

Running head: THE EFFECT OF GRAPHING CALCULATORS USE

The effect of Graphing Calculators Use
on Students' Understanding of the Derivative at a Point

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Author Note

This research paper is part of my doctoral dissertation under the supervision of Dr. Alfinio Flores at Arizona State University.

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Abstract

The present study examined the effect of the use of graphing calculators on students' understanding of the concept of the derivative at a point. It investigated whether or not the graphing calculator with its visual representation helps students construct an appropriate concept image of the derivative at a point. The participants in this study were 71 undergraduate students enrolled in two classes (24 in the experimental and 47 in the traditional) of calculus courses at two different universities. Students in both classes took an 11-item pretest then completed a posttest. In addition, eleven students participated in a one-on-one interview with the researcher. The results showed a statistical significant difference between the two classes at the end of the study.

The effect of Graphing Calculators Use
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Several researchers studied the difficulties that students face dealing with the derivative concept (Orton, 1983; Ferrini-Mundy & Graham, 1994). In their study, Ferrini-Mundy and Graham (1994), noted that students could not form a relationship between the tangent line and the derivative of a function, that is students were unable to connect the graphical and the symbolic representations of the derivative.

Although out of one hundred ten students who participated in Orton's (1983) study, fifty were studying to become mathematics teachers, Orton found that students in the sample had many difficulties dealing with the derivative concept. They were unable to connect the concepts of average rate of change to instantaneous rate of change. When they were asked about the rate of change at a point, they responded with the functional value. They had difficulty finding the rate of change at a point using a graph; they simply showed difficulty interpreting and using the graphical representation of the derivative.

The calculus reform movement emerged in response to a major crisis in the teaching of calculus. Part of the reasons for the teaching calculus crisis have been (a) students' increasing drop-out and (b) failure rates in calculus courses. Many students who finished calculus courses were unable to use the knowledge they acquired to their benefit in real life (Frid, 1994).

The National Council of Teachers of Mathematics [NCTM] Standards (1989) called for a mathematics curriculum that “emphasizes conceptual understanding, multiple representations and connections, mathematical modeling, and mathematical problem solving” (p. 125).

To improve students' performance in calculus, the new courses focused on teaching the concepts, not only the techniques. Concepts were introduced algebraically, graphically, and numerically. Some of these courses incorporated writing, others relied on computers and graphing calculators (Culotta, 1992).

Having technology available and instead of introducing the derivative by considering the limiting slope of secant lines as Orton (1983) did, Tall (1989) proposed the idea of *local straightness* of graphs. Locally straight means that with enough magnification, the graph of a function f at a point a looks like a straight line near a . It is local because it holds only in a small neighborhood of the point a . He proposed an instructional sequence in which students will investigate the local straightness of different graphs and relate that to other representations of the derivative concept.

In a comparison between the two strategies of Orton (1983) and Tall (1989), Fiske (1995) used two groups for his study, the first group followed Tall's strategy (Tall's group), while the second one followed Orton's strategy (Orton's group). Fiske found that Tall's group demonstrated stronger understanding of the concepts of slope, rate of change, derivative, and symbolic differentiation than the Orton's group. On the other hand, Orton's group was better at sketching the graphs of derivatives of functions. Both groups achieved a strong visual representation of the derivative as a function.

Graphing calculators allow students to graph functions easily and also provide students with a feature that helps them magnify the graph at any given point. The graphing calculators offer access to solving a wide range of problems that need not be limited anymore by the students' algebraic and arithmetic skills (Demana & Waits, 1988). The use of graphing tools in mathematics

instruction allows students to explore a wider range of function types. The tool provides students with easy access to multiple representations. It shifts students' attention from routine computation toward the examining and exploring of mathematical patterns, promoting an elementary analytical visual approach, and promoting the use of graphs to monitor progress in problem solving (Dick, 1992; Fey, 1989).

In reviewing other research studies, Simonsen and Dick (1997) found that the graphing calculator enhanced students' problem solving abilities and it equipped students with the ability to visualize problems using multiple representations. By using the graphing calculator, it was also possible to minimize the time spent on symbolic manipulation skills and instead, increase the time spent on problem solving and applications. Students took more responsibility for their own learning, and the dynamics of classroom activity shifted to more discussion, inquiry, and cooperative learning with continued use of graphing calculators, as well as with the use of a variety of other technological tools.

