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Geometric Explorations with Dynamic Geometry Applications based on van Hiele Levels

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ABSTRACT: The purpose of this paper is to present classroom-tested geometry activities based on the van Hiele geometric thinking levels using dynamic geometry applications. The other ideas behind the activities include teacher questioning, active student participation, and student-centered decision-making. During the lessons student teachers engaged in self-exploration and reinvention of geometric relations. It was evident from the episodes that students raised their level of geometric thinking by building on their current geometric understanding.

Key Words: dynamic geometry, van Hiele levels, teacher questioning

INTRODUCTION

Traditional elementary and middle school geometry curricula focus on having students learn list of definitions and properties of shapes. This focus is misguided. Instead of memorizing properties and definitions, students should develop personally meaningful geometric concepts and ways of reasoning that enable them to carefully analyze spatial problems and situations (Battista 2001). Instruction should also aim at raising the level of students' thinking. Currently, the best description of students' thinking about two-dimensional shapes is the van Hiele theory of geometric thinking (Battista, 2002).

Improving students' geometric thinking levels is one of the major aims of mathematics education since geometric thinking is very important in many scientific, technical and occupational areas as well as in mathematics. For example, in Turkey one third of the mathematics questions in the university entrance examination contain geometric content (Olkun, Toluk, Durmus, 2002). However, geometry is often neglected in the school mathematics especially in the elementary period. Some possible reasons for this negligence would be the lack of resources such as concrete materials, computer software, and the lack of knowledge and expertise about how to use computers and other materials for instructional purposes. National Council of Teachers of Mathematics (NCTM) suggests, in Principals and Standards for School Mathematics, that interactive geometry software can be used to enhance student learning (NCTM, 2000).

The purpose of the present article is to present geometry activities for elementary school students based on the van Hiele geometric thinking levels (van Hiele, 1986) using a dynamic geometry application, Geometers' Sketchpad. The instructional activities were designed to emphasize students learning through explorations instead of teaching a specific mathematical content. Therefore, the other ideas behind the activities include teacher questioning, active student participation, and student centered decision-making (Hannafin; Burruss & Little, 2001). First, we will give a glance of theoretical frameworks that are useful in understanding and leading students' geometric thinking and learning. Then, we are going to cite several classroom episodes in which students made some progress and raised their level of geometric thinking.

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THEORETICAL FRAMEWORKS

Teacher Questioning

Questioning is a teacher tool that is used to guide or direct student attention toward the exploration and reinvention of mathematics (Martino & Maher, 1999). Teacher questioning involves active student participation in the discussion of mathematical ideas. Therefore, both the content and the timing of the question are of importance.

First, teacher invites students to express their thinking in an inquiry-based classroom environment. Teachers, then make informed decisions about students' mathematical thinking to lead subsequent discussions. Individuals are challenged to consider their solutions through questions asked by the teacher and their classmates. In terms of the content of the question, for instance, it is important for the teacher to ask more open-ended questions aimed at conceptual knowledge and problem-solving strategies. These can contribute to the construction of more sophisticated mathematical knowledge by students (Martino & Maher, 1999). In order to use teacher questioning effectively, the teacher must be knowledgeable about the content domain, and possess the ability to distinguish between student imitation and student reinvention of a mathematical idea. As a result, a task assigned by a teacher's question should provide students with the opportunity to reinvent mathematical ideas through both exploration and the refining of earlier ideas (Martino & Maher, 1999; Middleton, Poynor, Toluk, Wolfe, & Bote, 1999).

The timing of the question is another concern that a teacher should take great care. Teachers can contribute to the process of conceptual change through the vehicle of asking timely questions in support of student reinvention and extension (Martino & Maher, 1999). Instead of asking all the relevant questions first and waiting for the students to respond, teacher can use the questioning tool whenever it is relevant for the topic under discussion and waits for students to formulate their answers. If, however, the students in a class are at different levels of thinking, then teachers can provide activity sheets containing some directions as well as questions for students to think about. Teacher should be ready to answer or redirect student questions during the explorations. In sum, asking good and timely questions is an important task of teachers in the process of learning and teaching mathematics that requires knowledge about both mathematics and children's learning of mathematics.

Van Hiele Geometric Thinking Levels

School geometry presented in an axiomatic fashion assumes that students think on a formal deductive level. However, this is usually not the case, and they lack prerequisite understandings about geometry. We must provide teaching that is appropriate to the level of children's thinking (van Hiele, 1999). Van Hiele postulated that students progressed through a sequence of stages in geometric reasoning. The first level is Visual, which begins with nonverbal thinking. A student at the first stage, visualization, can identify shapes by appearance but doesn't recognize the specific attributes or properties of an object. S/he may, for example, be able to identify a square, but not recognize that it has 4 equal sides. Students can classify shapes on the bases of their geometric form.

