The Mistakes Made by the Students Taking a Calculus Course in Solving Inequalities

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ABSTRACT

This study tries to analyze the students' performances and explore the mistakes done by the students taking Calculus course when are finding solution sets for inequalities. To these purposes, an examination was given to science students who have taken calculus course at a Turkish University. The results showed that the students are not successful in solving inequalities. The mostly observed mistake was to multiply both sides of inequality by expression that includes variable without paying attention to the sign of this expression.

Key Words: Inequalities, Solving inequalities, Students' mistakes.

1. Introduction

The concept "inequality" and finding a solution set for an inequality is an important issue in Calculus as well as for all the fields of mathematics. Because, the main concern of a calculus course includes the concept of "function" and analyzing the properties of these functions. The analysis of some properties of functions requires the solutions of inequalities. For instance, in order to find the domains of the functions $f(x)=\sqrt{x^2-2x+2}$ and $g(x)=\log\frac{x}{x+3}$, it is necessary to find the solution sets for the inequalities $x^2-2x+2\geq 0$ and $\frac{x}{x+3}>0$. Similarly, inequality solutions are required to determine the monotonicity and concavity of functions by the use of derivative (Sandor 1997). For instance, for the analyzing the monotonicity of function $f(x)=\frac{x^2+2x+4}{2x}$, it is necessary to find solution sets for the inequalities $\frac{x^2-4}{2x^2}>0$ and $\frac{x^2-4}{2x^2}<0$, for the concavity- the inequalities $\frac{4}{x^3}>0$ and $\frac{4}{x^3}<0$. Therefore; Calculus text books introduce the concept of inequality before they cover functions and their properties (Adams 2003, Thomas & Finney 1996). As these examples show, the students' performances in solving inequalities directly affect their success in Calculus courses. As a result, this situation triggered a considerable number of studies on the inequalities topic (Tsamir, Tirosh, Almog 1998; Tsamir&Almog 2001; Tsamir&Bazzini 2004; Linchevski&Sfard 1991; Dreyfus&Eisenberg 1985).

On the basis of these considerations, this study aims at analyzing the students' performances and exploring the mistakes done by the students taking Calculus course in a university in Turkey while finding solution sets for inequalities.

2. Method

The subjects of this study are 129 first year students attending Physics, Chemistry and Statistics Departments at the Faculty of Science of a university in Turkey and also taking Calculus course. This group, which has already been taught how to solve inequalities by using algebra and geometric methods, is administered a five-question exam

The Exam Administered to the Subjects

1. Find the solution set for the inequality $\lvert x+1\rvert > \lvert x-3\rvert$
In this question, the students are expected to find the solution by considering four situations occurred due to the properties of the absolute value.
2. Find the solution set for the inequality $\frac{2}{x-1} \geq 5$.
In this question, the students are expected to find the solution for the inequality payig attention to the sign of x-1.
3. Find the solution set for the inequality $-2x^2 + 5x + 3 < 0$.
In this question, the students are expected to find the set where the second degree polynomial get negative values.
4- Find the solution set for the inequality $\frac{x}{2} \geq 1 + \frac{4}{x}$.
In this question, the students are expected to find the solution by transforming this inequality to a rational expression after certain calculations and by considering the signs of numerator and denominator.
5- Find the solution set for the inequality $49x^2 \leq 64$.
In this question, the students are expected to find the solution set by keeping the equality $\sqrt{x^2} = \lvert x\rvert$ in mind.

During the assessment process of this exam, the criteria for the correct answer was “the correct solution sets as well as the correct calculations made by the students”. The data gathered was presented and interpreted as percentages.

3. Results and Discussion

Table 1 shows the students' performances in the exam administered.

