

## **Elementary school children's number sense**

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### **Abstract**

*This study on children's number sense involved 750 children (374 boys, 376 girls) from Grades 4 through 7. The children were given a 10-item paper and pencil number sense test, to be completed in 30 minutes. Based on the answer scripts, 64 students were interviewed, to clarify their answers to the items. Both the written answers and oral explanations given by the children were then analyzed. The findings seem to indicate that while there was no appreciable difference between the sexes vis-a-vis number sense, generally number sense seemed to decrease as students progressed through the grades, as evidenced by an almost complete absence of estimation procedures, and an increasing reliance on algorithmic procedures as students progressed from Grade 4 to Grade 7.*

Experienced teachers have noticed that children have a tendency to hand in homework exercises without checking whether the answers they obtained were sensible. In other words, children don't seem to be using number sense when they submit answers that are patently absurd or wrong. Number sense, according to Reys & Reys, McIntosh, Emanuelsson & Johansson, and Der, (1999) refers to "the general understanding of number and operations, along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful and efficient strategies for managing numerical situations" (p. 61). While it seems to be the case that preservice teachers themselves have some problems with number sense (Menon, 2004), preservice teachers have seldom been exposed to children's lack of number sense. This study arose out of a belief that preservice teachers needed to be aware of children's number sense, in order to understand children's way of answering math questions.

### **Participants and Methodology**

This study involved 750 children (374 boys, 376 girls) from Grades 4 through 7. Preservice teachers from one group of mathematics methods classes/sections I taught administered the 10-item paper and pencil number sense test (Appendix 1), to the children. For each item, students were given space on the test paper itself, to give written

explanations for their answers. They were given 30 minutes to complete the test. After checking the answer scripts, 64 students were interviewed, to clarify their answers to the items. The preservice teachers analyzed both the written answers and oral explanations given by the children, and submitted their findings as a requirement for their mini research assignment. I then analyzed and synthesized these reports. This analysis and synthesis is what I am reporting in this paper.

The 10 items came from refining items I had used in previous studies (e. g. Menon, 2004), which, in turn, were partly based on items from Reys & Reys et al, (1999). The components of number sense, together with the relevant item number assigned to test them, are given next:

1. To make mathematical judgments (J), for example, by determining appropriateness and sufficiency of information--item #1 & 2.
2. To develop useful and efficient strategies for managing numerical situations (E), for example, by using estimation and number relationship--item #s 3 to 5.
3. The general understanding of number and operations (U), especially those related to fractions and decimals--item #s 6 to 10.

### **Results and discussion**

The results of the study are summarized in 16 Tables (Tables 1 through 16). Tables 1 through 3 give the results of 131 grade 4 boys, 146 grade 4 girls and the 277 grade 4 students. Tables 4 through 6 give the results of 166 grade 5 boys, 157 grade 5 girls and the 323 grade 5 students. Tables 7 through 9 give the results of 68 grade 6 boys, 62 grade 6 girls and the 130 grade 6 students. Tables 10 through 12 give the results of 9 grade 7 boys, 11 grade 7 girls and the 20 grade 7 students. Tables 13 through 15 give the results of 374 boys from grades 4 to 7, 376 girls from grades 4 to 7, and the 750 students from grades 4 to 7. Table 16 gives the percentage of responses according to components of number sense. Reference is made to these tables in the discussion that follows.

From Tables 1 and 2, it can be seen that the grade 4 girls have generally done a little better than the grade 4 boys. Only on item 7, the question about which fraction was larger, have the girls done worse, being 6 percentage points below the boys. But in terms of item by item performance, there does not seem to be too much difference

between the performance of the boys and girls. From Table 3 it can be seen that items 4 and 5 that involved addition and subtraction of two digit numbers did not pose much difficulty for the students, as 86% of them got these correct. But the interview results indicate that only about 12.5% or an eighth of the 64 students (and the majority of these students were from grade 6 and 7) used the relationship between addition and subtraction to get the answer. In other words, they used an algorithm, rather than number sense in order to solve these.

If we examine the grade 4 students' responses to the other 8 items, we notice that the percent of correct responses range from 22 to 53, with items 6 to 10 (those on numerical operations of fractions and decimals) scoring between 22 and 36, item 3 (on estimating the age in days) scoring 41% and items 1 & 2 (on making judgments on information given) scoring 50% & 53% respectively.

