Special characteristics of engineer students' knowledge of functions

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Abstract: In our college of technology engineer students are studying mathematics for 3 semesters. During this time they can attain that mathematical knowledge that they will use later in professional studies. Theory of functions takes great part in mathematical knowledge that is necessary condition to going on.

Our courses have been building using their knowledge brought from the high school. For correct appreciation of these, we examined our new students. The test had been taking in second week of the first semester. The ability of attaining function was controlled using Vollrath' theory.

In this paper we relate our experiences. These observations make possible useful conclusions for future programs in mathematics teaching.

Keywords: functions, knowledge of functions

Introduction

Everybody who is dealing with mathematics or have dealt with it sometime must have experienced that a lot of people consider this a very difficult subject, even though they are intelligent and hardworking they cannot cope with it. To the cultivation of this subject, the most important thing is to form the abstract notions. The mathematic notions are one of the most abstracts. To form a notion in mathematics we need some examples for the notion and also some counter examples. If the students can't associate the notions to the examples, they cannot really bring up the notion itself.

There is a qualitative difference between the mechanic and the intelligent learning process. Mathematician and psychologist, Richard R. Skemp reached the conclusion in one of his research that the examined people can learn much better with the intelligent learning process and with the help of notional structures and they can remember for it for much longer time. In the majority of the subjects, especially in mathematics the materials of the lessons are built upon one another, because everything that we learn is in connection with something else that we have learnt. [see 6]

Concepts and cognition

The concept is thinks', events' and acts' substantive characteristics in one mental unit. (Hungarian Encyclopaedia 1999)

To think of mathematical principles, notions and conceptions you need to represent the notions somehow. There are two kind of representation: outer (objective, pictorial, symbolic) and inner. Outer representations are observable, but the other is not. We can conclude to the quality of them through outer representation. We can only digest a notion, principle or conception if it becomes the part of our representation network. The level of understanding is marked by the number, strength and stability of connections. The pictures, examples, experiences take a prominent part in forming the effective concept image. [see 1] [see 9]

Understanding is one of the basic conditions of effective learning. According to Skemp to understand something is like assimilating to the appropriate schema (notional structure). The appropriate inner organisation of the notional structure can help understanding, after all you can remember better for an integrated notional structure than separated principles. [see 6]

L. S. Vigotsky also emphasises the connection between the notions. According to him the children's whole mental development's key is in the formation of academic notions. He argued that a notion in the psychological point of view is meaning of word. So the concept development is meaning of word development. With the conceptual development the structure of the system also develops. [see 8]

According to Vollrath the student has acquired the notion if he has these verifiable abilities [see 10]:

1. Able to give a definition of the notion.
2. Able to decide that an object is connected to a certain notion or not (notion identification).
3. Able to enumerate (construe) some examples for the given notion (notion realisation).
4. Knows the notion's characteristics.
5. Able to use the notion and its characteristics to explain given situations and problems.
6. Able to place the notion in the hierarchical structure (knows the sub- and hyperordinated notions).

The Development of notion of real function

Function is one of the most important basic notion of mathematics. The real notion of the real function began to emerge in the 17^{th} century, started by the works of Descartes primarily. Under the notion of function they understood the kind of algebraical correspondence that can be used for calculating a certain variable to another one. The notion of function is connected to derivability and integrability.

The discovery of the Taylorian arithmetic series was a new step in the notion of functions. Next to derivability and integrability the rectifiablity into power series was a new development connected to the name of Euler and he began to call it an analytical function. A function is an analytical expression, which consist of variables and constants in some ways. The "some ways" refer to the Taylor sequence. On the curve of an analytical function there cannot be breakings.

The great mathematicians of the 18^{th} century (Lagrange, Euler) thought that analytical functions are enough to explain the phenomena of nature. The problems of vibrating chord and conduction of heat showed that it is not the case. Fourier did not connect the notion of function to only one analytical expression, but to the representabilty of the Fourier sequence. He did not break away from the expression, just broadened the interpretation of the function. Though he allowed that the curve could have a terminate line breaking in one interval.