At the next stage, analysis (analytic level), students are able to see and identify the properties of some specific objects. S/he can recognize that a rectangle has four sides and four right angles. Opposite sides are equal. Here language is important for describing shapes. At the third level, the informal deduction level, a child can make logical arguments about the attributes themselves or relations among attributes. She might reason, for example, "a square is a rectangle since it has the opposite sides equal, and has four right angles." The fourth and fifth levels in the van Hiele Theory are Formal deduction and rigor. These stages are not likely to be achievable by students at elementary school.

The ***van Hiele*** levels aren't age-dependent; your students will be at different stages, but a good geometry lesson will be accessible to all, allowing them to work at their own level of development.

Instruction intended to foster development from one level to the next should include sequences of activities, beginning with an exploratory phase, gradually building concepts and related language, and culminating summary activities that help students integrate what they have learned into what they already know.

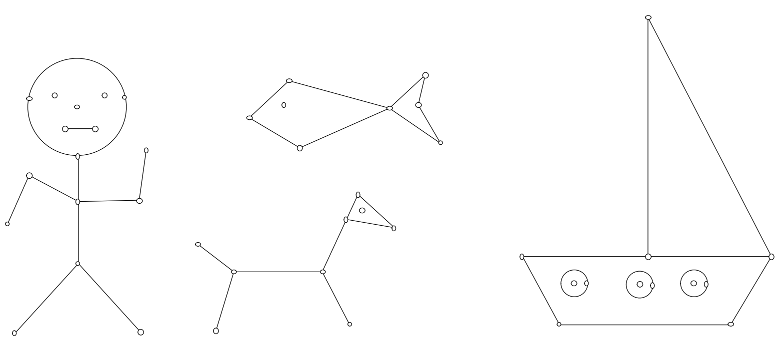
SAMPLE GEOMETRY ACTIVITIES PRESENTED TO STUDENTS

The Geometers' Sketchpad is a suitable dynamic environment in which students can explore geometry according to their geometric thinking levels. It provides students with shape-making objects that can be manipulated on screen. Teachers can prepare intro level activities to get their students familiar with the software environment. Then, they themselves can continue their explorations with little help from their teachers. The rest of this paper will mention such activities and some prospective teachers' reactions to these kinds of activities.

1. Visual level activities

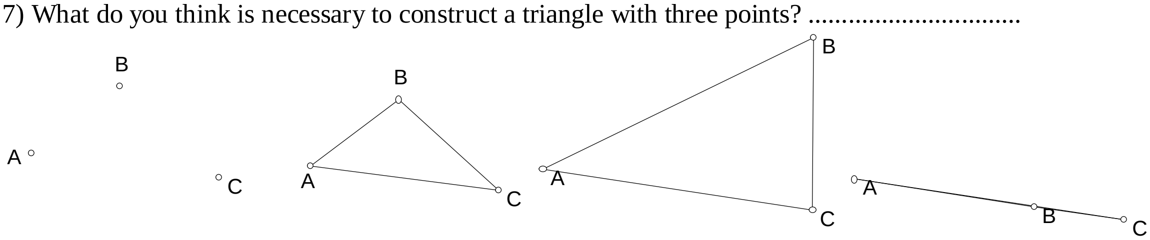
a) Drawing geometric forms

Draw pictures of objects such as toys and pets you saw before as shown below.



b) Constructing geometric shapes

1) Put any three points on the sketch and label them A, B, and C.
2) Construct segments between the points. What shape did you get? ....
3) Drag point B to the left or right. What changes? ....
4) Drag point C to the left or right. What changes? ....
5) Drag point A to the left or right. What changes? ....
6) Carry the point B to the AC line. What changes in the figure? ....
7) What do you think is necessary to construct a triangle with three points? ....