The purpose of this study was to investigate whether or not the use of graphing calculators in teaching calculus would help students develop appropriate concept images of the concept of derivative at a point. Two main questions of this study have been formulated as follows:

Question 1: Is there a difference between the concept images that are held by the students who are using graphing calculators and the students who are not using the calculators?

Question 2: To what extent can a student make connections between the graphical, symbolic, and numerical representations of the derivative at a point?

Method

Participants

The participants in this study were 71 undergraduate students enrolled in two different first semester calculus classes at two state universities in U.S.A (47 in the traditional class, and 24 in the experimental class). The two classes were chosen with the approval of their instructors. Students were enrolled in each class by regular registration procedures. Students chose their own class. The students in both classes were asked to volunteer to be part of the study and were told about the study and its purposes; confidentiality was granted for all participants; and the researcher obtained releases from the students in order to use the data collected for academic purposes.

Moreover, six students from the traditional class and six from the experimental class agreed to be interviewed. Only five students from the experimental class showed up. The interviews took place in an office in the campus of each university. The researcher met with each student individually. At the time of the interviews, both classes had already covered the first three chapters of the textbook and most of chapter four. They had already covered the concepts that were needed for this study. These chapters covered functions, limits, tangents, velocities, rate of change, derivative at a point, the derivative as a function, differentiation rules, maximum and minimum, and curve sketching.

Treatment

The instruction in the experimental class with the help of the graphing calculator emphasized the connections between symbolic, visual, and numeric representations. For example, the concept of function was introduced numerically by a table of values, visually by a graph, and

algebraically by an explicit formula. While the traditional class lacked the visual representation of calculus concepts that can be achieved using graphing calculators. They had the chance to see the graphs in the book and the ones drawn by the instructor on the blackboard. Both classes used the same calculus book written by Stewart (1999). This calculus book begins with a detailed study of functions, limits, derivatives, the derivative formulas, applications, and a study of integration. The textbook was written in a way that could be used with or without graphing calculators.

Instruments

Pretest

The pretest was a 50-minute test that contained 11 questions (see Appendix 1) that were mainly related to the derivative concept. Some of these questions asked the students to find the average rate of change, the derivative value at some given points, and what is meant by the concept of the derivative at a point. The test aimed at collecting as much information as possible about the students' concept images of the derivative concept. Some of the questions for this test were taken from research studies related to the derivative, such as Orton (1983), and Zandieh (1997).

The following are two questions from the pretest:

- Q1. a. Explain how the average rate of change of a function f can be used to find the instantaneous rate of change of f at a point a .
- b. Give a geometric interpretation of the instantaneous rate of change.
- Q2. What does a speedometer reading of 50 mph at $t = 1$ tell you? Is there a difference between the speedometer reading and the velocity at $t = 1$? If yes, what is the difference?

Posttest

The questions in the posttest were exactly the same as the pretest. The purpose of the posttest was to see which group had a better understanding of the derivative concept after the treatment. Another purpose of the posttest was to provide direct comparisons with the pretest and to obtain a detailed picture of each student's understanding of derivative.

Interview

The one-on-one interview questions (see Appendix 2) allowed the researcher to obtain more information about the students' understanding of the derivative concept. The researcher adapted the questions that were used in the interview from research literature relating to the derivative such as Orton (1983), and Zandieh (1997). The interview questions were designed to obtain information about the development of a student's concept image of the derivative. They were designed to provide information that would address the research questions mentioned earlier. The following are two examples from the interview questions:

Q1. What does the concept of derivative of a function at a point mean to you? Mention as many ideas and specific points as you can.

Q2. Given a curve C with an equation $y = f(x)$. How can you find the tangent to C at the point P $(a, f(a))$?

In order to minimize any influence on the students' responses, the interview protocol put limits to the interaction between the researcher and the students. When a student gave an answer to a question, the researcher would only ask questions to clarify the student's answer. For example, the researcher would respond to the student and ask for more clarification. "How can you find the slope?", and "you used the term 'tangent', can you tell me what you meant?" The

researcher was careful not to offer the students any guidance nor to address any of their questions regarding their answers during the interview.

