

Preservice Teachers' Subject Matter Knowledge of Mathematics

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Sixty four preservice teachers taking a mathematics methods class for middle schools were given 3 math problems: multiply a three digit number by a two digit number; divide a whole number by a fraction; and compare the volume of two cylinders made in different ways from the same rectangular sheet. They were to a) solve them, explaining their solution, b) classify them as easy, of medium difficulty, or difficult, explaining the rationale for their classification, and c) explain how they would teach/help children to solve them. Responses were classified under three categories of subject matter knowledge, namely traditional, pedagogical, and reflective. Implications of these categories to effective math teaching are then discussed.

Seeing teaching as a complex skill, many researchers (Aubrey, 1997; Ball, 1988; Baturó & Nason, 1996; Eraut, 1994; Ma, 1999; Prestage, 1999; Shulman, 1986; and Tamir, 1988) have offered definitions of teacher knowledge that contribute to such a complex skill, and have also reported on teachers' subject matter knowledge (e.g. An, Kulm, & Wu, 2004; Ball & Bass, 2000; Burton, Daane, & Giesen, 2008; Lamb & Booker, 2003; and Rowland, Huckstep & Thwaites, 2004). In this paper, I focus on preservice teachers' subject matter knowledge of mathematics, drawing on Ma's (1999) profound understanding of fundamental math (PUFM), and Eraut's (1994) and Prestage's (1999) three phases of subject matter knowledge.

According to Eraut (1994), teachers go through the following phases of subject knowledge:

1. *Professional traditions*, where teachers rely on their own learning of the subject, and on the tradition of "this is the way it is/was usually done."
2. *Practical wisdom*, where teachers modify their teaching of the subject as a result of their experience in working with schoolchildren in the classroom.
3. *Deliberate reflection*, where teachers reflect on the actual content of the subject they are teaching, apart from how it is taught.

In this paper, I have re-named these categories as *traditional*, *pedagogical*, and *reflective*, respectively.

Method

Right at the beginning of the quarter, 64 preservice teachers, from two sections of a mathematics methods class for middle schools, were given 3 math problems: multiply a three digit number by a two digit number; divide a whole number by a fraction; and compare the volume of two cylinders made in different ways from the same rectangular sheet. They were to a) solve them, explaining their solution, b) classify them as easy, of medium difficulty, or difficult, explaining the rationale for their classification, and c) explain how they would teach/help children to solve them. Responses were then classified under three

categories of subject matter knowledge, namely traditional, pedagogical, and reflective.

Data analysis and discussion

The 1st question was divided into three parts, as shown next:

la. Evaluate 456×78 , explaining your reasoning/showing all working.

I b. State a word problem corresponding to 456×78 .

lc. Describe how you would teach/help a child to evaluate 456×78 . The results of these questions are shown in Table 1.

Table 1: Results of Question 1

Question #	Correct answer	Incorrect answer/ Not done	Procedural explanation	Conceptual explanation
la	61 (95%)	3/0	55 (86%)	6
Ib	48 (75%)	15/1	NA	NA
Ic	58 (91%)	0/6	46 (72%)	12

From Table I, it can be seen that although 95% of the preservice teacher could do the multiplication to get the correct answer, 5% of them got it wrong. As for explaining the reasoning, 86% of them relied on a procedural explanation, such as stating the steps for the multiplication algorithm.

About 25% of them could not write down a word problem that corresponded to the symbols needed to represent and solve the problem. Among the incorrect word problems were:

1. How many candies can you give 78 children if you had a total of 456 candies to give?
2. There are 456 pencils, and 78 erasers in the classroom. If we multiply the 456 pencils and the 78 erasers, how many pencils and erasers will we have in total?

When asked how they would help children to evaluate 456×78 , 72% of them restated the algorithm, while about 18% mentioned the use of manipulatives, or other means to develop an understanding of the algorithm.

From the perspective of *learner-knowledge*, and *teacher knowledge* (Prestage & Perks, 1999), the results for Question I a seem to indicate that the majority of the preservice teachers had not been able to transform their learner-knowledge to teacher-knowledge. That is, they could not transform the knowledge they had as students who had to pass math exams, to teachers, who had to have a much deeper knowledge, to teach math. That is, they were at the traditional phase of subject matter knowledge of multiplying numbers, relying on the procedures they themselves had learned as schoolchildren.