Dirichlet did not connect the notion of function to any curve or to any formula just to the unambiguous association. To give an f(x) function one principle is enough argued Dirichlet. In a certain set of numbers every x member is associated with only one definite y = f(x) value. He also gives an example to illustrate this function, which was impossible to be drawn up: this is the Dirichlet function:

$$f : R \mapsto R, \quad x \mapsto \begin{cases} 0, & if \quad x \in Q \\ 1, & if \quad x \in R \setminus Q. \end{cases}$$

Not only the mathematicians of the age, but even some of the present day students dislike this kind of "artificial" functions. People think about funtcions as they thought in the 18^{th} century. That is why people have difficulties with understanding the definitions of the 20^{th} century's functions. [see 2]

The diverse story of teaching the functions in Hungary

In middle-level education at the curriculum of the year 1899, the functions graphical representation had a bigger space, curves of the second order became part of the curriculum. In 1906 the National Secondary School Teachers Association established the Hungarian Mathematical Reform Committee with 24 members to secure the values of mathematics, and the modern teaching of it. Thanks to their collaboration there was a simplified notion of function and function analysis in the curriculum of 1926.

The school books which were introduced in 1956 prepared the way to the notions carefully, they wanted to educate the students to mathematical thinking.

The curriculum which were introduced around 1974 in the primary schools had a focus on geometry not just on arithmetics. The problem–centred mathematics teaching was also getting central place. The five topics in 1974 curriculum are: Sets and logics; Numbers and algebra; Geometry and measurements; Relations, functions and sequences; Combinatorics, probability and statistics. In the Basic curriculum of 1995 mathematics is a distinct educational area. The topics in the Basic curriculum: Thinking methods; Numbers and algebra; Functions and sequences; Geometry and measurements; Probability and statistics. [see 7]

Research

In our college the engineer students acquire the needed mathematical knowledge in 3 semesters, which they will use in their field work, and at their professional subjects. We measured the fresher's memories about the functions. (When we do not say other, then its domain is the possibly widest subset of *R*.) I want to examine whether their knowledge of functions should be confirmed or not. At the beginning of the research, my hypothesis was that the fresher's knowledge was not satisfactory, so they should spend much more time dealing with functions because a stitch in time saves nine.

91 fresher contributed to the research at the second week of the fall semester. The test was about vollrathian theory of notions of functions, it tested whether they acquired them or not. The test consisted of 6 tasks. The test focused on: functions, odd and even functions, periodic functions, functions of the second order, inverse functions. 50 minutes were available.

1. Write down the definition of function.
2 a. Select the odd functions:

$f(x)=5^x$, $f(x)=x^3-11$, $f(x)=\sin x$,

$f(x)=\cos x+\frac{\pi}{2}$,

$f(x)=\frac{1}{x}$, $f(x)=|x-5|$, $f(x)=|x|-5$.

b. Examine this function in the point of view of periodicity!

$f(x)=\cos x-\cos ax \qquad a\in R\setminus Q$

3. Give at least 5 examples of functions and 2 counter examples!
4. Judge these statements.
   A. Every monotone decreasing function has two zero point.
   B. $f(x)=2^x$ is a second order function.
   C. Inverse of the function $f(x)=\frac{x}{3}-\frac{2}{3}$ is function $f^{-1}(x)=3x+2$.
   D. $f(x)=(x+2)^2$ is an even function.
5. We cut a 20 cm line segment into two part. On the top of both we draw a regular triangle. Draw up the function which shows how the area of the regular triangles depends on the 20 cm line segment length. Find out that how long should be the line segment if want to get the minimal sum of triangles' area. [see 3]
6. Classify the elementary real functions.

The students average performance in the tasks:

Task	1	2 a	2 b	3	4	5	6
Performance	71%	59%	3%	61%	57%	8%	47%

Analysis of the task

In task 1, 78 of the students could give a definition. In the rest of the students the papers show that they can not understand the notion.