Procedure

The present study was a two group study that used both qualitative and quantitative methodology where data were collected in a university freshman calculus course. The first group used a graphing calculator as an aid in the learning of the derivative concept. The second group received traditional instruction of the derivative concept (graphing calculators were not allowed). No attempt was made to balance the two groups equally according to age, sex, or any other factors; the two groups represent two ordinary university classes as they are.

At the beginning of the semester, the researcher explained to the two instructors the purpose of the study. The pretest was conducted at the beginning of the semester, the posttest and the interviews were carried out at the end of the research period.

Analysis

To analyze the quantitative data a t test was used to determine if there was a significant difference between the two classes on the pretest at the beginning of the study, since it was not possible to randomly assign students to the two groups. Moreover, an analysis of covariance (ANCOVA) was performed. ANCOVA was used to measure if there was a significant difference between the two groups and was also used because it takes into account possible initial differences between the two groups. Pretest scores were entered as the covariate, and the posttest scores were entered as the dependent variable.

For the qualitative data, a color-coding scheme was used to analyze the interview transcripts. Different colors were used to represent the different references given by the students

for the derivative at a point: slope, rate of change, velocity, and symbolic. In addition, there was a color for misstatements that included the incorrect expressions that students used to describe the derivative at a point. In the analysis, The researcher focused on students' ability in stating the formal definition of the derivative of a function at a point, and how to apply the definition for a specific function at a specific point. Also another focus was on the relationship between the average rate of change and instantaneous rate of change. Moreover, the researcher focused on students' ability to describe the connections between the different representations of the derivative concept.

RESULTS

The purpose of this study was to investigate whether or not the use of graphing calculators in teaching calculus would help students develop appropriate concept images of the concept of derivative at a point. We will start by presenting the statistical results, then the interviews of 11 students will be analyzed, and finally a summary of the results comparing the two groups will be given.

Statistical Results

Statistically, Table 1 and Table 2 provide summaries of the means and standard deviations for both groups in the pretest and posttest.

Since it was not possible to assign students to both groups randomly, the pretest results were examined to determine whether there was a statistical difference between the two groups. An independent samples t test for the equality of means showed that there was no significant difference between the means of the two groups at the start of the study $t=1.08$, $df=69$, $p=.28$;

but a t test showed that there is a significant difference between the posttest means between the two groups, $t = 5.03$, $P < .0001$

An ANCOVA analysis (see Table 3) was conducted with the posttest as the dependent variable and with the pretest as the covariate to see if there was any significant difference between the two groups. These results show that the posttest scores were significantly different at the .05 level of significance. This means that the experimental group had better understanding of the concepts presented than the traditional group. The adjusted posttest mean for the traditional group was 8.11, and for the experimental group was 13.18.

Interview results

Students' responses during the interviews provided an insight on their concept image of the derivative. The analysis focused on the following aspects of the derivative: formal definition, instantaneous rate of change, tangent lines, numerical representation, and graphical representation.

The focus will be on students' interpretations of the derivative at a point and their ability to describe the connections between the different representations of the derivative. The analysis will start with the six students from the traditional group and then the analysis of the five students from the experimental group will be given. It is also worth noting that students were not allowed to use graphing calculators during the interviews except for arithmetic calculations.

The research questions focused on the difference between the concept images that are held by the students in the two groups and the connection between the different representations that students can make of the derivative at a point. First we will look at the images that were held by students in both groups, then we will look at representations that the students actually used.

The images that students held about the derivative at a point varied from one student to another in both groups. In the traditional group, the most dominant image of the derivative at a point was the slope of the tangent line at that point (4 out of 6). The formal definition came second (3 out of 6), only two of them used the formal definition correctly without any algebraic mistakes. Only two students mentioned rate of change.

For the experimental group, slope also was the most dominant image (4 out of 5). Similar number of students mentioned instantaneous rate of change. Only two students mentioned the formal definition, and only two students used the definition correctly without any algebraic mistakes.

The results demonstrate that the slope of the tangent line was the most dominant image for the derivative at a point for both groups. This supports other research findings that indicated that students depend on their images not on the formal definition in dealing with the concept. In this study, students relied mostly on their visual image of the derivative concept.

