty in identifying the operations needed to solve arithmetic word problems. Besides, many studies outside Seychelles have reported that pupils who are proficient in arithmetical computations may not necessarily solve arithmetic word problems with the same proficiency (Lyons, 1994; Nesher, 1976). Despite the fact that a lot of work has been done in this field to alleviate the pupils' weak performance on this part of the curriculum, the phenomenon remains a pedagogical puzzle.

Several explanations have been provided as to why pupils find arithmetic word problems difficult to solve. Ineffective instruction has been among these explanations (Cardelle-Elawar, 1992; Carpenter & Moser, 1984; De Corte & Verschaffel, 1987; Essen & Hamaker, 1990). Dellarosa, Kintsch, Reusser, and Weimer (1988) contended that pupils' failure on arithmetic word problems is due to a lack of linguistic knowledge. This situation is even more problematic when the word problems are expressed in the learners' second language (Abedi & Lord, 2001; Bernardo, 2002; Cuevas, 1984; Maro, 1994). Linguistic knowledge is among those other internal constructs such as the processing skills of pre-requisite knowledge, and cognitive ability which hinder word problem solving ability (Dellarosa et al., 1988; Kintsch & Greeno, 1985). Other explanations relate to the semantic structure of the word problems (Carey, 1991; Carpenter, Hiebert & Moser, 1981; Carpenter & Moser, 1984; Christou & Philippou, 1998; Lopez, 1992; Nesher & Hershkovitz, 1994; Shalin, 1987; Wilson, 1967) and some less influential factors like: context (Hiscock, 1993; Lopez, 1992), binary steps involved in the problems (Huinker, 1990), and superfluous information in the problems (Dunbar, 1995). Although less successful, some researchers have used attitudes and motivation (Wenger, 1992) and gender (Zambo, 1990) to explain why primary pupils failed on arithmetic word problems.

This paper draws on findings from a study conducted in seven primary schools in Seychelles about pupils' proficiency in one-step arithmetic word problems to discuss the roles of semantic structures of the problems on the pupils' ability to identify the operation required to solve them. This will in essence, establish a basis for remedial instructions to be institutionized and pedagogical reflections to be effectuated.

Theoretical Underpinning

The Effect of Semantic Structure of Problems on Solution Processes

Effect of semantic structure of arithmetic word problems on the pupils' solution and thought processes, particularly for addition and subtraction has received a lot of attention (Carey, 1991; Carpenter, et al., 1981; Carpenter & Moser, 1984). A pioneer study relating structure of verbal problems to performance is the work reported in Wilson (1967) and presented arguments as to why pupils should be made to understand structures underlying word problems. The teaching implication of Wilson's (1967) argument is that word problems could be taught meaningfully and systematically, thus ensuring that the different components [structures] are given sufficient consideration during instructions. It has also been advocated by Lopez (1992), Nesher and Hershkovitz (1994), and Shalin (1987) that the semantic structures of the problems influence solution processes. In other word, the pupils interact differently to problems of different semantic structures. For instance, it has been established over years that additive structures is more popular among the pupils than the multiplicative structure (Christou & Philippou, 1998). It has also been demonstrated that problems that involve relational statements (the compare problems) are more difficult for young children to solve than problems that do not contain such statements (Kintsch & Greeno, 1985).

Carpenter and Moser (1984) showed that young children's strategies for subtraction problems are strongly influenced by the semantic structure underlying the problems. Those children operating at the material and verbal levels tended to solve each subtraction problem with the strategy that closely models its semantic structure. De Corte and Verschaffel (1987) reported a similar finding relating to the pupils' strategies for addition problems. As the children's conceptual knowledge changes with growth, they become more flexible in their choice of solution (Carey, 1991). De Corte and Verschaffel (1987) observed that the pupils' strategy for word problems also depends on the sequence in which the known quantities are introduced in the text.

Effects of multiplicative semantic structure on performance were reported in Bell, Fischbein & Greer (1984) and, in Christou and Philippou (1998). Bell, et al. (1984) noted that situations that could be conceived as repeated addition were easier than the others. Partition-type problems were the most favored division-type problems, and fractional-quotition problems were among the least popular among the pupils. Christou and Philippou (1998) observed that rate problems were the hardest multiplicative problems.

Mulligan and Mitchelmore (1997) investigated how young children formed and developed intuitive models of multiplication, and how these models are related to the semantic structure of word problems. They conducted a longitudinal study on a group of girls as they progressed from Grade 2 to Grade 3. The pupils were interviewed at the beginning and at the end of the each school year. At the first interview they were not yet exposed to multiplication. The problems used in the study were set in familiar contexts and all of them involved whole number data and answers. Analysis conducted on the correct responses showed three intuitive models emerging from multiplication problems (direct counting, repeated addition, multiplicative operation) and four from the division problems (direct counting, repeated addition, repeated subtraction, multiplicative operation). The results also showed great variation in the use of models and consistent progression of solutions from Grade 2 to Grade 3 (Mulligan & Mitchelmore, 1997). That provided tentative but valid evidence of how young children's intuitive understanding of whole number multiplication and division develop.