**Table 1.** *Item number, number of Grade 4 boys (n = 131) with correct and incorrect responses, and percent correct by item.*

Item #	# correct	# incorrect	% correct
1	62	69	47
2	73	58	56
3	53	78	40
4	110	21	84
5	115	16	88
6	28	103	21
7	52	79	40
8	24	107	18
9	34	97	26
10	35	96	27

**Table 2.** *Item number, number of Grade 4 girls (n = 146) with correct and incorrect responses, and percent correct by item.*

Item #	# correct	# incorrect	% correct
1	77	69	53
2	75	71	53
3	61	85	42
4	128	18	88
5	122	24	84
6	40	106	27
7	49	97	34
8	37	109	25
9	37	109	25
10	53	93	36

**Table 3.** *Item number, number of Grade 4 students (n = 277) with correct and incorrect responses, and percent correct by item.*

Item #	# correct	# incorrect	% correct
1	139	138	50
2	148	129	53
3	114	163	41
4	238	39	86
5	237	40	86
6	68	209	25
7	101	176	36
8	61	216	22
9	71	206	26
10	88	189	32

On comparing these grade 4 results with those of grade 5 students (Table 6), it was surprising to find that, for almost every item, the latter did worse than the former. The interview results indicate that the grade 5 students are more likely to resort to procedures and algorithms, and less likely to use number sense, possibly as a result of more “drill and practice” in school. Just as for grade 4 students, the grade 5 girls did generally better than the grade 5 boys (Tables 4 & 5).

**Table 4.** *Item number, number of Grade 5 boys (n = 166) with correct and incorrect responses, and percent correct by item.*

Item #	# correct	# incorrect	% correct
1	48	118	29
2	71	95	43
3	77	89	46
4	129	37	78
5	131	35	79
6	37	129	22
7	62	104	37
8	31	135	19
9	49	117	30
10	84	82	51

**Table 5.** Item number, number of Grade 5 girls ( $n = 157$ ) with correct and incorrect responses, and percent correct by item.

Item #	# correct	# incorrect	% correct
1	53	104	34
2	67	90	43
3	70	87	45
4	132	25	84
5	124	33	79
6	45	112	29
7	54	103	34
8	28	129	18
9	39	118	25
10	82	75	52

**Table 6.** Item number, number of Grade 5 students ( $n = 323$ ) with correct and incorrect responses, and percent correct by item.

Item #	# correct	# incorrect	% correct
1	101	222	31
2	138	185	43
3	147	176	46
4	261	62	81
5	255	68	79
6	82	241	25
7	116	207	36
8	59	264	18
9	88	235	27
10	166	157	51

From Tables 7 & 8, the girls in grade 6 did better than the boys in only 3 items, namely items 2, 5, and 10. This was contrary to the findings for grade 4 and grade 5 students, where the girls generally did better than the boys. The grade 6 students did better than grade 4 and grade 5 students on the items that required judgment, possibly because of increased maturity and experience. But for items 7 to 10, grade 6 students did worse than the grade 4 and grade 5 students. It seems to be the case, then, that *as the number of years in schools increase, there is an increasing reliance on algorithms and procedures.*

**Table 7.** Item number, number of Grade 6 boys ( $n = 68$ ) with correct and incorrect responses, and percent correct by item.

Item #	# correct	# incorrect	% correct
1	42	26	62
2	43	25	63
3	36	32	53
4	62	6	91
5	60	8	88
6	21	47	31
7	24	44	35
8	12	56	18
9	18	50	26
10	21	47	31

**Table 8.** Item number, number of Grade 6 girls ( $n = 62$ ) with correct and incorrect responses, and percent correct by item.

Item #	# correct	# incorrect	% correct
1	37	25	60
2	46	16	74
3	31	31	50
4	53	9	85
5	55	7	89
6	13	49	21
7	21	41	34
8	10	52	16
9	14	48	23
10	24	38	39

On comparing Tables 3, 6, and 9, we can see that the 6<sup>th</sup> graders have generally fared better in items 1 through 5, and done about the same in items 6 through 9. Surprisingly enough, the percent of correct responses for item 10 (placing of decimal point in a product) is about the same for grade 4 students as for grade 6 students, but is higher for grade 5 students. The better results of grade 5 compared to grade 4 students in this item could be explained by the extra exposure the former get in decimals. Perhaps the poorer results of grade 6 compared to grade 5 students on this item could be explained by an over-reliance on procedures, such as “count the number of places..?”