For the numerical representation, which deals with students' ability to find an estimate of the derivative at a specific point given a table of functional values near that point, all students in the traditional group except one mistakenly used the functional value at that point to represent the derivative at that point (see Episode 1). All students in the experimental group except one came up with a correct estimation of the derivative at the given point. This finding indicates that for this group of students, the emphasis on the visual and numerical representations of the derivative at a point helped students in the experimental group to deal with these representations (see Episode 2).

Episode 1

Okay, I'm just looking at all of the given numbers of x below 2 which are 1.99 and 1.98. All of the numbers are starting at about 7.76 and the numbers are increasing 7.88 and then at 2 it reaches 8 and then above 2 I'm looking at 2.01, 2.02, the numbers are getting larger from 8, above 8, 8.1, 8.2. So it's suspected as x values get bigger than 2 the value starts getting bigger and bigger away from 8 and as they get smaller than 2 the values get more and more away from 8 in the other direction so I would say that the limit, my initial response would be 8.

Episode 2

I'd go with the secant line, saying that as the distance between two points approaches 0, the secant line would be the tangent line. I'd mention that - I'd have to do this on a graph the closer the two points are the better derivative equation you will get. The closer the two points are the better grade of equation you will get so on your table the closer the two values are the better derivative at the point. Zooming in also kind of goes with this, because you get a better graph as far as two points together

The second research question focused on students' abilities to make connections between the graphical, symbolic, and numerical representations of the derivative at a point. For the students in the experimental group, concepts were introduced graphically, numerically, and symbolically. Students in this group had the opportunity to visualize the derivative by zooming in on different graphs to find the derivative. In the interview, when students were presented with graphs of a given function under different magnifications near a specific point, most of them found the slope. One student from the traditional group, who had an experience with graphing calculators, mentioned "zoom in", he stated "you're given a window- like a calculator kind of". While three students from the experimental group mentioned "zoom in", actually, one of them stated "If you zoom in and you get a straight line", this indicates that the student is implementing

the local straightness idea by zooming in so that a curve looks like a line near a specific point(see Episode 3).

Episode 3

If you zoom in and you get a straight line, the value of x changes and the value of y changes. By finding the slope of the line, the derivative will be 2.

Only one student from the traditional group mentioned the connection between the average rate of change and the instantaneous rate of change. He explained how to find instantaneous rate of change at a point by evaluating average rate of change over an interval that contains that point (see Episode 4). Four students from the experimental group brought up that information in their interview (see Episode 5). Moreover, two of them gave the graphical representation of average rate of change as the slope of the secant line. This finding suggests that students in the experimental group were able to better connection between the average rate of change and the instantaneous rate of change than the ones in the traditional group.

Episode 4

Well, okay, the way I would do it - if you were given two points, because I guess you would need two points to know the average rate of change. If you have a function and you had two points and you knew the average rate of change between the two, which would be just connecting them just like your slope like

$\frac{y_2 - y_1}{x_2 - x_1}$ and that would be your slope between the points.

So if you knew the average rate of change between two points $x + h$ and x and you want to find the instantaneous rate of change at x , what I would do is just take the average rate of change and instead of having them over 10 x value points decrease them and do 5 and then decrease them again and do 2 and keep

decreasing your difference between your two x values in the $\frac{y_2 - y_1}{x_2 - x_1}$. That's how I

did it - on our last test we had a question that dealt with the ball going up and down, and it asked for the instantaneous rate of change - like the velocity of the ball at 3 seconds and I didn't remember to take the derivative of the height

function, so I just took two points along that function that were so incredibly close together and just found the average rate of change between the two, and I think that the actual value was -32 meters per second and I got -31.9.

Episode 5

Well if you found the average rate of change between two points and then as it gets closer and closer to the one point, you get closer and closer to the instantaneous rate of change.

The data also suggests that the use of graphing calculators and the emphasis on the visual and numerical representations of the derivative did not affect students' ability in the experimental group to develop proficiency with the use of symbolic definition of the derivative at a point and the differentiation rules. In fact, approximately the same number of students in both groups mentioned the symbolic definition correctly. Likewise, the same number of students had the ability to use the definition correctly to find the derivative of a given function at a specific point (see Episode 6). Students in both groups mastered differentiation rules and used them easily in finding the derivative of a given function at a specific point. Moreover, instantaneous rate of change was mentioned by more students in the experimental group than the traditional group.