(1) (2) (3-4-5) (6)

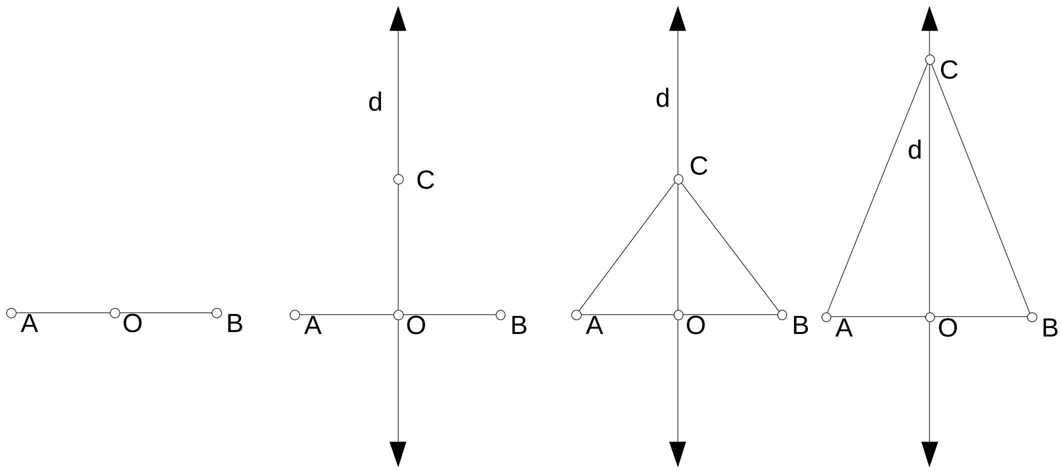
2. Analytical level activities

a) Examining the geometrical properties:

Draw a line segment AB. Determine its midpoint (O). Construct a line which is perpendicular to line segment AB and passes through this midpoint. Label it with d. Construct a point C on d but not on AB. Construct segments among A, B, and C. What can you say about the ABC triangle? ....

Move point C on line d. Which properties of the triangle change and what stays the same? ……………………

… … ……

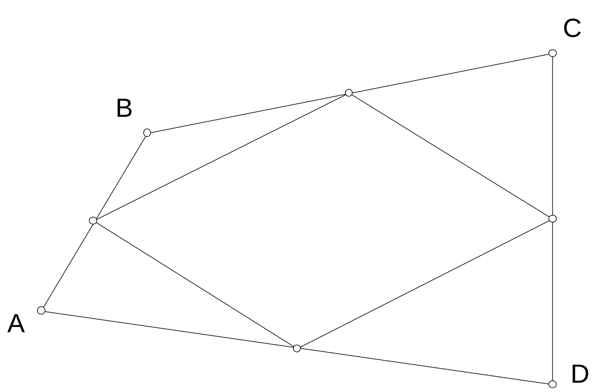


3. Informal deduction level activities

a) Constructing an arbitrary quadrilateral:

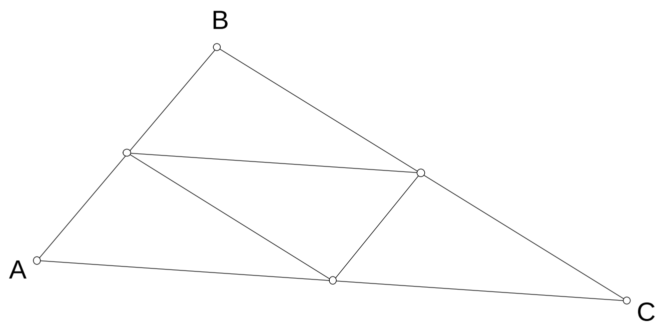
Draw any quadrilateral. Find the midpoint of each side. Connect the midpoints of adjacent sides. Observe the figure obtained. What does it look like? ....
Is it by chance? …………………………………………………………………………………
Is it true if the quadrilateral is not convex? ....
Is it true if the lines of the quadrilateral intersect each other? ....
How does the inner quadrilateral change with respect to the changes of outside quadrilateral?

First guess! Then test your conjecture ……………………………………………………………



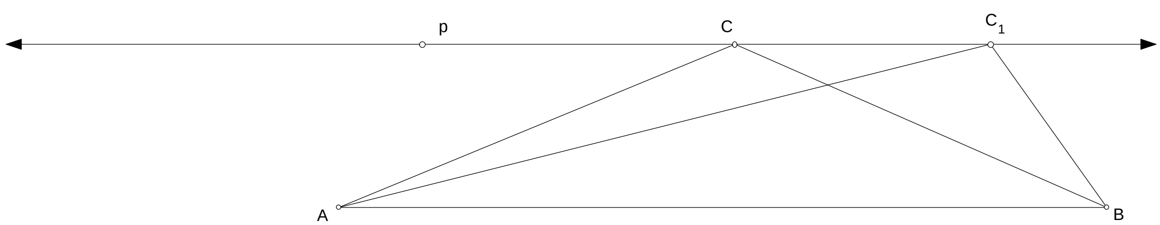
b) Constructing an arbitrary triangle:

