

## **Exploring preservice teachers' understanding of two-digit multiplication**

### *Abstract*

*About 70 preservice teachers were given a two-digit multiplication problem, solved in three different ways, by using three separate algorithms. Questions were asked to ascertain their understanding of the fundamental mathematics on which these algorithms were based. Findings indicate that a profound understanding of mathematics needed to explain why the algorithms work, is lacking among these preservice teachers. These findings and suggestions to redress this lack of understanding, will be discussed.*

Researchers from different countries (for example, Andrews & Hanley, 1997; Aubrey, 1997; Baturu & Nason, 1996; Bell, 1990; Goulding & Suggate, 2001; Ma, 1999; Rowland, Martyn, Barber, & Heal, 2001) have investigated teachers' subject matter knowledge in mathematics. Most of the research has examined teachers' mathematical knowledge in a number of domains. This study, however, focuses on preservice teachers' understanding on an algorithm that is of fundamental importance in computation, namely the algorithm for multiplying 2-digit numbers. The reasons for focusing on just the domain of multiplying 2-digit numbers are: it is fundamental and very familiar; it is a good place to start to clarify the difference between having procedural knowledge and having conceptual knowledge; and it can make clearer the connections between abstract mathematical laws and principles and their application to common computations.

### **Method**

I gave two math methods classes (77 students in all) three tasks, namely, three different ways of multiplying 2-digit numbers. These tasks were first written on the chalkboard, and oral questions were posed, to which written responses had to be given.

For each of these three tasks, the preservice teachers were to first write the answers to these questions *individually*. Then, *after* completing *all three* tasks, they were to get together in groups of 4 to 5, and write the responses, as a *group*.

## Tasks

### Task 1: The Standard Algorithm

I first used the standard algorithm and asked some questions about the partial products, thus:

$$\begin{array}{r}
 65 \\
 \times 34 \\
 \hline
 260 \text{ ————— } 4 \times 65 \text{ (4 groups of 65)} \\
 195 \text{ ————— } 3 \times 65 \text{ (3 groups of 65)} \\
 \hline
 2210 \text{ ————— } 7 \times 65 \text{ (7 groups of 65)}
 \end{array}$$

The discussion went something like this: The first partial product is obtained by multiplying 65 by 4, that is,  $4 \times 65$ , or 4 groups of 65. Do you agree? The second partial product, 195, is obtained by multiplying 65 by 3, that is,  $3 \times 65$ , or 3 groups of 65. Do you agree? So, the final product, 2210, is obtained by adding 4 groups of 65 to 3 groups of 65. That is, 2210 is the same as  $(4 + 3)$  groups of 65 or 7 groups of 65. But we started off with  $34 \times 65$ , or 34 groups of 65. Hence, we seem to be missing  $34 - 7$ , or 27 groups of 65. So, what happened to the missing 27 groups of 65?

### Task 2: Alternative Algorithm I

For Alternative Algorithm I, I showed them the following, saying that I obtained the partial products by multiplying “from the left,” and not “from the right,” as in the standard algorithm.

I asked them whether this alternative method of multiplying would work or not, and to justify their answer. For example, could we do the following (and why or why not)?

$$\begin{array}{r}
 65 \\
 \times 34 \\
 \hline
 1950 \\
 260 \\
 \hline
 2210
 \end{array}$$

Task 3: Alternative Algorithm 2

Next, I asked them to look at the same product, obtained by yet another method. First they have to explain how the partial products were obtained, and then discuss whether they can justify whether the method is generalizable. The algorithm follows:

$$\begin{array}{r}
 65 \\
 \times \underline{34} \\
 1820 \\
 240 \\
 \underline{150} \\
 2210
 \end{array}$$

**Results and discussion**

The category of responses, together with the percent of responses, is given in Table 1.

Task #	Response category	# of responses		% of responses	
		Ind	Gp	Ind	Gp
1	Correct explanation	47	70	61	91
	Nothing missing	30	7	39	9
2	Correct explanation	40	65	52	84
	Don't know	37	12	48	16
3	Correct explanation	30	45	39	58
	Don't know	47	32	61	42

**Table 1**

From Table 1, it can be seen that for Task 1, 61% of the preservice teachers understood the mathematics behind the standard multiplication algorithm. But that still leaves 31%, or almost a third of them, unsure about the mathematical basis for this algorithm.

As for Task 2, about half of them understood the mathematics behind the algorithm, while the other half did not. Even among those who understood it, very few could explain the reasons, using appropriate math terminology such as the commutative principle or the distributive principle.