Episode 6

I am substituting $x + h$ into the function and find the value $f(x+h)=(x+h)$ squared. Then I also find the value of $f(x)$ at $x=1$ which is 1. So it will be limit of $(1+h)$ squared minus 1 over h as h goes to 0. Simplifying $(1+h)$ squared. I will get $1+2h+h$ squared. Simplifying it will be limit of $2h+h$ over h as h goes to 0. The answer is 2.

Results Summary

Summary of Traditional Group Results

Analysis of 6 interviewed students in the traditional group indicated the following:

1. Students constructed a visual image of the derivative at a point as the slope of the tangent line at that point.
2. Students mastered differentiation rules and used them in finding the derivative of a given function at a specific point.
3. Most of the students were not able to deal with the derivative at a point numerically.
4. Most of the students were not able to use the symbolic definition of the derivative at a point correctly.
5. Most of the students were not able to make the connection between the average rate of change and the instantaneous rate of change.

Summary of Experimental Group Results

Analysis of 5 interviewed students in the experimental group indicated the following:

1. The most dominant image for students of the derivative at a point was the slope of the tangent line at that point.
2. Students were able to use differentiation rules correctly.
3. Most of the students evaluated the derivative given a table of functional values and different magnifications of a given graph.
4. Most of the students were not able to use the symbolic definition of the derivative at a point without algebraic mistakes.
5. Most of the students made the connection between the average rate of change and instantaneous rate of change.

These findings suggest that the experimental group students could form better connection between the different representations of the derivative at a point in comparison with students in the traditional group.

Conclusion and Recommendations

The findings of this study suggest that the use of the graphing calculator and the emphasis on the visual and numerical representations of the derivative concept help students develop a concept image that includes different representations of the derivative with better connections among these representations. Understanding the derivative as a three layers (the ratio, the limit, and the function) is important. More emphasis should be given on this while teaching the derivative concept. Students should understand the formal definition through their learning experience; that definition should be emphasized and connected with the other representations. The first layer plays a major role in learning the derivative concept; it should be emphasized more and presented in different representations. This study suggests that practices of teaching calculus concepts should change to achieve a comprehensive concept image of the derivative concept that includes all the different representations.

References

- Culotta, E. (1992, February 28). The calculus of education reform. *Science*, 225, 1060-1062.
- Demana, F., & Waits, B. (1988). Pitfalls in graphical computation, or why a single graph isn't enough. *The College Mathematics Journal*, 19, 177-183.
- Dick, T. (1992). Super calculators: Implications for the calculus curriculum, instruction and assessments. In T. J. Fey & C. R. Hirsch (Eds.), *Calculators in mathematics education* (pp. 145-157). Reston, VA: National Council of Teachers of Mathematics.
- Ferrini-Mundy, J., & Graham, K. (1994). Research in Calculus learning: understanding of limits, derivatives, and integrals. In J. J. Kaput, & E. Dubinsky (Eds.), *Research issues in undergraduate mathematics learning* (pp. 31-45). Washington, DC: Mathematics Association of America.
- Fey, J. (1989). School algebra for the year 2000. In S. Wagner, & C. Kieran (Eds.), *Research issues in the learning and teaching of algebra* (pp. 199-213). Reston, VA: NCTM.
- Fiske, M. (1995). A comparison of the effects on student learning of two strategies for teaching the concept of derivative (Doctoral dissertation, The Ohio State University, 1994). *Dissertation Abstracts International*, 56(10), 125A.
- Frid, S. (1994). Three approaches to undergraduate calculus instruction: Their nature, and potential impact on students' language use and sources of conviction. *Issues in*

Mathematics Education, 4, 69-100.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

Orton, A. (1983). Students' understanding of differentiation. *Educational Studies in Mathematics* 14(3), 235-250.

Simonsen, L., & Dick, T. (1997). Teachers' perceptions of the impact of graphing calculators in the mathematics classroom. *Journal of Computer in Mathematics and Science Teaching*, 16 (2/3), 239-268.

Stewart, J. (1999). *Calculus early transcendentals* (4th ed.). Pacific Grove, CA: Brooks/Cole Publishing Company.

Tall, D. (1989). Concept images, generic organizers, computers, and curriculum change. *For the Learning of Mathematics*, 9(3), 37-42.