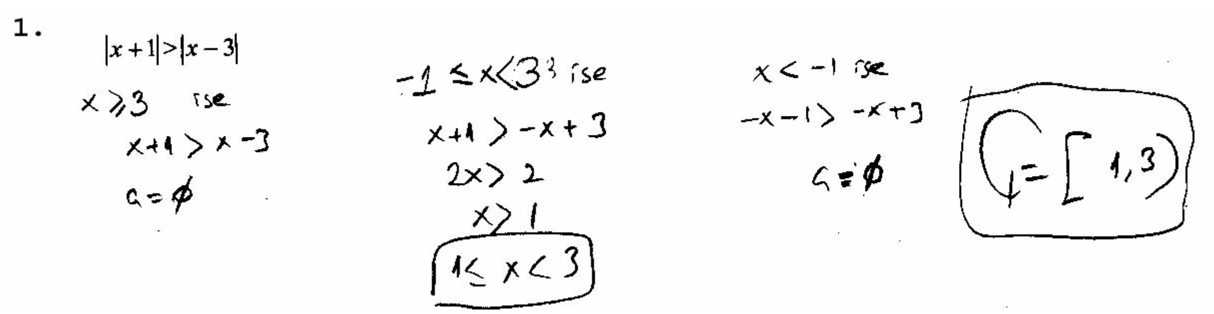
Table 1: Students' Performances

The questions	1st question	2nd question	3rd question	4th question	5th question
The percentages for the correct answers	**55**	**36**	**48**	**25**	**69**

- 55 % of the students gave the correct answer to the first question. In other words, 45 % of the students were not able to solve the inequality correctly. In this question, the students could have found the solution set by analyzing the following options separately.

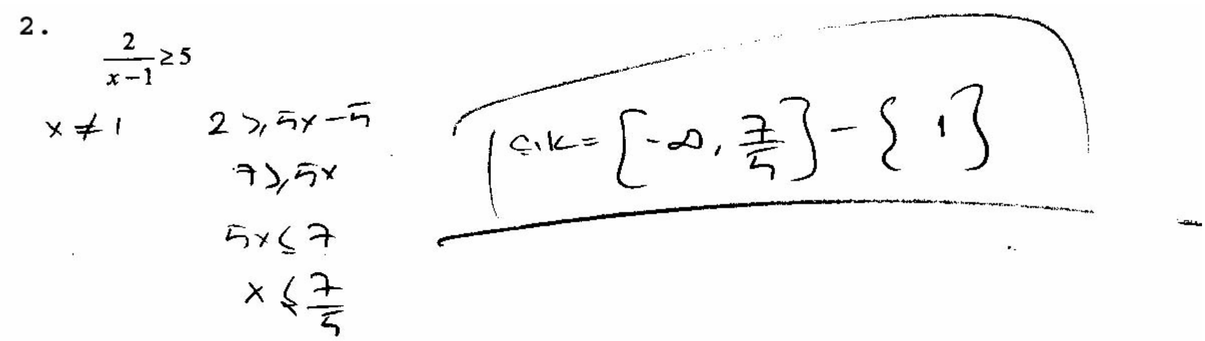
$$x \geq -1 \quad \text{and} \quad x \geq 3,$$
$$x \geq -1 \quad \text{and} \quad x < 3,$$
$$x < -1 \quad \text{and} \quad x \geq 3,$$
$$x < -1 \quad \text{and} \quad x < 3$$

When the calculations made by the students who gave the wrong answer were analyzed, it was realized that students found the solution set according to the options $x \geq -1$ and $x < 3$ and for other options they found an empty set. One of the solutions made by using this way was given in Sample 1.