- Construct any triangle. Join the midpoints of sides. What can you say about the area of the new 4 triangles? … … … … …………………………………………………………………………………
- Are these areas equal? Is it possible that they do not equal for some situations? First guess! Than test your conjecture ………………………………………………………………………………………
- Prove your claim. ………………………………………………………………………………



c) Area of a triangle

- Draw a line segment AB. Determine a point, C out of this line. Construct another line (p) passes through this point and parallel to line AB. Determine another point C_1 on this line which does not fit point C. Construct the ABC and ABC_1 triangles. What can you say about the area of these two triangles? ....
- After measuring the area of the ABC triangle, move point C through the line p. Can you observe any changes with the area?
- Why? ……………………………………………………………………………………



After examining the preceding activities we required student teachers to produce similar activities for elementary school students as well as for their own. The following edited episodes illustrate some activities produced by prospective teachers as well as their reflections on the activities they studied.

STUDENTS EXPLORATIONS and REFLECTIONS ON ACTIVITIES

Episode 1

Activity 1

- Draw several polygons starting from a triangle.
- Measure and calculate the sum of the interior angles of each polygon.
- Make a conjecture based on the evidences you gathered. ....
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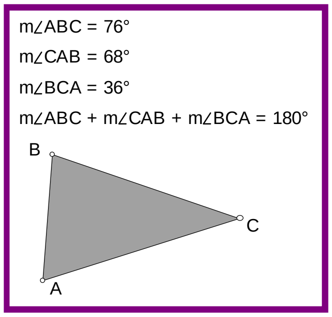


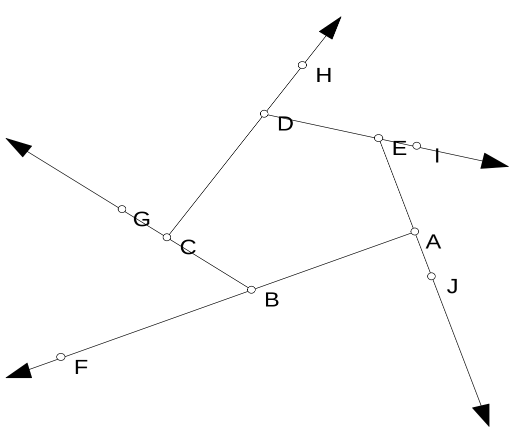
A Student' s Explorations and Reflections on Activity 1

In this case, starting with the triangle, we construct some polygons. We measured the angles of each side of polygons and we added them in order to find the sum of the interior angles. Then, we drug any vertices of a polygon. And we observed that each of the angles is changed but, the sum of interior angles is not changed. In other words; changing the size of polygons causes the angle measures to change, however, the sum of angle stays the same.

Episode 2

Activity 2

- Construct the rays AB, BC, CD, DE and EA to form a pentagon.
- Adjust the points if necessary so that pentagon ABCDE is convex.
- On the rays, outside the pentagon, construct the points F on AB, G on BC, H on CD, I on DE and J on EA. You may need to drag them into position, outside the pentagon.
- Measure the exterior angles JAB, FBC, GCD, HDI and IEA. Calculate the sum of these angles.
- Now move parts of the pentagon to see if the sum changes. (Make sure that the pentagon remains convex).
- Is the sum of these exterior angles the same for any pentagon?
- Try similar constructions using half-lines to make triangles, quadrilaterals, hexagons or other polygons, with a set of exterior angles.
- What are the sums of all the exterior angles in the polygons you have investigated?
- Is the sum any related to the number of sides?



A Student' s Explorations and Reflections on Activity 2

The exterior angle is always the same that measures 360. I think it is always the case since the sum of the exterior angles makes a complete turn. The sum is not any related to the number of sides. If the number of sides increases, the size of each angle decreases.