Comprehending Arithmetic Word Problems

A number of problem solving models have been developed and reported by Briars & Larkin (1984), Fuson (1992), Hegarty, Mayer and Monk (1995) and, Kintsch and Greeno (1985). The model presented in Hegarty, et al. (1995) assumes that the solver uses two distinct paths in comprehending a text: The direct translation approach and a problem model approach. The text is processed in increments. At each increment, the solver reads a statement containing the information about one of the variables. In constructing a text base the solver represents the prepositional content of this statement, and integrates it with other information in his or her current representation. At the second stage of comprehension the solver is guided by the goal of solving a mathematics problem and constructs a representation that is referred to as the mathematics-specific representation. It is at this stage that the solvers using the two approaches differ. At the third stage, once the problem solver has represented the information that he or she believes to be relevant in solving a problem, he or she is ready to plan the arithmetic computations necessary to solve the problem. A solver who uses the direct translation approach bases his or her solution on keywords while one who uses problem model approach has a richer representation on which to base the solution plan.

Regardless of which path a solver takes, the task of comprehending word problems is the most critical and represents the threshold of successful solutions. The mental representation of the problems is formed from comprehending the different relationships of quantities and sets in the problem and it is the basis of a choice of operation (Kintsch & Greeno, 1985). The process of constructing a problem representation involves mapping the verbal statement onto an existing schema. Thus, the schema comprises a vehicle for the comprehension of the semantic relations underlying a given text and its mathematical structure, and it acts as a generalised frame for action in a given context. Ultimately, differences in difficulty between arithmetic word problems of various semantic structures may be accounted for by differences in the complexity of the available schema (Nesher & Hershkovitz, 1994).

The study of Christou & Philippou (1998) sought to verify whether a developmental trend could be traced in the pupils' ability to solve one-step problems and whether variables like structures and operations could explain the difficulty of the problems. The results showed that four hierarchical levels exist among additive and multiplicative word problems and that problem solving ability develops with age but the relative difficulty inherent in each problem-type was grade independent but was affected by the nature of the situation and the operations involved in the schemes (Christou & Philippou, 1998).

The purpose of the present study was to ascertain how well primary school pupils in Seychelles could identify operations of arithmetic word problems of various semantic structures, compare such ability among pupils of different grade levels, and ascertain whether pupils demonstrated in their choice a preference for a particular operation.

Methods and Procedures

The Subjects

The pupils (n = 530) were drawn from seven primary schools in Seychelles. In each school an intact class from each of the Primary 4, Primary 5 and Primary 6 grade levels was selected. The resulting sample was Primary 4 (n_4 = 170), Primary 5 (n_5 = 181), and Primary 6 (n_6 = 179).

Description of the Test

The test consisted of fifty two one-step word problems of six different semantic structures (see Table 1) based on the categories proposed by Nesher, Greeno and Riley (1982) for the additive problems and Kouba (1989), Mulligan and Mitchelmore (1997) for the multiplicative problems. Each item consisted of the stem and four distractors. The distractors were made up of one of the four basic operations and the numbers mentioned in the stem. The numbers used were whole numbers presented in familiar context. This was done to reduce cognitive loads such as linguistic, and computation on the problems. An example of one item (Item 9) is shown in the figure below.

9) A tray can take 6 eggs. There are 24 eggs. How many such trays are needed?

Which of the four workings will give you the answer?

A	$6 + 24$	☐	B	6×24	☐
C	$24 \div 6$	☐	D	$24 - 6$	☐

Fig 1 Example of one item of the test

Moreover, when writing up the distractors cares were taken to avoid hints or tricks in the answers. For example; when the solution model involved the operation of division or subtraction, it was ensured that the bigger number be always written first. The otherwise case would appear unusual to the pupils at this age so they might automatically reject the distractors even if they could not solve the problem.

Table 1

Examples of problems belonging to each semantic structure tested

Additive structures		Multiplicative structures	
Semantic structures (No. of items)	Example of word problems (Item Number)	Semantic structures (No. of items)	Example of word problems (Item Number)
Combine (12)	Ann has 3 rupees. Peter has 9 rupees. How many rupees do they have altogether? (Item 10)	Partition (5)	9 girls share 18 balloons equally among themselves. How many does each one get. (Item 52)
Compare (13)	In a group there are 5 men more than the women. There are 15 men in the group. How many women are there? (Item 14)	Quotition (5)	16 children are sitting in groups of 8. How many groups are there? (Item 1)
Change (12)	Lyn has 9 sweets. She gives 3 sweets to her best friend. How many sweets does Lyn have now? (Item 11)	Repeated multiplication (5)	In a box of chocolate there are 15 bars of chocolate. There are 5 boxes. How many bars of chocolate are there altogether? (Item 26)

Procedures

The test was administered to the pupils under exam conditions. All the three classes within a particular school had the test administered to them at the same time but in separate rooms. However, not all schools took the test on the same day. The tasks in the test required the pupils to read the scenario in the stem and select from four solution models the one which will give them the answer to the problem. Selection of the correct solution model would mean identification of the correct operation to solve the problem.