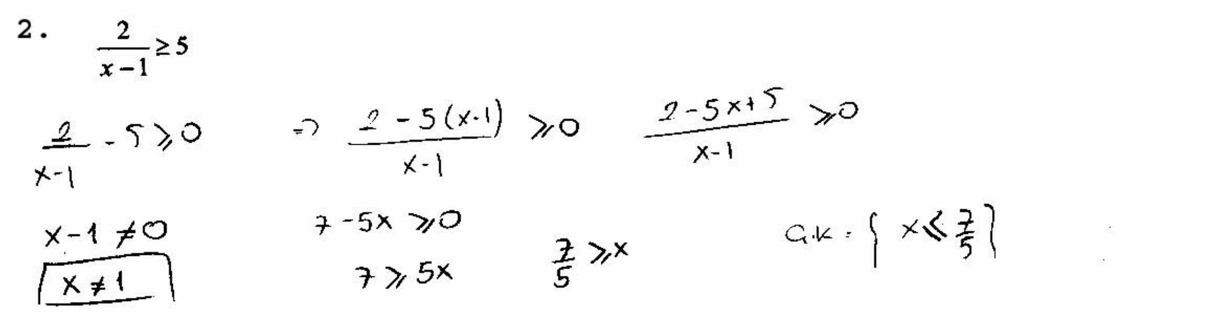
Sample 1: *The solution given by one of the students who failed to give the correct answer to the first question.*

- The second question was given a correct answer by 36 % of the students. According to this result, we might conclude that a considerable number of students failed to answer this question correctly. As given the Sample 2.1., the students obtained the inequalities $2 \geq 5(x-1)$ and $x \leq \frac{7}{5}$ by multiplying both sides by x-1 in $\frac{2}{x-1} \geq 5$. Since the students did not examine the situations x < 1 and x >1, they could not find the correct answer.



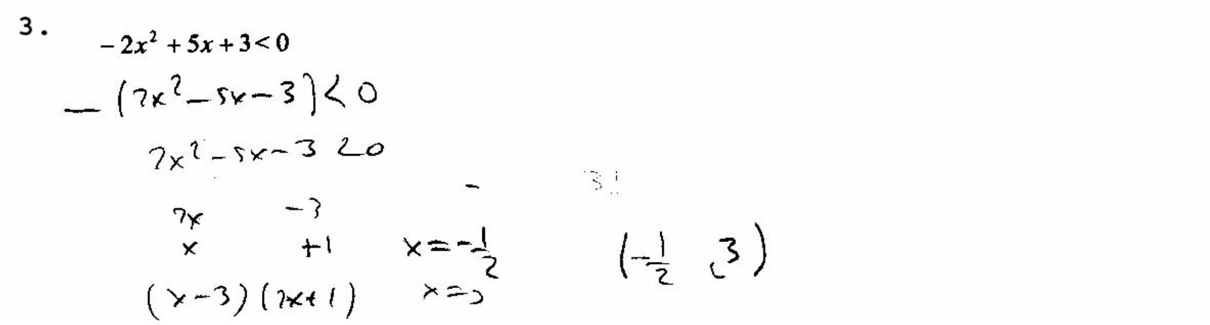
Sample 2.1. : *The solution given by one of the students who failed to give the correct answer to the second question.*

Another mistake done in this second question was to find incorrect solution set by using the inequality $7-5x \geq 0$. This inequality was found by multiplying both sides of the inequality $\frac{7-5x}{x-1} \geq 0$ by x-1. One of the solutions made by using this method was given in Sample 2.2.



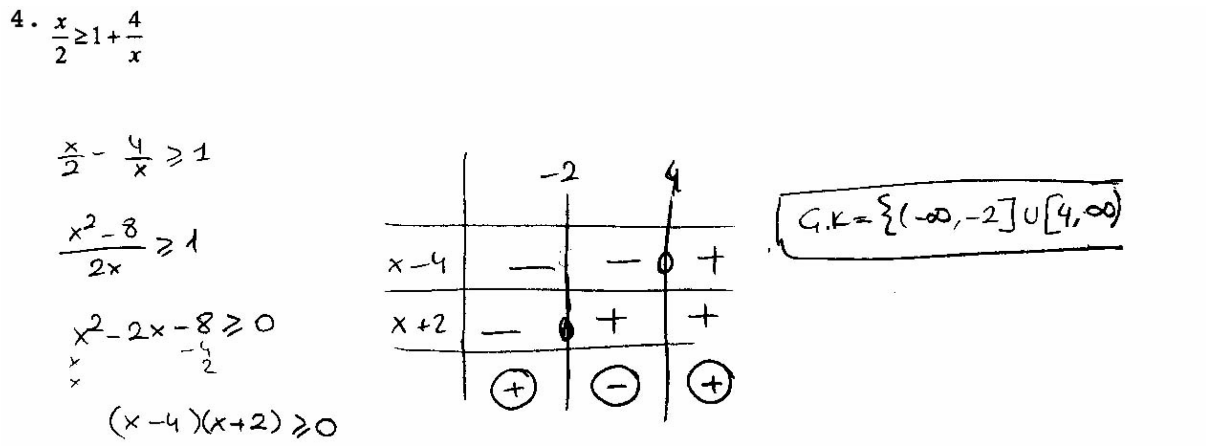
Sample 2.2. : *The solution given by one of the students who failed to give the correct answer to the second question.*

