

Textbooks, Word Problems, and Student Success on Addition and Subtraction

Sinan Olkun

Zülbiye Toluk

Abant Izzet Baysal University, Turkey

Abstract: Improving childrens' problem solving skills is an important aim of mathematics education. Since textbooks are widely used in classrooms as a source of mathematical learning it is necessary to look for their adequateness in developing such skills. With this purpose in mind first, elementary school mathematics textbooks were examined with respect to the different meanings of addition and subtraction. Second, 3rd and 4th graders were surveyed to determine their successes on these types of addition and subtraction word problems. Last, statistical procedures were run to investigate any relationship between the two. Results showed that textbooks did not adequately represent all types of addition and subtraction problems. Similarly, students were usually less successful on the problem types underrepresented in textbooks. Implications for mathematics education at the elementary level were discussed.

1. Introduction

Improving childrens' problem solving skills is an important aim of mathematics education. While solving problems students not only use their mathematical knowledge they already gained (Wyndhamn ve Saljö, 1997) but also improve their knowledge and understanding leading them to a better mathematical insight. Therefore, problem solving should be used as the basis for teaching mathematical concepts so that students construct their own knowledge (Peterson, Fennema & Carpenter, 1989). In this sense, for example, carefully chosen word problems can provide a rich context for learning addition and subtraction concepts (Greer, 1997).

The research literature about the meanings of operations derived from standard word problems has been firmly established (Carpenter, Moser & Bebout, 1988; Carpenter & Moser, 1984; Peterson, Fennema & Carpenter, 1989). Up to now, eleven different meanings of addition and subtraction have been defined as represented in word problems (Van de Walle, 1998). However, the practical implications of this knowledge base still need to be investigated. For example, textbooks often include only some of those problem types, thus limiting children's ability to learn the operations meaningfully (Greer, 1997; Peterson, Fennema & Carpenter, 1989).

2. Background literature

The meanings of operations stem from the student strategies that reflect the semantic structure of the word problem (Carpenter & Moser, 1984). In other words, students initially choose the strategies relevant to the actions and relationships described in problems. There are four main categories based on these relationships: join, separate, part-whole, and compare. Additionally, some subcategories are determined according to which quantity is unknown in the mathematical sentence modelled from a simple word problem. A basic mathematical sentence has three quantities: start, change, and result. Altogether makes eleven types of word problems (see Appendix A).

For young children not all these problems are alike. Some, especially start unknown problems, are relatively difficult (Peterson, Fennema & Carpenter, 1989) since it makes it hard to start to mathematically model the problem situation. While solving different word problems students not only experience the relationships between language and mathematical processes but also experience sense making and mathematization of the reality (Greer, 1997; Reusser & Stebler, 1997; Wyndhamn & Saljö, 1997). In order to produce an acceptable

solution for a problem, students should be able to attach meaning to what is happening in the problem. In other words, children should be able to establish appropriate relationships among the quantities which is provided by the problem context using their real world knowledge. For this, they need enough opportunities to experience with all kinds of problem types.

The purpose of this study is threefold: One is to examine whether elementary school mathematics textbooks adequately include the standard word problems representing different meanings of addition and subtraction. The other is to determine student success on the predetermined eleven types of problems. The last is to look for any relationship between the two: textbook inclusion of the problem type and the student success on the problem type. Findings of this study will also show the applicability of this knowledge base to students in the Turkish language and culture.

3. Method

In order to achieve the first purpose, the eleven types of word problems, which were all standard type (Peterson, Fennema & Carpenter, 1989), were translated into Turkish. In order to control any change in meaning, an English teacher translated the problems back into English (see Appendix A). This table is used to determine the distribution of word problems used in textbooks according to the problem types. We also looked for nonstandard problem types but there were no nonstandard problems in the textbooks examined (see Verschaffel, De Corte & Vierstraete, 1999; Greer, 1997 for examples of nonstandard word problems). That's why, these problem types were not included in the study. A widely used Mathematics textbooks series, all from the same publisher, for grades 1 through 5 were reviewed by 5 undergraduate students who took Mathematics Methods I and II courses and additionally familiarized with the related literature by the researchers. Each reviewer independently read all of the word problems in textbooks and tabulated them into predetermined categories. All but a few word problems were put in the same categories by different reviewers. Those misplaced word problems were collectively reread and put in an appropriate category after reaching an agreement by all of the reviewers and researchers. We included all word problems that can be solved using addition and subtraction of natural numbers and excluded such symbolic expressions as " $34 + 23 = ?$ " and such phrases as "what is the number if it is 3 less than 10?"