Presentation of the Results

The pupils' responses to the items were scored and facility ratios, the proportion of pupils who responded correctly to each item, were noted. A mean facility ratio of each category of word problems was worked out and the results are tabulated below.

Table 2

Average facility ratios by semantic structure by grade levels

	Grade levels		
Semantic Structures	4	5	6
Combine	0.69	0.79	0.87
Change	0.59	0.73	0.83
Compare	0.47	0.56	0.68
Partition	0.54	0.68	0.78
Quotition	0.46	0.59	0.73
Repeated groups	0.42	0.56	0.71

The values presented in Table 2 show differences in the facility ratio of problems of the various semantic structures. Word problems belonging to the combine and change semantic structures have higher facility ratios than word problems of other semantic structures. At all the three grade levels the pupils seem to relate best to problems belonging to combine and change structure as they do to problems of other structures. This lead to the conclusion that when word problems are presented to the pupils it is very much likely that they would solve the combine or the change problems but may ask for the operation they would need to solve problems of semantic structure compare or the repeated group categories.

The effect of semantic structures on the difficulty level of the items is well demonstrated in the findings. ANOVA for a single within-subject independent variable (repeated measures) conducted on the pupils' overall scores on the categories of word problems showed a significant difference in the means of the different categories of problems. $F(5, 2645) = 142.2$, $p < 0.001$. A paired t-test showed that the pupils performed better on additive word problems ($M = 68.1$, $SD = 23.1$) than on multiplicative word problems ($M = 60.8$, $SD = 27.6$). $t(529) = 9.7$, $p < 0.001$. Further analyses were made to compare groups; either schools or classes of particular schools. The mean and standard deviation of each group are illustrated in Table 3.

Table 3

<u>Mean and standard deviation of scores by school by grade level</u>

	Grade Levels Scores (%)		
School ID	Primary 4	Primary 5	Primary 6
1	59.5	83.7*	86.0*
	19.3	**10.8**	**14.3**
2	59.2	71.7	84.8
	19.9	**15.9**	**12.2**
3	41.9*	45.9*	67.6
	17.3	**17.8**	**19.1**
4	69.0*	74.1*	87.5
	20.3	**17.4**	**12.1**
5	42.7*	57.0*	58.7*
	21.9	**25.7**	**23.3**
6	35.3	58.3*	69.1*
	19.3	**19.2**	**21.5**
7	55.0*	56.1*	66.3*
	16.4	**18.4**	**20.6**

* indicates classes within the corresponding school with no significant difference of means

The results show some interesting observations. First, they indicate that the pupils' ability to identify the operation needed to solve arithmetic word problems is different among schools. Analysis of variance conducted on the results for difference in the school means shows that the seven schools performed significantly different on the test $\underline{F}(6, 523) = 23.38$, $\underline{p} < 0.05$. Calculation of eta-square (η^2) the correlation ratio between the schools and the overall scores, shows that 21% of the variations in scores is accounted for by difference in the schools.

Although the results show that many lower graders outperformed some higher graders on the test, the holistic view indicate an increase in competency with increase in grade level, $\underline{F}(2, 527) = 49.1$ ($\underline{p} < 0.001$) with no homogeneous means as revealed by Tukey' s HSD. However, further post hoc analyses conducted within each school for differences in the grade level means show that a significant improvement in the scores with an increase in grade levels was observed in only results of School 2. On the other hand results of School 5 and School 7 indicate that all the three grade means were homogeneous. Results of School 3 and School 4 indicate that no difference exists in the P4 and P5 means and results of School 1 and 6 show no significant difference in the means of P5 and P6.

The question as to whether pupils show preference to specific operation when they did not know the correct operation was answered by analyzing the distractors of the fifteen most difficult items. In each case, the correct operation and the percentage pupils choosing the most popular distractor for the items were noted. In other words, this gave an indication of which operation that most pupils tended to choose when they do not know the correct operation (see Table 4).