• In the third question, nearly 48% of the students found the correct solution set. In other words, 52% of the students were not able to find the correct solution set. These students mostly focused on the negative and positive values of the polynomial $2x^2-5x-3$. They reached the incorrect answer since the solutions to the equations $-2x^2+5x+3=0$ and $2x^2-5x-3=0$ are the same. A sample for this situation was given in Sample 3.



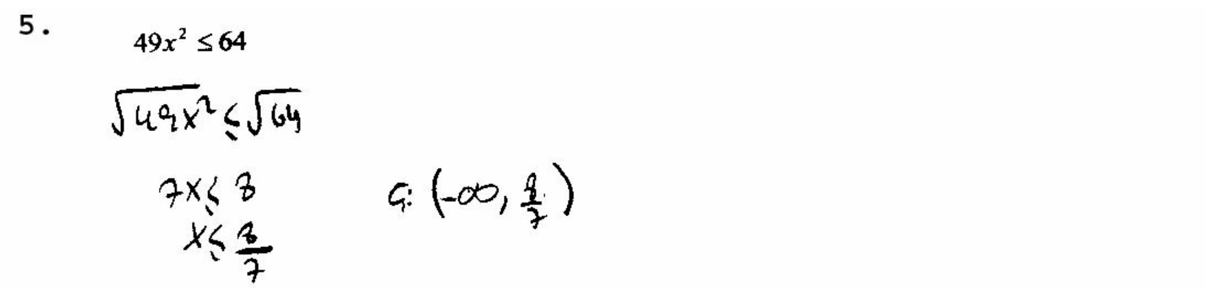
Sample 3 : *The solution given by one of the students who failed to give the correct answer to the third question.*

• The correct solution set for the fourth question was found by 25% of the students. The majority of the students (75%) gave the wrong answer to this question. These students obtained $\frac{x}{2}-\frac{4}{x}-1\geq 0$ from the inequality $\frac{x}{2}\geq 1+\frac{4}{x}$. Later they obtained the inequality $\frac{x^2-2x-8}{2x}\geq 0$ by making the denominators equal. Next, ignoring the possibility that x may take both negative and positive values, they multiplied x by both sides of the last inequality and found the solution set for $x^2-2x-8\geq 0$. Unfortunately, this was the incorrect solution set. The following sample is explaining this situation.



Sample 4: *The solution given by one of the students who failed to give the correct answer to the fourth question.*

• Almost 69 % of the students were successful in finding the correct answer to the fifth question. When compared with other questions, the students can be said to have been more successful in this question. The mostly observed mistake in this question was given in Sample 5.



Sample 5: *The solution given by one of the students who failed to give the correct answer to the fifth question.*

When the overall student performances are considered for all five questions, the students cannot be said to have been successful in finding the correct solution sets for inequalities. We believe that this situation might negatively affect their success in Calculus courses.
In addition, when the calculations of all the students (those who found or failed to find the correct solution set) are analyzed, it was observed that the students only perceive the solution to the problem as a series of algebraic calculations without considering what the solution set

of an inequality means. In other words, they did not check whether an “x” real number in the solution set proves the inequality. If the students had followed this checking procedure, they could have found some clues to help them find the correct solution set.

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