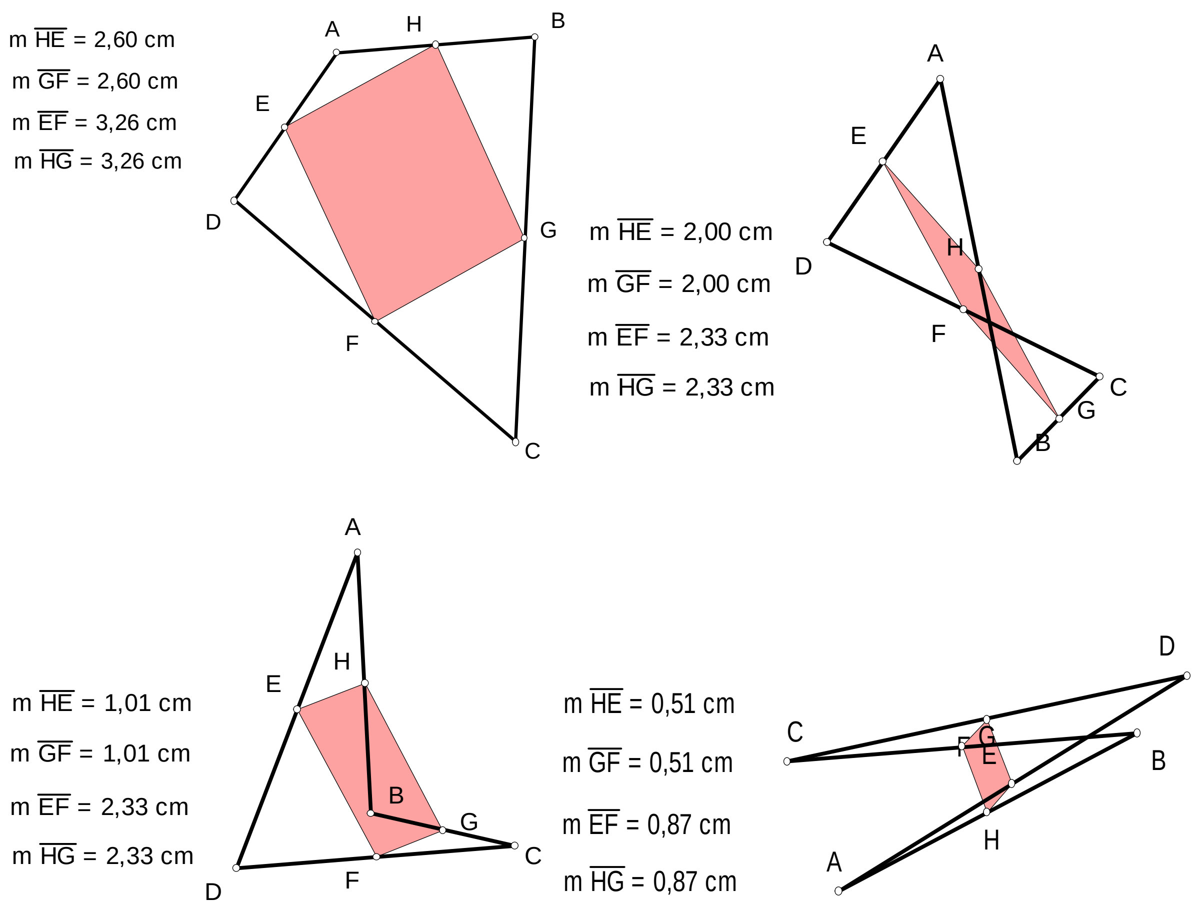
Episode 3: From informal deduction to formal deduction

Activity 3

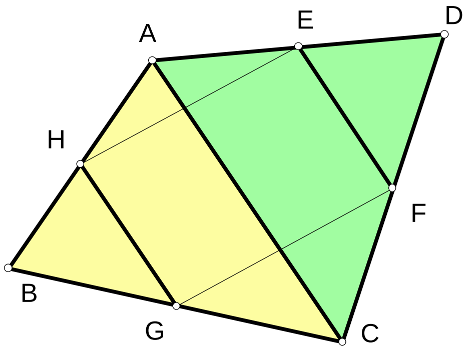
- Draw any quadrilateral. Find the midpoint of each side. Connect the midpoints of adjacent sides. Observe the figure obtained. What does it look like? ....
- Is it by chance? ……………………………………………………………………………
- Is it true if the quadrilateral is not convex? ....
- Is it true if the lines of the quadrilateral intersect each other? ....
- How does the inner quadrilateral change with respect to the changes of outside quadrilateral?
- First guess! Then test your conjecture ……………………………………………………

A Student' s Explorations and Reflections on Activity 3

The midpoints of the sides of a quadrilateral make up a parallelogram. When the vertices of the quadrilateral are dragged, the inner quadrilateral still remains as a parallelogram. This rule is true even for a concave quadrilateral and the quadrilateral that its lines intersect each other.