The results presented in Table 4 shows some interesting observations. What is striking from the table is the consistency in the way pupils at three different grade levels responded to the items. The most popular wrong answer is generally similar for all the three levels except for Items 37 and 46. In 10 out of 17 cases the pupils' choice of operation shows consistency in the general structure of the word problems. For instance, if the items were additive their most popular wrong distractor would be an additive operation. The same applied for the multiplicative items (See Items 20 38, 29, 47, 6, 7, 25, 34, 9 and Item 46 for Grade 4 and 5). Four out of the seven remaining items also show some interesting pattern. The pupils tended to select the operation of addition in favour of all other operations.

Table 4

The mode distractors for the fifteen most difficult items by grade level

		Grade level		
Item	Correct operation	Primary 4	Primary 5	Primary 6
20	+	€ (44%)	€ (48%)	€ (56%)
38	+	€ (44%)	€ (45%)	€ (40%)
15	+	× (33%)	× (37%)	× (46%)
29	÷	× ((38%)	× (41%)	× (30%)
22	×	+ (45%)	+ (38%)	+ (23%)
4	÷	+ (38%)	× (25%)	+ (24%)
47	€	+ (26%)	+ (30%)	+ (22%)
14	€	÷ (24%)	÷ (23%)	÷ (16%)
37	€	+ (25%)	÷ (19%)	× (11%)
46	+	× (19%)	€ (25%)	€ (24%)
50	×	+ (49%)	+ (38%)	+ (25%)
6	÷	× (26%)	× (19%)	× (22%)
7	÷	× (31%)	× (25%)	× (18%)
26	×	+ (39%)	+ (27%)	+ (17%)
25	€	+ (24%)	+ (22%)	+ (9%)
34	€	+ (45%)	+ (27%)	+ (13%)
9	÷	× (27%)	× (22%)	× (17%)

Discussion and Conclusion

The overall picture reveals that for one-step arithmetic word problems the pupils related better to the combine and change problems and had great difficulties to relate to problems of the compare semantic structure. This is similar to what Kintsch and Greeno (1985) reported. Word problems that contain relational statements are more difficult for children to solve than word problems that do not contain such statements. The pattern of pupils' performance on the different categories of one-step problems was consistent with other previous studies (Carpenter, 1985; Carpenter & Moser, 1984; De Corte & Verschaffel, 1987; Kouba, 1989).

The pupils also related better to the additive than to multiplicative problems. This difference is most significant at P4 level than other level. This may be due to the fact that at P4 the pupils have had very little exposure to such types of word problems than at P6 level. More to that the observation that the additive problems were better solved than the multiplicative problems may be explained by the fact that additive problem appeared more often in a pupils' environment at this age. Secondly, the addition and subtraction word problems given to the pupils generally include extensive quantities only, quantities that can be directly represented. Multiplication and division problems involve both extensive and intensive quantities, quantities that are derived from other quantities such as bottles per crates. Thus, the problem schemata for multiplication and division problems would have to be more complex than those required for addition and subtraction (Carpenter, Ansell, Franke, Fennema & Weisbeck, 1993).

There was evidence that to some extent the pupils could map the problems to their different overall structures. When analysis of distractors are conducted on the pupils' responses to the seventeen most difficult items, it reveal that in most cases when an item required the operation indicating an additive or multiplicative structure, the most popular wrong answer would be one of the same semantic structure. Nevertheless some further analysis of distractor of other items within that group of seventeen, also indicator a strong tendency for the pupils to select the operation of addition when they could not identify the correct operation.

It has to be made clear that pupils' choice of operation in solving simple arithmetic problems as contended by Bell, et al. (1984) may be influenced by a number of factors most of which unfortunately has not been investigated in this study. Bell et al. (1984) claimed that many of these factors are due to misconception such as multiplication always makes things bigger, or division must be of a larger number by a smaller number. These misconceptions are transmitted even to higher grades if they are not corrected at an early stage (Bell, Swan, & Taylor, 1981). Future research will certainly be needed to investigate these misconceptions amongst the pupils in Seychelles. Nevertheless this research highlights why the semantic structures of arithmetic word problems need to be taken into consideration when materials for this part of the curriculum are being devised.

The finding about pupils' reaction to arithmetic word problems of various semantic structures has to be interpreted with care. There is a possibility that the teachers are putting emphasis on just few types of problems but neglecting the other. Or perhaps they are unaware of the different categories of word problems thus they do not treat them in their teaching. A more comprehensive study is therefore needed to verify how teachers in the schools of Seychelles handle the teaching of arithmetic word problem. More to that, case studies could be devised in the better performing schools to determine whether some of the good teaching models could be adopted.

Due to the performance observed it is also being suspected that the pupils lack other skills relating to solving word problem such as problem interpretation and representation. This gives rise to more related research such as an investigation of the pupils' cognitive and thought processes while solving word problems. It would also be of interest to know factors that influence the pupils' choice of operation in solving arithmetic word problems. Since the pupils use English as a second language it would also be necessary to investigate how this has had an impact on their learning of word problems.

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