Zandieh, M. (1998). The evolution of student understanding of the concept of derivative (Doctoral dissertation, Oregon State University, 1997). *Dissertation Abstracts International*, 58(08), 3056A.

Table 1

Experimental Group Pretest and Posttest: Means, and Standard Deviations

Dependent Variable	M	SD
Pretest	6.96	3.97
Posttest	13.58	5.06

Table 2

Traditional Group Pretest and Posttest: Means, and Standard Deviations

Dependent Variable	M	SD
Pretest	6.04	3.04
Posttest	7.70	4.45

Table 3

Analysis of Covariance on Dependent Variable

Source of Variation	SS	df	MS	F	Sig. of F
Within cells	885.52	68	13.02		
Regression	616.14	1	616.14	47.31	.000
Group	401.90	1	401.90	30.86	.000
Total	2051.18	70	29.30		

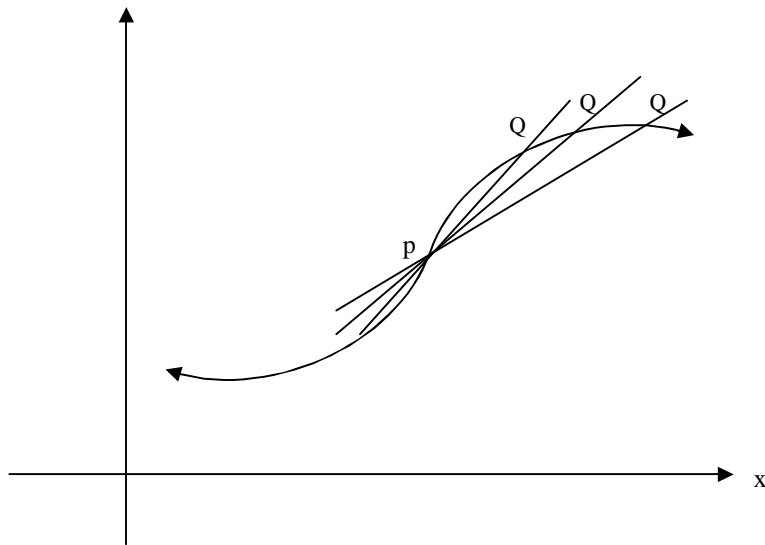
Appendix 1

Pretest

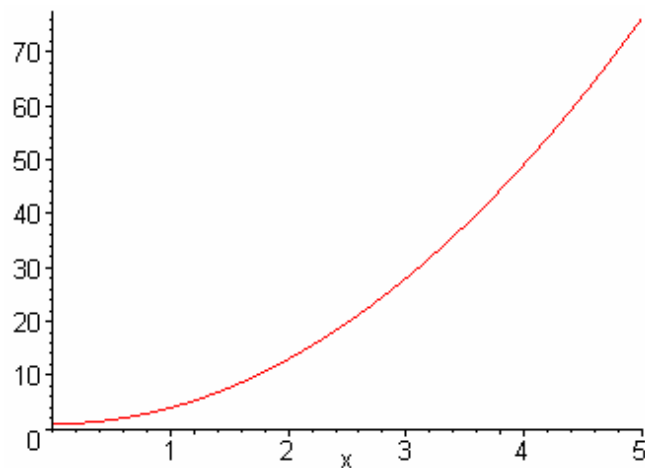
Student Name:.....**Date:**.....

1. Simplify. $\frac{(x+y)^2 - x^2}{y}$

2. Explain in as much detail as you can, what happens to the slope of the secant PQ as the point Q tends toward P.



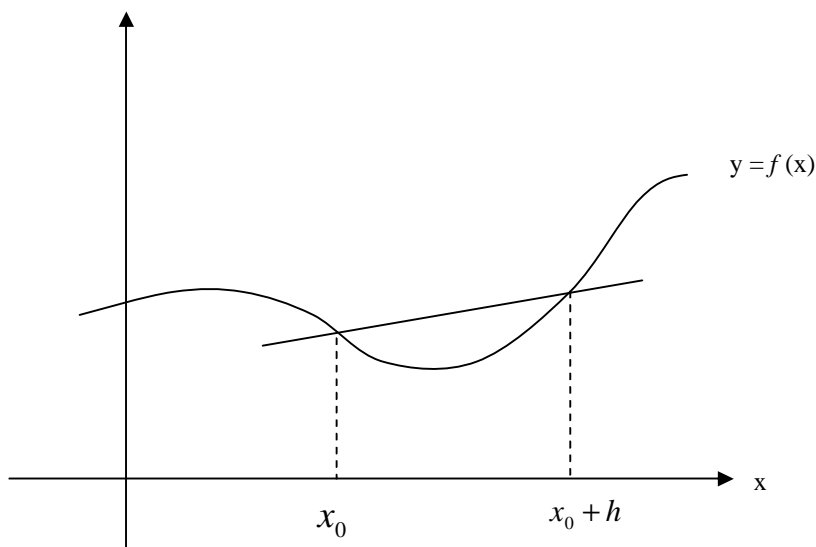
3. The graph below represents the function $y = 3x^2 + 1$, from $x = 0$ to $x = 5$.