**Table 9.** *Item number, number of Grade 6 students (n = 130) with correct and incorrect responses, and percent correct by item.*

Item #	# correct	# incorrect	% correct
1	79	51	61
2	89	41	68
3	67	63	52
4	115	15	88
5	115	15	88
6	34	96	26
7	45	85	35
8	22	108	17
9	32	98	25
10	45	85	35

Tables 10, 11 and 12, for grade 7 students seem to give quite different results from that of students from grades 4, 5, and 6. A caveat is in order here, because of the small number (20) of grade 6 students in the study. Even so, it is interesting that, for items 6, 8 & 10 (those that dealt with the number of fractions/decimals between a pair of fractions/decimals, & the placement of the decimal point), not ONE student in grade 7 got it correct. As for items 3 and 4, interview results indicate that more and more students use the relationship between numbers and operations as they grow older. For example, out of a total of 64 interviewees, only 1 grade 5 student used these relationships, while 3 grade 6, and 4 grade 7 used these relationships.

**Table 10.** *Item number, number of Grade 7 boys (n = 9) with correct and incorrect responses, and percent correct by item.*

Item #	# correct	# incorrect	% correct
1	2	7	22
2	6	3	67
3	5	4	56
4	6	3	67
5	7	2	78
6	0	9	0
7	4	5	44
8	0	9	0
9	6	3	67
10	0	9	0

**Table 11.** *Item number, number of Grade 7 girls (n = 11) with correct and incorrect responses, and percent correct by item.*

Item #	# correct	# incorrect	% correct
1	8	3	73
2	7	4	64
3	2	9	18
4	11	0	100
5	11	0	100
6	0	11	0
7	7	4	64
8	0	11	0
9	0	11	0
10	11	0	100

**Table 12.** *Item number, number of Grade 7 students (n = 20) with correct and incorrect responses, and percent correct by item.*

Item #	# correct	# incorrect	% correct
1	10	10	50
2	13	7	65
3	7	13	35
4	17	3	85
5	18	2	90
6	0	20	0
7	11	9	55
8	0	20	0
9	6	14	30
10	11	9	55

From Tables 13 & 14, it seems that there was no appreciable difference between the performance of boys and that of girls.

**Table 13.** *Item number, number of Grade 4-7 boys (n = 374) with correct and incorrect responses, and percent correct by item.*

Item #	# correct	# incorrect	% correct
1	154	220	41
2	193	181	52
3	171	203	46
4	307	67	82
5	313	61	84
6	86	288	23
7	142	232	38
8	67	307	18
9	107	267	29
10	140	234	37

**Table 14.** *Item number, number of Grade 4-7 girls (n = 376) with correct and incorrect responses, and percent correct by item.*

Item #	# correct	# incorrect	% correct
1	175	201	47
2	195	181	52
3	164	212	44
4	324	52	86
5	312	64	83
6	98	278	26
7	131	245	35
8	75	301	20
9	90	286	24
10	170	196	45

From Table 15, only items 2, 4, & 5 showed more than 50% of the students getting correct answers. Even so, just over half of the students had the correct answer for item 2, on the age of the cab driver, given some irrelevant information about the ages of the passengers. Also, the seemingly high percent of students getting correct answers (a little over 80%) belie the fact that the majority of them used algorithms mechanically, without making use of the relationships involved among the numbers. In other words, about 90% of them did NOT use  $48 + 37 = 85$ , to deduce the answer to  $49 + 36$ , and  $38 + 47 = 85$  to deduce the answer to  $85 - 47$ . Rather, they just added 49 and 36, and subtracted 47 from 85, using the traditional addition and subtraction algorithms. Hence, about 80% of the students used inefficient strategies for managing these numerical situations, and were over-reliant on algorithms.

**Table 15.** *Item number, number of Grade 4-7 students (n = 750) with correct and incorrect responses, and percent correct by item.*

Item #	# correct	# incorrect	% correct
1	329	421	44
2	388	362	52
3	335	415	45
4	631	119	84
5	625	125	83
6	184	566	25
7	273	477	36
8	142	608	19
9	197	553	26
10	310	440	41

As for the items that were done correctly only by about a fourth or less of the students, these were items 6, 8, & 9, with corresponding percents of 25, 19, and 26. It

seems that about 75% of the students do not understand the densely packed nature of rational numbers. Most of them believed there were no fractions between  $\frac{2}{5}$  and  $\frac{3}{5}$  (item 6) because the next (whole) number after 2 (the numerator of  $\frac{2}{5}$ ) is 3 (the numerator of  $\frac{3}{5}$ ). Similar reasons were given for saying that there were no decimal fractions between 1.52 and 1.53 (item 8). Answers to item 9, on the best estimate for  $292 \times 0.96$ , indicate the common misconception that multiplication results in a larger number.