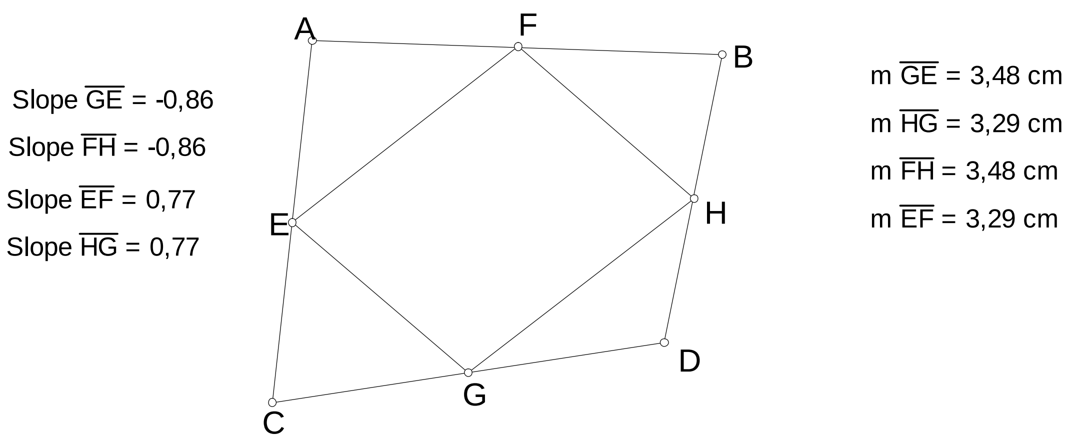
Student's Conjecture: As it is seen from the figures, the lengths of the opposite sides of the small quadrilateral are equal to each other and this is enough to show that the quadrilateral is a parallelogram. So we can conclude that the quadrilateral obtained by joining the mid points of the sides of any quadrilateral is a parallelogram. At first, this result seemed very strange for me. Than I tried to find the proof of this result and I have found it. Hear is the proof:



Student's Proof: I drew one of the diagonals of the quadrilateral. Than I remembered that if the midpoints of the two sides of a triangle connected, this line segment will be parallel to the third side of the triangle. So in the ADC triangle the line segments EF and AC are parallel to each other. By the same way in the ABC triangle the line segments GH and AC are parallel to each other and so does EF and GH. When I did the same things for ABD and ADC triangles that are obtained by constructing the second diagonal of the quadrilateral, I have found that the line segments HE and GF are also parallel to each other. Therefore, the EFGH is a parallelogram.

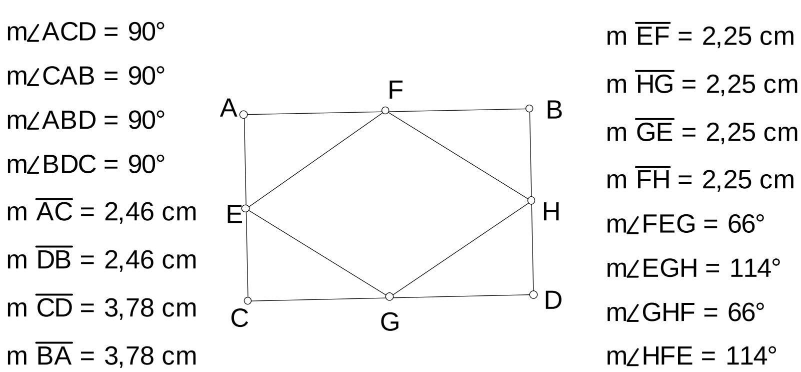
Another Student's Explorations and Reflections on Activity 3

I construct the midpoints of all of the sides of a quadrilateral. Then I connect the midpoints to form a midpoint quadrilateral. I think this midpoint quadrilateral must be a parallelogram. To confirm this I measure the slopes of the opposite sides of EGHF quadrilateral, and I found that the slope EF = the sl ope of GE; the slope of EG = the slope of FH .



When I dragged one of the vertices, I found some special quadrilateral by measuring the lengths, slopes and the angles;

a) If the ABCD is a rectangle then the midpoint quadrilateral ABCD is a rhombus.

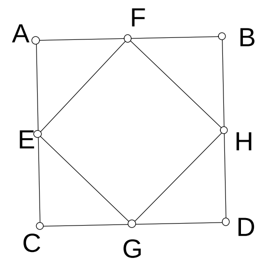


Since ABCD is a rectangle, AB = CD and AC = BD. E, F, H, G are midpoints of the sides of the rectangle ABCD. AB is equal to CD because ABCD is a rectangle, and F and G bisect AB and CD. So CG = GD = BF =

FA because they are all one-half of equal segments. Angles A, B, C, and D are all equal to 90 degrees because quadrilateral ABCD is a rectangle. Therefore triangles AEF, ECG, GDH, and HBF are all congruent by side angle side. Since these triangles are equal the sides EG, GH, HF, FH are all equal because they are corresponding parts of congruent triangles. I measured the lengths and the angles as shown above. Therefore quadrilateral EGHF is a rhombus.