For the second purpose, names and objects in problems were replaced with the ones authentic to Turkish language and culture. A paper and pencil worksheet, which included a word problem from each category, a total of 11 word problems, was produced. Double digit numbers, whose sum did not exceed 99, were used in problems. Students were provided enough space following each problem on the worksheet. A total of 60 students, 30 from third and 30 from fourth grade, were surveyed with the worksheet. Students were provided only paper and pencil to solve the problems with no time limitation. Students' answers were analyzed according to the operation they chose for the problems and the result they found. If the students chose a correct operation but got the answer incorrect (i.e., procedural mistake) we considered their answer correct since we were more interested in the appropriate choice of operations.

To serve the last purpose of the study, statistical procedures were applied to determine any possible relationship between the distribution of problem types in textbooks and student success on these problem types.

4. Results

The results of the study are presented under four subheadings. These are: (1) the distribution of word problems in textbooks over grades, (2) the distribution of word problems in textbooks

according to problem categories, (3) student success on the eleven-type problems, and (4) the relationship between the distribution of word problems in textbooks and the student success on problem types.

4.1. The distribution of word problems in textbooks over grades

In the textbooks examined, there were separate units for addition and subtraction. Word problems were placed generally at the end of each unit. However, there were also some sections such as “problems for arithmetic operations” that included different and combined word problems. Although very few, there were word problems scattered through the units to explain the procedures for the operations. Rather, they were mainly used for the application of learnt procedures.

There were 304 word problems in textbooks for grades one through five. Four of the problems were not included in the analysis since they were incomplete (i.e., grammatically incorrect). The distribution of the remaining 300 word problems over grades is shown in Figure 1. As shown in Figure 1, word problems that can be solved using addition and subtraction operations are almost evenly distributed over grades except 3rd and 5th grades. In 3rd and 5th grades, many word problems are replaced with such numerical expressions as “ $23 + 12 = ?$ ” and such phrases as “what is the number which is 38 more than 29?” As stated earlier, these expressions were not considered as word problems. Additionally, in 4th and 5th grades, there are combined word problems instead of one-step addition and subtraction problems. These problems were included in the analysis as many times as the number of operations they required to solve.

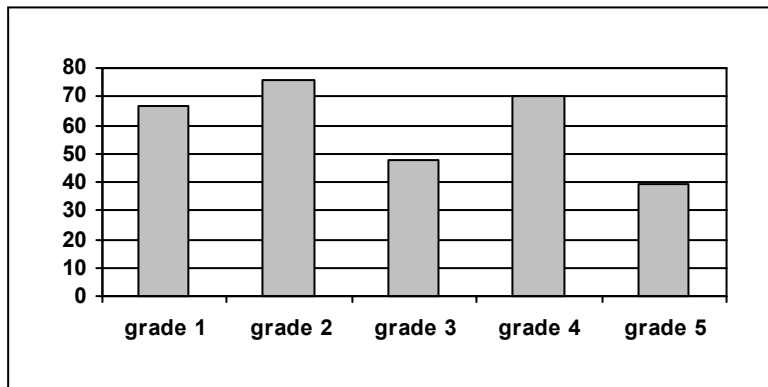


Figure 1. Distribution of word problems over grades

4.2. The distribution of word problems in textbooks over the categories

All of the 300 word problems could be categorized into the eleven predetermined categories. Since the distribution of the categories over grades was random and there were not much differences between the grades, the data were pooled together. Then the distribution of the word problems over categories was obtained (see Figure 2).