Some students' answer for task 1:
"Those set of points in the space, those conceive as a rule."
"Special relations." (It is right, but not enough.)
"Aggregate of unique relations."
"Let's make A and B one non empty sets. We image one element of set B to one element of set A from certain property." (It is not clear and exact enough.)

"A and B sets are given. Mapping from A and B is that no more that one element of set A is mapping to one element of set B."

In task 2 a. only four students could give the parity of all the functions. (The students got 1-1 point per good answer so the average is 59%.) The other 87 could not really pick which is even and which is not. I suppose they could not remember the definition also. [see 6] The functions which consisted an odd number were considered an odd function by many of them.

The students average performance in task 2 a:

Functions:	Answer "Odd"	Answer "Even"	Answer "Non-odd"
$f(x)=5^{x}$	54%	8%	38%
$f(x)=x^3-11$	50%	3%	47%
$f(x)=\sin x$	84%	5%	11%
$f(x)=\cos x+\frac{\pi}{2}$	50%	3%	47%
$f(x)=\frac{1}{x}$	60%	3%	37%
$f(x)=\lvert x-5\rvert$	24%	8%	68%
$f(x)=\lvert x\rvert-5$	27%	3%	70%

2 b: the condition ($a \in R \setminus Q$) was not understandable for a lot of students (although number set were repeated last week) or some students just ignored it, the wrong answers are due to this. The majority of the students did not even want to prove whether their answer is good or not.

In task 3 everybody could give an example to the function, but there were very few counter examples. We can conclude thus that they do not understand the notion itself, so they cannot use it in a new environment. They must have swotted the definition of a function.

Among the statements of the 4th task only the last 2 was problematic for the students. Only 8 students gave perfect answers. The $f(x)=(x+2)^2$ function was considered an odd one by most of the students. It is maybe because of the wrong analogy of even number equals even function, or maybe because of their deficiencies in function transformation. That is why problems with the task 2 a.

The students average performance in task 4:

Statements:	Answer "True"	Answer "False"	No answer
A. Every monotone decreasing function has two zero point.	0%	100%	0%
B. Is a function of the second order $f(x)=2^{x}$?	8%	92%	0%
C. Inverse of the function $f(x)=\frac{x}{3}-\frac{2}{3}$ function $f(x)=3x+2$.	18%	79%	3%
D. Is an even function $f(x)=(x+2)^2$?	85%	15%	0%

In task 5 most of the students only got to the drawing up of the function (90% of the students drew figure), or did not even start it. They can not manage it and they could not conceive an easier affine task, whose result is employable in this task. [see 4] The connection between the two orderly triangle's area and the two lines were very difficult to guess. Only 6 of them could do it. For the second question none of the students could prove their point. Maybe it is because their knowledge did not become here a system and that is why they could not really cope with it. [see 1] [see 6] It seems that a practical work was more difficult.

Some students' answer for task 5:

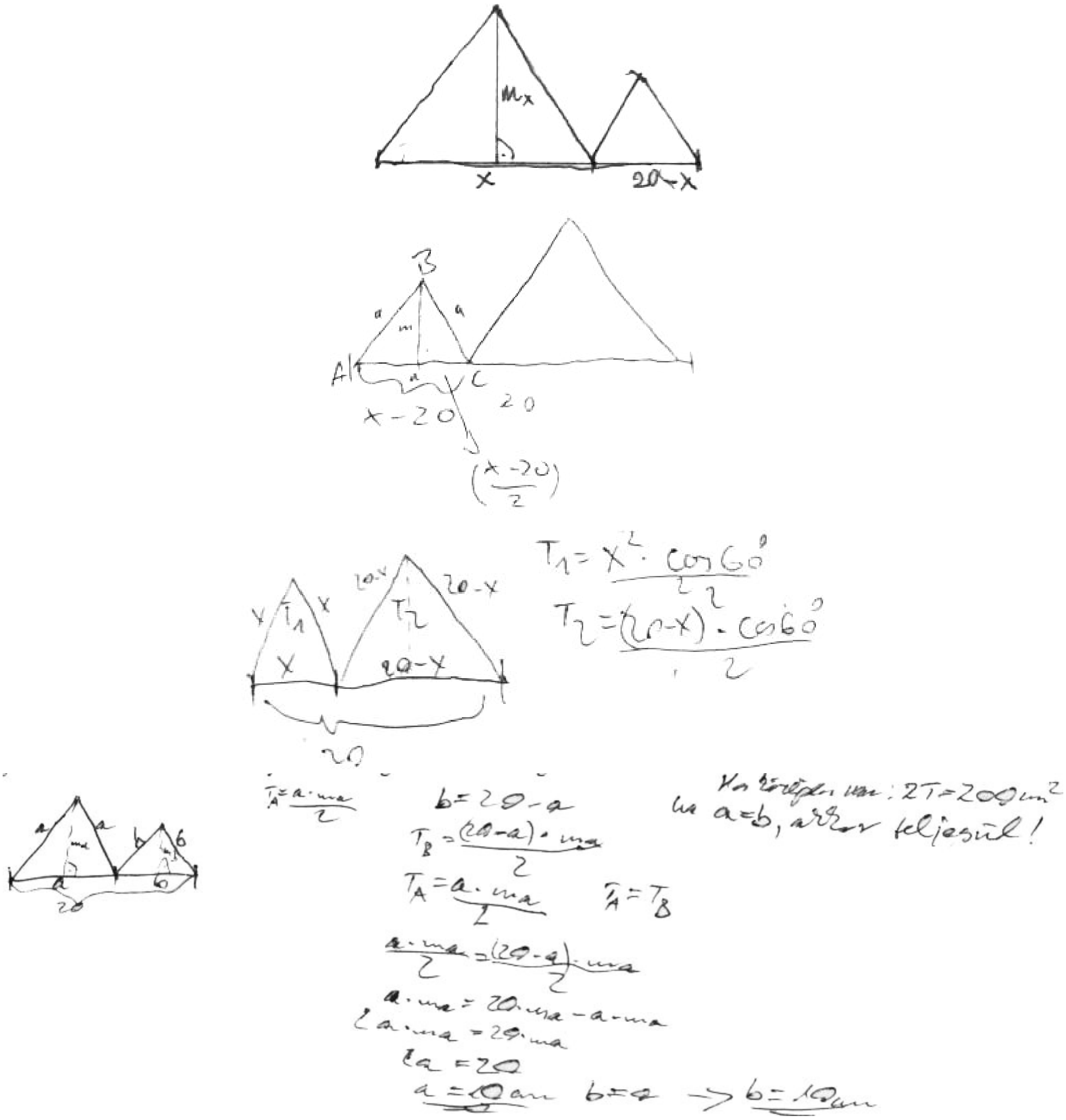

Figure 1

In task 6 the low efficiency indicator shows that the student could not really see the connection between the functions, although one way of understanding is seeing the connections. [see 6]

We also examined the type of secondary school of the student and also the year of the Matura exam at 18 in the mirror of their accomplishments.

31% of the students went to secondary grammar school, 55% went to technical high school and 14% of them were a vocational school student.