b) If the ABCD is a square, then the midpoint quadrilateral ABCD is also a square.

m∠ACD = 90°
m∠CAB = 90°
m∠ABD = 90°
m∠BDC = 90°
m $\overline{AC}$ = 2,46 cm
m $\overline{DB}$ = 2,46 cm
m $\overline{CD}$ = 2,46 cm
m $\overline{BA}$ = 2,46 cm

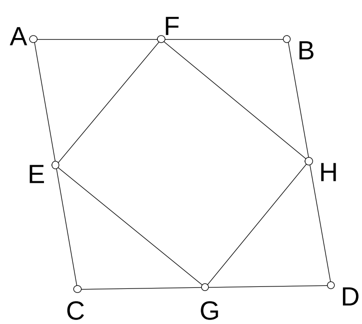


m $\overline{EF}$ = 1,74 cm
m $\overline{HG}$ = 1,74 cm
m $\overline{GE}$ = 1,74 cm
m $\overline{FH}$ = 1,74 cm
m∠FEG = 90°
m∠EGH = 90°
m∠GHF = 90°
m∠HFE = 90°

ABCD is a square. All sides are equal and all angles equal 90^o. If I connect the consecutive midpoints of each side, they form a square and four right isosceles triangles. The triangle AEF has one right angle, and since AE = EF, AEF and AFE angles have the degree of 45. Similarly, CEG and CGE also have the degree of 45. So GEF triangle is a right triangle. And in the same way EGH, GHF, HFE triangles are the right triangles and EG = GH = HF = FE are equal and therefore the inside quadrilateral is a square.

c) IF the ABCD is a rhombus, then the midpoint quadrilateral ABCD is a rectangle.

m $\overline{BA}$ = 3,36 cm
m $\overline{CA}$ = 3,36 cm
m $\overline{CD}$ = 3,36 cm
q = 3,36 cm
m∠FEG = 90°
m∠EGH = 90°

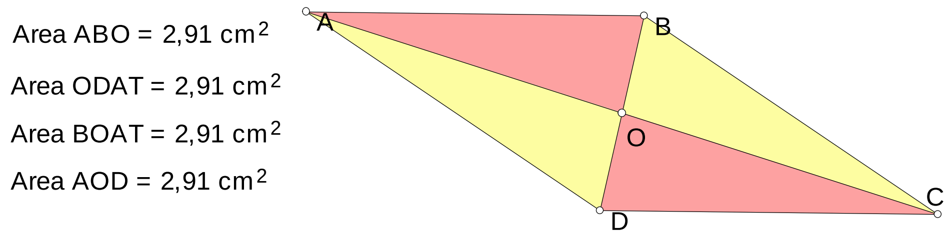


Quadrilateral ABCD is a rhombus, therefore angle A equals to angle D, angle B equals to angle C, and all sides are equal. Points F, E, G, and H are midpoints which AE = EC = CG = GD = HD = HB = BF = FA because they are all one-half of equal segments. Triangles HGD and AEF are congruent by side angle side. Similarly triangles ECG and BFH are also congruent by side angle side. Angle EGH = angle HFE = angle FEG. Angles GEC, EGC, BHF, and BPQ are all equal because they are base angles of identical isosceles triangles. Angles DGH, DHG, AFE, and AEF are also equal because they are base angles of identical isosceles triangles. Therefore because DHG+BHF is supplementary to angle FHR, DGH+EGC is supplementary to EGH, GEC+AEF is supplementary to angle FEG, and AFE+BFH is supplementary to EFH, angles PHG, HGE, GEF, and EFG are equal because all their supplements (DHG+BHF, DGH+EGC, GEC+AEF, AFE+BFH) are all equal. Therefore since there are four equal angles in a quadrilateral and there are only 360 degrees in any quadrilateral each angle must equal 90 degrees. Therefore quadrilateral FHGE is a rectangle because it has four angles equal to 90 degrees. Therefore this is a rectangle, because there are two pairs of equal parallel sides, and all angles are right angles.

Activity 4 (Student produced)

- Compare the areas constructed between the diagonals of a parallelogram
- Make a conjecture and try to justify your conjecture

Student's Conjecture: The areas constructed by the diagonals of a parallelogram are equal to each other.