Task 3 seemed to pose the most difficulty, as evidenced by the fact that only 39% could justify it, and give an explanation in terms of the answer being obtained by adding the partial products in a different order.

The percents discussed above are for the individual responses. If we turn to the percents for the group responses, it can be seen that there was marked improvement in the explanations/justification for all three tasks. Even though the improvement seems to be less for Task 3 than for the other two tasks, nevertheless, there seems to be a distinct

advantage in letting these preservice teachers participate in group discussions. This result seems to agree with what was found in studies of cooperative learning, such as those of Kumar & Harizuka (1997), Mulryan (1995), and Slavin (1990).

### **Redressing the lack of understanding**

Even though the lack of understanding of the mathematical principles undergirding 2-digit multiplication was of concern, it was later determined that a discussion along the following lines seemed to make significant progress in their understanding.

Initially, for the traditional algorithm, many of the preservice teachers could not give a reasonable explanation for the “missing” groups of 65. After some discussion, a few would venture the answer that the 2<sup>nd</sup> partial product, 195, is actually 1950, the product of 30 and 65, not 3 and 65, and therefore, no groups of 65 were missing. Then we went on to discuss the importance of emphasizing place value in multiplying.

For the alternative algorithm 1, after some discussion, they see that this is just multiplying by the tens first, and then the ones. In the traditional algorithm, we multiply by the ones first, and then the tens. So, since we have only changed the *order* in which we obtain the partial products, but not the value of the partial products themselves, the final product should remain the same.

Next, we discussed how the distributive and commutative laws apply to these two multiplication algorithms. We wrote the following:

$$\begin{aligned} 65 \times 34 &= 65 \times (4 + 30) \\ &= (65 \times 4) + (65 \times 30) \text{---Distributive Law} \\ &= 260 + 1950 \end{aligned}$$

= 2210, and this is the traditional algorithm

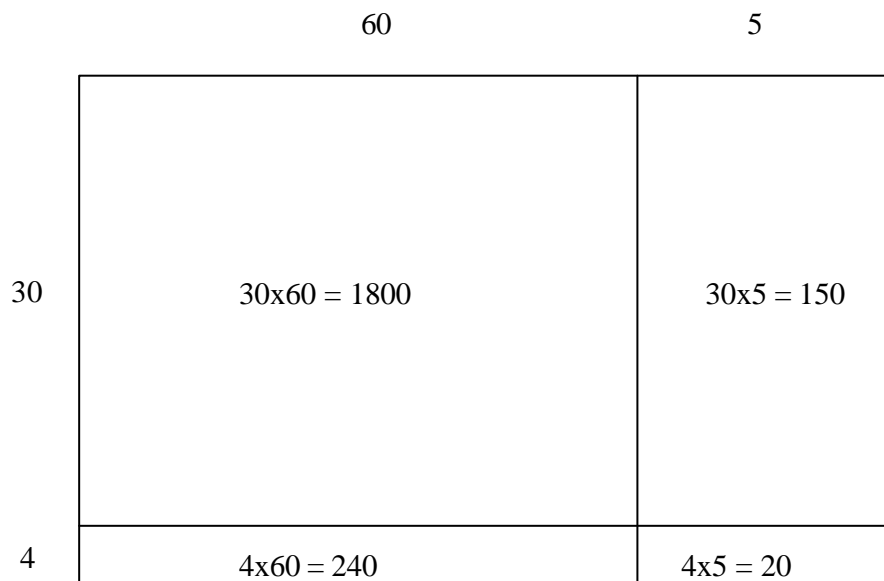
$$65 \times 34 = 65 \times (4 + 30) = 65 \times (30 + 4) \text{---Commutative Law}$$

$$= (65 \times 30) + (65 \times 4) \text{---Distributive Law}$$

$$= 1950 + 260$$

= 2210, and this is the 2<sup>nd</sup> algorithm

To give a visual representation of what is involved in 2-digit multiplication, I then drew upon the area representation for multiplication. For this,  $65 \times 34$  was broken up into  $(60 + 5)(30 + 4)$ , and these were shown to represent the sides of a rectangle whose area is obtained by multiplying these lengths. The diagram (Figure 1) shows a rectangle subdivided into 4 smaller rectangles, with the partial products being obtained from a combination of the subdivided rectangles.



**Figure 1**

In the traditional algorithm, the first partial product, 260, is obtained by adding 240 to 20 (the areas of the two smaller rectangles at the bottom of Figure 1), and the 2<sup>nd</sup>

partial product 1950, is obtained by adding 1800 to 150 (the areas of the two rectangles at the top part of Figure 1). In the 2<sup>nd</sup> algorithm, we get the partial products by first considering the two rectangles at the top, and then those at the bottom of Figure 1.