- What is the value of y when $x = a$?
 - What is the value of y when $x = a + h$?
 - What is the change in y as x increases from a to $a + h$?
 - What is the average rate of change in y in the x -interval from a to $a + h$?
 - Can you use the result of (d) to obtain the instantaneous rate of change of y at $x=2.5$? If so, how? If not, why not?
4. a. Explain how the average rate of change of a function f can be used to find the instantaneous rate of change of f at a point a .
- b. Give a geometric interpretation of the instantaneous rate of change.

5. The diagram below is used to illustrate the following definition for the derivative

$$f'(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h} \text{ provided this limit exists.}$$



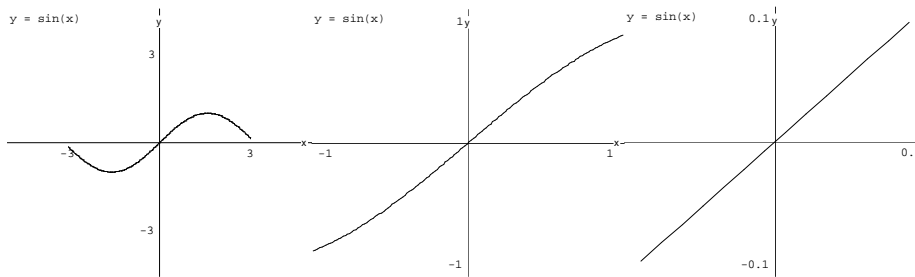
- a. At which point or points of the graph does the formula measure the instantaneous rate of change, and how do you know?
 - b. Use the definition above to find the derivative of $f(x) = x^2$ at $x = 1$.
6. What does the concept of derivative of a function at a point mean to you? Mention as many ideas and specific points as you can.
7. What specifically do we mean when we say that the derivative with respect to x of $f(x) = x^2$ at $x = 1$ is 2?

8. The following table shows values of $f(x) = x^3$ near $x=2$. Use it to estimate $f'(2)$.

x	1.98	1.99	2.00	2.01	2.02
x^3	7.762392	7.880599	8	8.120601	8.242408

9. What does a speedometer reading of 50 mph at $t=1$ tells you? Is there a difference between the speedometer reading and the velocity at $t=1$? If yes, what is the difference?

10. The following diagrams gives you successive graphs of the sine function with x in radians, with smaller and smaller scales, near the point $(0,0)$. Estimate the value of the derivative of $\sin x$ at $x=0$, using these graphs and explain your answer.



Magnifications of the graph of $\sin x$ near $x=0$

11. Find the derivative of each of the following

a. $f(x) = (x^3 + x)^3$ $x = -1$

b. $y = \frac{3x}{(x^3 + 1)}$ $x = 0$

Appendix 2

Interview Questions

1. What does the concept of derivative of a function at a point mean to you? Mention as many ideas and specific points as you can.

Probing Questions:

1. Can you add more ideas? Do you remember anything else?
2. What is the definition of the derivative at a point?
3. Give the exact symbolic mathematical definition of the derivative at a point? Write it please.
4. Using the definition find the derivative of $f(x) = x$ at $x = 1$.
5. If the student doesn't mention instantaneous rate of change, he will be asked if he heard about it, and how does that relate to the symbolic definition of the derivative?
6. Can you give a physical example of the derivative at a point? If yes, give one? What about velocity?
7. Can you relate all what you mentioned about derivative to each other?
8. If the function represents velocity, what would the derivative at a point represent?
9. You mentioned slope in your answers. What does it mean? Can you draw a picture of what you are saying?