Student's justifications for the conjecture: After finding this result I have tried to find a reason i.e. a proof for this and here is the proof. The area of the triangle ABD and the triangle BCD are equal to each other since their bases (AB and DC) and the heights to these bases are equal to each other (the heights are equal to the height of the parallelogram). Now, it remains to prove that ADO and AOB have equal areas. I know that the diagonals of a parallelogram intersect each other at their midpoints. This reminds me the rule that the ratio between the areas constructed in a triangle is directly proportional with the base lengths of them. Therefore, since DO and OB are equal to each other, their areas are also equal.

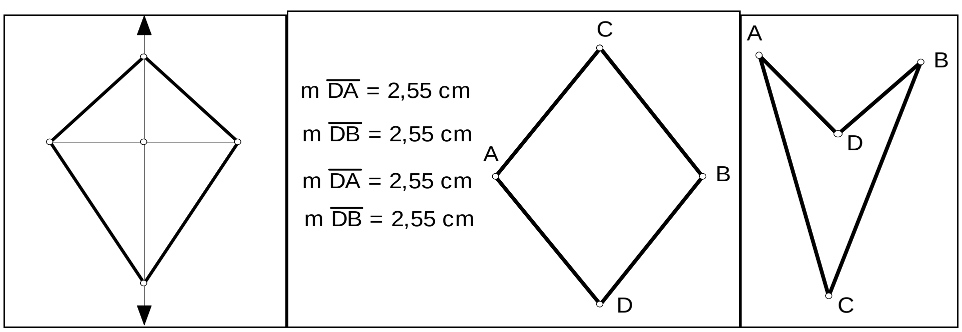
Activity 5

- Try to construct a deltoid (Remember you should so construct the deltoid that when you change the size of the shape by dragging any point the deltoid still remains as a deltoid).

A Student's Explorations and Reflections on Activity 5

In order to construct a deltoid I followed these steps:

- Construct a line segment and find its midpoint.
- Construct a perpendicular line to this segment from its midpoint.
- Construct two points on the perpendicular line on anywhere you like.
- Connect the end points of the line segment with these two points.
- Hide the line segment, intersection point and the perpendicular line.



Step 1 Step 2 Step 3

Student's conjecture: Before doing this exercise, when I think 'what is a deltoid?' always an ordinary shape deltoid like in Figure 1 comes to my mind. I think that a deltoid should look like this. On the other hand, I have realized that we can construct very different shaped deltoids. A rhombus and a concave shaped quadrilateral like in Figure 3 is also a deltoid provided that it has a pair of consecutive sides equal.

Students thought for the future: I believe that, dynamic geometry is a very effective method to teach many things in geometry with well-designed activities. In the future, I want to teach geometry with the effect of visualization of geometric software to my students in order to make them to discover themselves and make their understandings more concrete. I read a research study done in Turkey. In the study, the students are observed that while they are working on the activities, they discover the mathematical relationships. Besides, it is observed that the self-confidence of the students increase when they discover new properties, relations and patterns and also they started to see that the geometry as an activity to discover new things. Now, I feel the same way.

DISCUSSION

We witnessed that students both enjoyed and learned much in such a class. We also observed that they found the class very engaging. It is evident in the activities they produced and in their reflections that different students approached the same problem from different angles and found various explanations for the same mathematical truth. In a teacher directed (instead of teacher guided) classroom this would be difficult if not impossible. It is also evident in their reflections that they transitioned from one geometric level to the next by studying on their own on the activities either they produced or teacher produced. While they were approaching the problems as if there was only one solution, during the activities however they discovered that there might have been more than one way of doing the same problem.

With the guidance of the instructor and students developed an approach in which they first formulate questions then make a conjecture about the possible outcomes, and then try to justify their conjecture based on their explorations. Some students also developed very formal mathematical proofs for their questions. These outcomes could have been obtained in a teacher directed classroom but the deepness of students' understanding and satisfaction would not have been the same. During the class students also have changed their conceptions of teaching, especially geometry teaching. In their reflections some of them criticized the way they were taught geometry in their school years and decided to use a more visual, open-ended, and exploratory approach in just the way we used in this class.

CONCLUSION

Shape and location knowledge is important for understanding the geometrical shapes and structure of the geometrical problems. So it is necessary for children to gain experiences related to the every location and formation of the shapes to investigate the geometric shapes. It is possible to provide such an environment with appropriate software by using dynamic geometry applications. Such work supports and encourages students' development and understanding of the property-based conceptual system used in geometry to analyze shapes. It encourages students to move to higher levels of geometric thinking instead of having to memorize a laundry list of shape properties (Battista, 2002). Also the theoretical approaches concerned with the development of the geometrical thinking of students should be internalized by the teachers and educators to provide a rich learning environment.

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