We expected an average of 20 to 30 problems representing each of the eleven categories since there were 300 hundred word problems ($300/11 \cong 27$). As presented in Figure 2, however, the first and fourth categories, all result unknown problems, were represented with more than 100 problems in elementary school mathematics textbooks. Similarly, the 10th and 11th categories, again result unknown problems, were the second most frequently used problem types. Other categories, the 2nd, 3rd, 5th, 6th, 7th, 8th, and 9th categories were not adequately

represented in textbooks. More strikingly, there was not even one problem representing the 8th category. There were also very few problems for the 5th, 6th, and 9th categories.

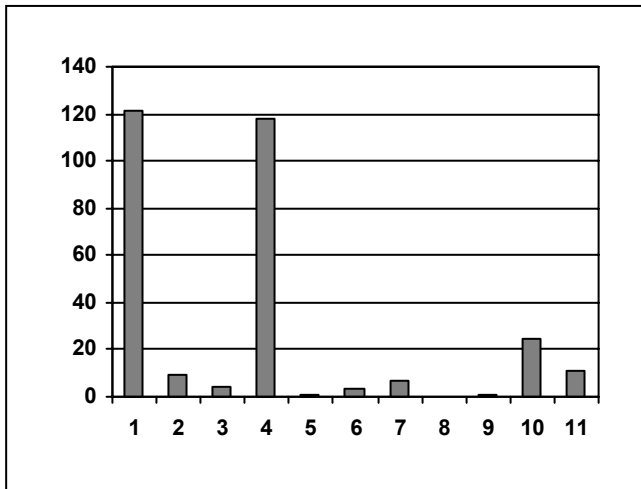


Figure 2. Distribution of word problems over categories

4. 3. Student succes on the eleven-type problems

Student succes on the eleven type problems is depicted in Figure 3. As shown, it ranged from 47% to 100% depending on the problem type, being the lowest on the category 8, as expected. Generally, fourth graders were slightly more successful than 3rd graders in all types of problems except the first category. Both 3rd and 4th graders were 100% successful on the first category. The variations in student success according to the categories followed almost the same pattern across the grades. All students were slightly less successful on subtraction (separate) problems than they were on addition (join) problems.

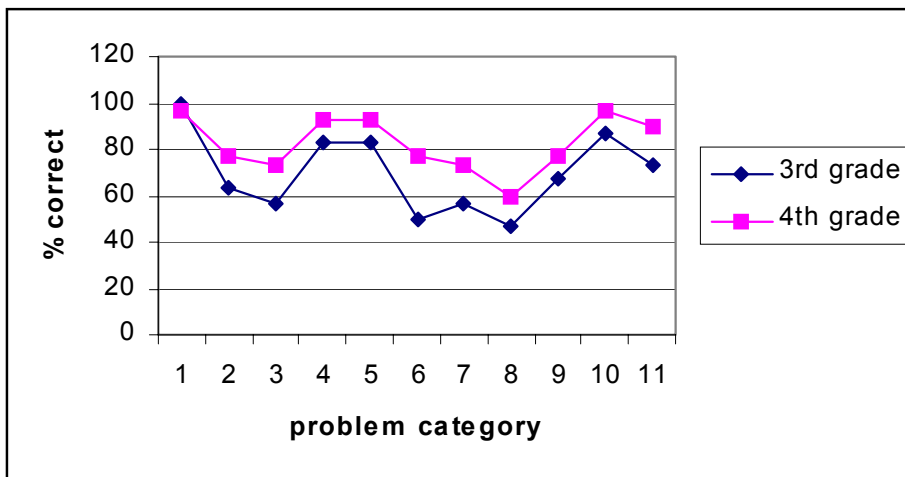


Figure 3. Student succes on the eleven type problems

4. Relationship b/w the distribution of word problems and students' success on problem types

The similarity of patterns between the number of word problems in textbooks according to categories and student success on these categories (see Figure 2 and Figure 3) suggested us to run statistical procedures to find out if any significant relationship exists between the two distributions. The analysis revealed that there was a statistically significant correlation at $\alpha=0.05$ level ($r=.629$, $p=.038$, and $n=11$).

5. Discussion and Conclusions

All of the word problems found in textbooks could be categorized into the eleven categories. That means all the actions and relationships described in English have correspondents in Turkish language. It can be said that the eleven category developed in English makes sense in Turkish, too.