For alternative algorithm 2, most of the preservice teachers were initially very perplexed. After some time, they noticed that the 1<sup>st</sup> partial product is seemingly obtained by multiplying 5 and 4 (vertically) to get 20 (the rightmost 2 digits in the 1<sup>st</sup> partial product), and 6 and 3 (vertically) to get 18 (the leftmost digits in the 1<sup>st</sup> partial product). The 240 is obtained by (cross) multiplying the 60 by 4, and the 150, by (cross) multiplying 40 by 5.

Then, on relating the partial products obtained by the alternative algorithm 2 to the 4 rectangles shown in Figure 1, the preservice teachers noticed that the 1<sup>st</sup> partial product, 1820 was actually the sum of 1800 and 20, the areas of the 2 rectangles, added “diagonally.” The 2<sup>nd</sup> partial product, 240, was the area of bottom, left-most rectangle, and the 3<sup>rd</sup> partial product, 150, was the area of the top, right, rectangle in Figure 1.

So, by relating the three different algorithms to the partial products shown in Figure 1, the preservice teachers could now see that it was only the order in which the partial products were combined and added, that made the algorithms look different, but that the resulting product was exactly the same.

To reinforce the idea that all three algorithms are just different ways of combining the partial products, I then ask the preservice teachers to try several 2-digit multiplications, using the 3 algorithms, to see whether they get the same answer. In most cases, they agree that all 3 algorithms result in the same answer. Then they point out that

in some cases, such as  $34 \times 42$ , the alternative algorithm 2 does not seem to work, for the partial products and the final product are as follows:

$$\begin{array}{r} 34 \\ \times \underline{42} \\ 128 \\ 60 \\ \underline{160} \\ 348 \end{array}$$

They point out that, on using the other 2 algorithms, they get 1428, and not 348. I agree with them that the product is 1428, but ask them to check the partial products they obtained by relating them to the rectangular model for multiplication. On doing this, they find that the 1<sup>st</sup> partial product is the sum of the areas of the two smaller rectangles, added diagonally, while the 2<sup>nd</sup> and 3<sup>rd</sup> partial products are just the areas of the two remaining small rectangles. Hence, for  $34 \times 42$ , the 1<sup>st</sup> partial product is  $(30 \times 40 = 1200) + (4 \times 2 = 8) = 1208$ , and NOT 128, as computed earlier. In other words, place value is still of paramount importance.

Then, I revisit the rectangular model for multiplication, and re-emphasize that for all three multiplication algorithms, the partial products are just different ways of combining the areas of the four small rectangles. Hence, they can now generalize that all three algorithms should lead to the same product. Thus, I have led them to “multiple and fluid conceptions” (Buchmann, 1984, p. 14) of multiplying 2-digit numbers, thereby deepening their understanding of 2-digit multiplication.

I also ask them to reflect on which algorithm would be easier for students, and why. Some say that the traditional algorithm is better, as the children won't get confused. Others say that the 3<sup>rd</sup> algorithm is better, because one completes all the multiplications

first, and only then does one add the partial products, whereas, in the 1<sup>st</sup> two algorithms (including the traditional algorithm), both multiplication and addition have to be done to get the partial products. Such a reflection encourages them to question taken for granted assumptions engendered by the *professional traditions* (Eraut, 1994), the tendency to teach as they were traditionally taught.

Finally, I assign a mini-study to my preservice teachers, where each preservice teacher has to give a 2-digit multiplication problem computed by using three different algorithms (very much like the algorithms they themselves were exposed to) to 2 children, and investigate the children's responses and submit a report. At the end of the investigation, the preservice teachers tell me that now their understanding of the mathematics behind 2-digit multiplication algorithms is much better.

## **Conclusion**

In this paper, I shared that my 77 preservice teachers initially had a difficult time understanding the mathematical basis for the 2-digit multiplication algorithm, by exploring three different algorithms (one familiar, and two unfamiliar ones). Then, I reported that when the preservice teachers worked together in groups to explain the mathematical basis for the algorithms, they did much better than when they responded individually. Finally, I shared the steps I took to redress the preservice teachers' lack of understanding, namely, by a) using a rectangular model to represent 2-digit multiplication, b) connecting the 3 different algorithms to the rectangular model, c) discussing why some 2-digit multiplication did not seem to give rise to correct answers when the third algorithm was used, and d) letting the preservice teachers conduct

investigations on students' understanding of the mathematical basis for 2-digit multiplication algorithms.

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