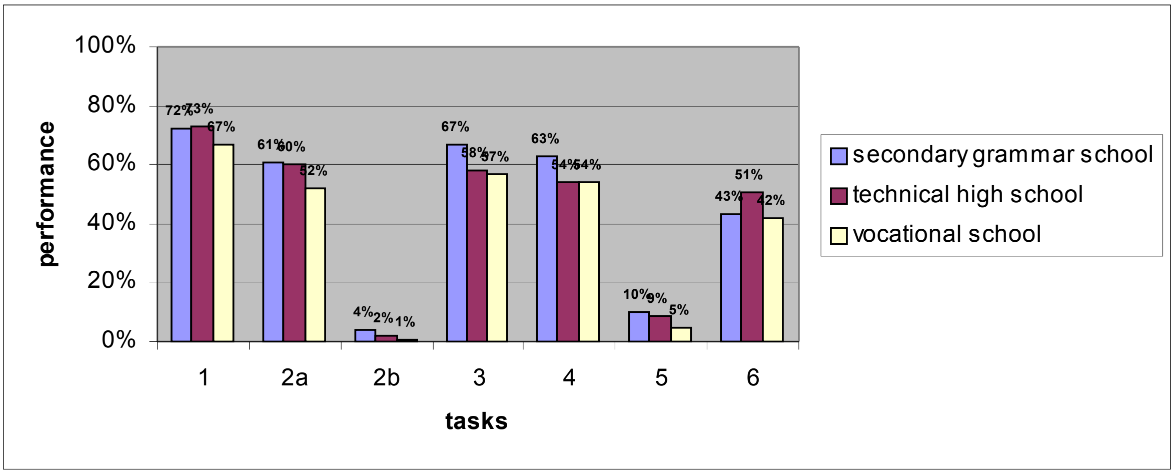

Figure 2: The connection between the performance and the type of school

With the exception of two tasks the students who used to go to a secondary grammar school performed a little better. In task one and six the students of the technical high school performed better. We can conclude that students from the secondary grammar schools have a better mathematical background and perhaps the students from the technical high schools have a better system view. (task 6)

55% of the students made their Matura exam in 2004, 31% in 2003 and 14% in 2002 or earlier.

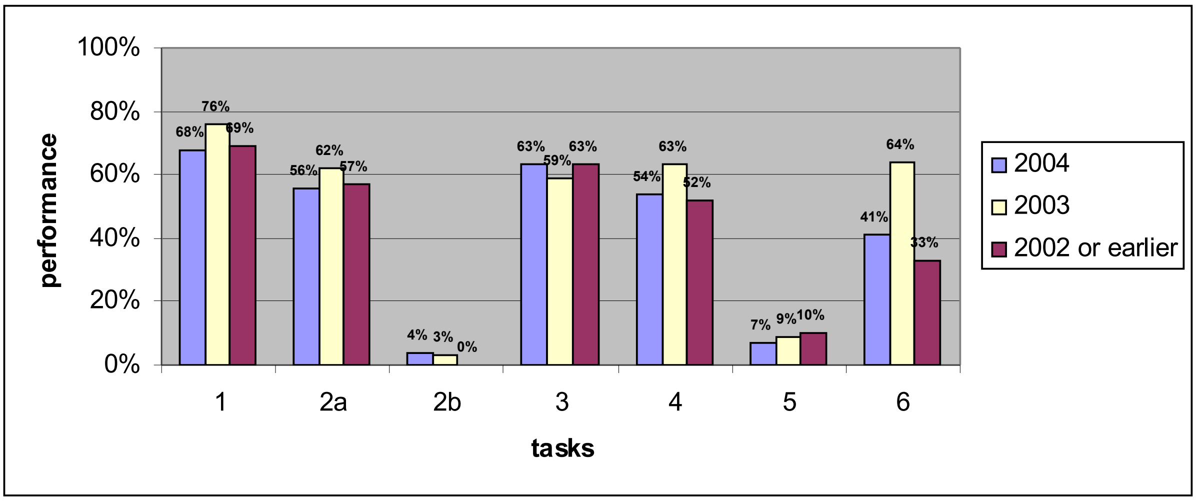

Figure 3: The connection between the performance and the year of the Matura exam

Students who made their Matura exam in 2003 are observably better at this test. The difference in the 6th task was the most salient. It is likely that these students spend their last year with learning and systemising their knowledge.

Conclusions

The results give us some food for thought in some of the tasks (task 2 b and 5), but we can trust that spending some hours with repeating, systemising and deepening the knowledge the deficiencies can be compensated.

Putting more emphasis on system theory and the connection between the notions, our concepts are more durative and more usable. [see 6]

Basis of all our knowledge is subject acquirement and reflectiveness. In mathematics the reflectiveness is much more important than acquirement of subject knowledge. So we have to cultivate reflectiveness too, beyond subject acquirement. [see 5]

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