* In the case of students who can't mention different descriptions of the derivative, they will be reminded about the ones they missed and will be asked to elaborate on them.. For example, Do you remember the formal definition for derivative? slope? instantaneous rate of change?

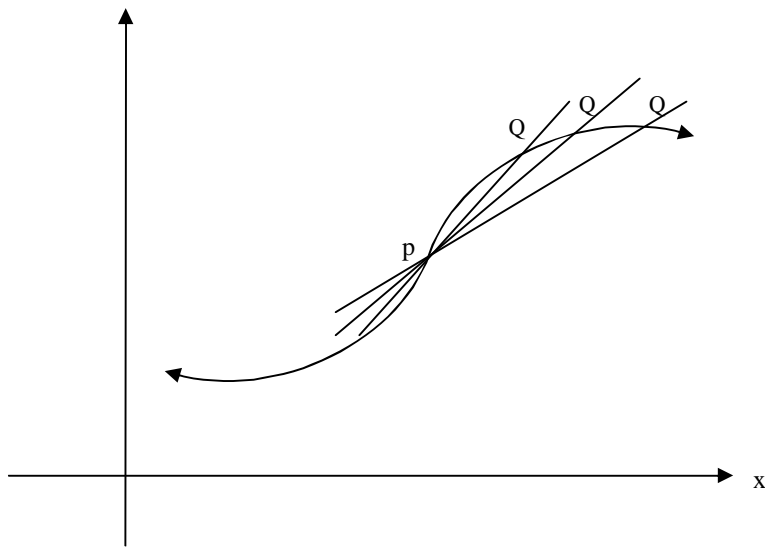
2. a. Explain how the average rate of change of a function f can be used to find the instantaneous rate of change of f at a point a .

b. Give a geometric interpretation of the average rate of change and the instantaneous rate of change.

3. Given a curve C with an equation $y = f(x)$. How you can find the tangent to C at the point $P(a, f(a))$.

Probing Questions:

1. What do you need to find an equation of a line?
 2. What is given? What is missing?
 3. How you can find the slope?
4. Explain in as much detail as you can, what happens to the slope of the secant PQ as the point Q tends toward P .



Probing Questions:

1. You said that the slope is increasing, why?

2. What is happening to the secant line in this case?
3. Can you give more information?
4. You mentioned tangent line. What does that mean? What is the relation between its slope and the slope of the secant lines?
5. Does the slope of the tangent involve an limiting process? Explain.

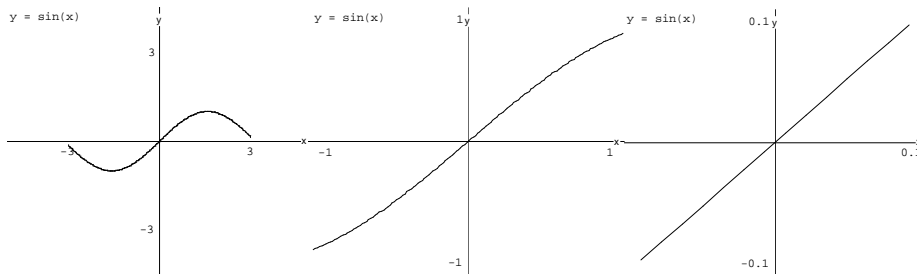
5. The following table shows values of a function $f(x)$. Use it to estimate $f'(2)$.

x	1.98	1.99	2.00	2.01	2.02
$f(x)$	7.762392	7.880599	8	8.120601	8.242408

Probing Questions:

1. Is this information enough? Do you need any other information?
2. Can you give a better estimate?
3. Can you give a general way to find an estimate of the derivative at a specific point given a table of values?
4. If you know that the function is $f(x) = x$, how do you relate your estimate to the actual value?

6. The following diagrams gives you successive graphs of the sine function with x in radians, with smaller and smaller scales, near the point $(0,0)$. Estimate the value of the derivative of $\sin x$ at $x=0$, using these graphs and explain your answer.



Magnifications of the graph of $\sin x$ near $x=0$

8. Find the derivative of each of the following

a. $f(x) = (x^3 + x)^3$ $x = -1$

b. $y = \frac{3x}{(x^3 + 1)}$ $x = 0$