For item 7, to find the largest fraction, only 36% of the students got the correct answer. Even among this 36%, about half of them chose  $\frac{4569}{4570}$  as the correct answer because “4570 is the largest number (among 4569, 4570, 500, and 501).” In other words, they focused only on the denominator, and obtained the right answer with the wrong reasoning! The other half of the 36% wrote nothing down, leading one to believe that they just guessed. The interview results indicated that very many did not have a clue how to find the largest of the 4 given fractions, that many guessed, and just as many used the wrong reasoning by focusing only on the magnitude of the denominator.

As for the remaining items, 41% answered item 10 (on the placement of the decimal point for the product of 15.24 and 4.5) correctly. About a half of this 41% actually worked out the product, and then placed the decimal point correctly, without using estimation. The other half used estimation to arrive at the correct placement of the decimal point. In other words, only about a fifth of the students used estimation to arrive at the correct answer. Once again, the over-reliance on algorithms and rules was evident, with little or no reliance on estimation and making sense.

For item 1, almost 56% of the students gave  $2\frac{1}{4}$  as the correct answer to the number of taxis required to take 9 passengers, if one taxi were only permitted to take 4 passengers. This indicates that they just worked on the computational aspect of the problem, without paying any attention to the context (and this was borne out by interview results). In other words, they failed to see that  $2\frac{1}{4}$  taxis is an unrealistic answer, and this again is an indication of poor number sense, as they could not make appropriate judgments based on all the available information.

For item 3, about 45% got it correct. But closer examination (during the interview) revealed that about a half of them actually computed their age in days by multiplying, and then rounded it down or up as the case may be. Once again, then, only about a fifth of the students used estimation to arrive at the correct answer, thereby revealing that almost 80% of the students were not using mental arithmetic or useful and efficient strategies for managing numerical situations. Some slight measure of comfort can be taken by the fact that slightly over half of them realized that there was insufficient information in item 2.

Given that these 10 items needed very little computation, it is troubling that (discounting the inflated 83% to 84% for items 5 & 4) the percent of correct responses indicating number sense ranged only from 19% to 52%.

**Table 16.** *Percentage of responses according to components of number sense*

Number sense component	Item #	# of correct responses	Maximum # of possible responses	%
J	1 & 2	717	1500	48
E	3 to 5	1591 (310)	2250	71 (14)
U	6 to 10	1106	3750	29

J: To make mathematical judgments.

E: To develop useful and efficient strategies for managing numerical situations.

U: The general understanding of number and operations.

From Table 16, it can be seen that J was about 48%, E was about 71%, and U was about 29%. In other words, it would seem that a majority of these students from grade 4 through 7 were unable to make appropriate mathematical judgments, but were able to develop useful and efficient strategies for managing numerical situations. However, given that 2 out of the three items under E, items # 4 & 5 were correctly obtained by

NOT using relationship by about 90% of the students, but by straight computation, a re-evaluation of the 71% is in order. From Table 15, then, the total number of correct responses for items 4 & 5 would be 10% 1256, or 126. This would result in 310 correct (number sense) responses for category E, out of 2250, giving the percent of correct responses in the E category to be 14% and not 71% as shown in Table 4 (these figures are shown in parentheses in Table 16). As for the category U, only about 29% of the students seem to and have a reasonable understanding of numbers and operations. In category J (for making mathematical judgments) these students had 48%, which implies almost a half of them still cannot make mathematical judgments. But this seeming proficiency in mathematical judgment (compared to U, the general understanding of number and operations), could be an artifact of maturation rather than number sense per se. Overall, then, the percent of correct number sense responses ranging from 19% to 48% seem to indicate that these students from grades 4 through 7 do not have a good grasp of number sense.