Word problems are extensively used in the textbooks examined but not in a systematic manner. For example, some of the categories such as the first and fourth categories are overrepresented while others are either underrepresented or not represented at all in textbooks. This may prevent students to develop a rich concept of addition and subtraction. It may also hinder the development of students' problem solving skills. In fact, we found that students were generally less successful on the problem types that are underrepresented in textbooks. However, this relationship could be incidental. Further investigation is needed in order to make a reliable inference.

Reader may argue that those problem types on which students were less successful are more difficult than the other problem types on which students were more successful. Even if this was the case, those students we surveyed were at the 3rd and 4th grade and should have experiences with a number of problem situations so that they can develop rich mathematical concepts of addition and subtraction. In other words, they could be dealing with more complex mathematical situations than simple addition and subtraction. According to The Turkish Primary School Mathematics Curriculum (MOE, 1998), for example, students in 3rd and 4th grade should do addition and subtraction with five and six-digit numbers as well as with fractions. Even though the sum of the numbers used in problems did not exceed 99, the student success was as low as 47 percent. Therefore, students' failure on some of the problem types appears to be due to the lack of experience with such situations rather than the difficulty with numbers. The finding that student success on problem types showed similar pattern as the distribution of the problem types in the textbooks supports such a claim.

Few of the students made procedural mistakes. Instead, many of them failed to choose the correct operation because they were unable to identify the mathematical relationships between the quantities mentioned in word problems. One reason for this failure might be the inadequate experience of students with different types of problem situations in early grades. Our review of textbooks support this hypothesis. The overemphasis on procedures in the Turkish Curriculum (MOE, 1998) and the textbooks should be questioned. However, additional research, especially experimental and qualitative research should be done to make a reliable conclusion. Further research should also include nonstandard word problems that take real world knowledge as the base for mathematical modelling (Greer, 1997).

Given the limitations of textbooks, teachers and prospective teachers should be provided with access to recent research on childrens' thinking in general (Peterson, Fennema & Carpenter, 1989) and different meanings of operations in particular. For example, they may be required to construct word problems relevant to the categories described in the literature. Additionally, relevant revisions in the curriculum seem to be necessary.

6. References

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Appendix A.

Categorization of standard addition and subtraction word problems

TYPE	Sample word problem
1. JRU*	Jon has 5 marbles. Jim gave him 3 more. How many marbles does Jon have altogether?
2. JCU	Jon had 5 marbles. Jim gave him some more. Now Jon has 8 marbles. How many did Jim give him?
3. JIU	Jon had some marbles. Jim gave him 3 more. Now Jon has 8 marbles. How many marbles did Jon have to begin with?
4. SRU	Jon had 8 marbles. He gave 3 marbles to Jim. How many marbles does Jon have now?
5. SCU	Jon had 8 marbles. He gave some to Jim. Now Jon has 5 marbles left. How many did he give to Jim?
6. SIU	Jon had some marbles. He gave 3 to Jim. Now Jon has 5 marbles left. How many marbles did Jon have to begin with?
7. CDU	Jon has 8 marbles and Jim has 5 marbles. How many more marbles does Jon have than Jim?
8. CLU	Jon has 3 more marbles than Jim. Jim has 5 marbles. How many marbles does Jon have?
9. CSU	Jim has 3 fewer marbles than Jon. Jon has 8 marbles. How many marbles does Jim have?
10. PWU	Jon has 5 blue and 3 red marbles. How many marbles does he have altogether?
11. PPU	Jon has 8 marbles. 5 of them are red, and the rest are blue. How many blue marbles does Jon have?

*JRU, join result unknown; JCU, join change unknown; JIU, join initial unknown; SRU, separate result unknown; SCU, separate change unknown; SIU, separate initial unknown; CDU, compare difference unknown; CLU, compare larger unknown; CSU, compare smaller unknown; PWU, part-whole whole unknown; PPU, part-whole part unknown.

Source: Adapted from Peterson, Fennema & Carpenter, 1989; Van de Walle, 1998.