### **Implications**

While I have not computed measures of reliability for this 10-item number sense test, I have used this instrument (or a slight variation of it) four times, with varying populations of students and preservice teachers, and have obtained similar results, results that are a cause for concern. Given that a number of items have been answered worse by students as their school experience increases, one has to ask whether teachers are unwittingly causing children to learn math through a reliance on drill and practice, without the profound understanding of fundamental mathematics (PUFM) suggested by Ma (1999). If that is the case, we may be preventing students getting a “greater facility with analyzing and making sense of data and deeper conceptual understanding of mathematics,” (p. 306, Olson & Berk, 2001). Even though the Principles and Standards for School Mathematics (NCTM, 2000) suggest teachers need to emphasize number sense, teachers seem unwilling to let go of the adage that “practice makes perfect,” with practicing algorithms made synonymous with “practice.” With standardized tests looming large in every teacher’s (and students’) life, how can a case be made for doing well in these exams and for getting a PUFM, even if we decrease drill and practice and increase conceptual learning and teaching? Is there sufficient data that students taught

with an emphasis on conceptual understanding do as well as or better in standardized examinations than those taught with an emphasis on drill and practice? Further research is needed to address this question.

## References

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**Appendix 1**  
**Number Sense Test**

Name: \_\_\_\_\_ Sex: \_\_\_\_\_ Date: \_\_\_\_\_

1. A taxicab is only allowed 4 passengers in one taxi. How many taxis would be needed to take 9 passengers? Circle your answer. Explain why you chose that answer. Draw pictures if you need to.
- A. 2
  - B.  $2\frac{1}{4}$
  - C. 3
  - D. Not enough information given

Explanation: \_\_\_\_\_  
\_\_\_\_\_

2. A cab driver picks up 4 passengers. The passenger's ages are 9, 11, 14 and 34. What is the age of the cab driver? Circle your answer. Explain why you chose that answer. Draw pictures if you need to.
- A. 20
  - B. 48
  - C. 68
  - D. Not enough information given

Explanation: \_\_\_\_\_  
\_\_\_\_\_

3. *About* how many *days* old are you? Circle your answer. Explain why you chose that answer. Draw pictures if you need to.
- A. 300
  - B. 3000
  - C. 30000
  - D. 300000

Explanation: \_\_\_\_\_  
\_\_\_\_\_

4. Given that  $48 + 37 = 85$ , what is the answer to  $49 + 36$ ? Circle your answer. Explain why you chose that answer. Draw pictures if you need to.
- A. 85
  - B. 86
  - C. 715
  - D. 4126

Explanation: \_\_\_\_\_  
\_\_\_\_\_

5. Given that  $38 + 47 = 85$ , what is the answer to  $85 - 47$ ? Circle your answer. Explain why you chose that answer. Draw pictures if you need to.
- A. 38
  - B. 42
  - C. 132
  - D. 817

Explanation: \_\_\_\_\_  
\_\_\_\_\_

6. How many different fractions are there between  $\frac{2}{5}$  and  $\frac{3}{5}$ ? Circle your answer. Explain why you chose that answer. Draw pictures if you need to.

- A. None
- B. One
- C. A few
- D. Lots

Explanation: \_\_\_\_\_  
\_\_\_\_\_

7. Which is the largest fraction? Circle your answer. Explain why you chose that answer. Draw pictures if you need to.

- A.  $\frac{4568}{4569}$
- B.  $\frac{4569}{4570}$
- C.  $\frac{499}{500}$
- D.  $\frac{500}{501}$

Explanation: \_\_\_\_\_  
\_\_\_\_\_

8. How many different decimal numbers are there between 1.52 and 1.53? Circle your answer. Explain why you chose that answer. Draw pictures if you need to.

- A. None
- B. One
- C. A few
- D. Lots

Explanation: \_\_\_\_\_  
\_\_\_\_\_

9. Without calculating the exact answer, circle the best estimate for  $292 \times 0.96$ . Explain why you chose that answer. Draw pictures if you need to.

- A. slightly more than 292
- B. slightly less than 292
- C. 292
- D. Cannot tell without calculating it

Explanation: \_\_\_\_\_  
\_\_\_\_\_

10. If a "broken" calculator displays 6858 as the answer to  $15.24 \times 4.5$ , where should you place the decimal point in the answer? Circle your answer. Explain why you chose that answer. Draw pictures if you need to.

- A. 6.858
- B. 68.58
- C. 685.8
- D. 0.6858

Explanation: \_\_\_\_\_  
\_\_\_\_\_