Also, 25% of them could not give a meaningful context for the symbols 456×78 , again indicating a procedural knowledge of algorithms, and a reliance on the traditional phase of subject matter knowledge.

From the results of Question 1e, it can be deduced that about 28% of them had pedagogical content knowledge, while 72% still seemed at the traditional phase of subject matter knowledge of multiplication of numbers. That is, the 28% of them not only knew the algorithm, but knew how to teach it in a developmentally appropriate manner to children.

The 2nd question was divided into three parts, as shown next:

- 2a. Evaluate $10000 \div 4/5$, explaining your reasoning/showing all working.
- 2b. State a word problem corresponding to $10000 \div 4/5$.
- 2c. Describe how you would teach/help a child to evaluate $10000 \div 4/5$.

The results of these questions are shown in Table 2.

Table 2: Results of Question 2

Question #	Correct answer	Incorrect answer/ Not done	Procedural explanation	Conceptual explanation
2a	45 (70%)	14/5 (22%/8%)	45 (70%)	0
2b	1	30/33 (47%/52%)	NA	NA
2c	16 (25%)	0/48	16 (25%)	0

From Table 2, it can be seen that although 70% of the preservice teacher could do the division to get the correct answer, 30% of them got it either wrong, or did not do it. As for explaining the reasoning, 70% of them (or all those who had the correct answer) relied on a procedural explanation, such as stating the steps for the division algorithm.

Only 1 preservice student could write down a word-problem that corresponded to the symbols needed to represent and solve the problem ("If \$1 0,000 was $4/5$ of the price of a car, what was the price of the car?"), while about 99% of them got it either wrong, or did not do it. All the incorrect word problems were associated with multiplication, not division.

When asked how they would help children to evaluate $10000 \div 4/5$, 25% of them restated the algorithm, while no one mentioned the use of manipulatives, or other means to develop an understanding of the algorithm. Indeed, 75% of them did not even attempt to do this question.

From the perspective of *learner-knowledge*, and *teacher knowledge* (Prestage & Perks, 1999), the results for Question 2a seem to indicate that the majority of the preservice teachers had not been able to transform their learner-knowledge to teacher-knowledge. That is, they could not

transform the knowledge they had as students who had to pass math exams, to teachers, who had to have a much deeper knowledge, to teach math. That is, they were at the traditional phase of subject matter knowledge of dividing a whole number by a fraction, relying on the procedures they themselves had learned as schoolchildren.

Also, about 99% of them could not give a meaningful context for the symbols $10000 \div 4/5$, again indicating a procedural knowledge of algorithms, and a reliance on the traditional phase of subject matter knowledge.

From the results of Question 2c, it can be deduced that none of them had pedagogical content knowledge, while 25% still seemed at the traditional phase of subject matter knowledge of division of numbers. That is, even though 25% of them knew the algorithm, none of them knew how to teach it in a developmentally appropriate manner to children.

The 3rd question was divided into three parts, as shown next:

3a. Suppose you are given a rectangular sheet of paper, with its length, L , twice its width, w . By folding the *length* of the paper into a circle, you get a cylinder V_L with the *width* of the paper as its height. Alternatively, by folding the *width* of the paper into a circle, you get a cylinder V_w with the *length* of the paper as its height. Explain whether the volume of the cylinders V_L and V_w are the same or unequal.



V_L



V_w

3b. Describe how would you teach/help a child solve this problem about the cylinders.

The results of these questions are shown in Table 3.

Table 3: Results of Question 3

Question #	Correct answer	Incorrect answer/ Not done	Procedural explanation	Logical explanation
3a	3 (5%)	51/10 (80%/16%)	2 (3%)	1
	Practical explanation	Incorrect expln/ Not done	Procedural explanation	Logical explanation
3b	35 (55%)	0/29 (0%/45%)	0	0

From Table 3, we can see that about 96% of them got the incorrect answer ("They have the same volume"), or did not attempt the problem. Of the 3 who got it correct, 2 used the formula for the volume of a cylinder, and one said that the radius, squared, contributes more to the volume of the cylinder than the height (a logical explanation). Such results indicate that their subject matter knowledge is woefully weak, even at the traditional, and learned-knowledge phase.

As for the results of Question 3b, it seems that they rely on pedagogical knowledge, as the 55% of them who explained it relied on a practical, hands-on approach, such as, "Fold the papers into cylinders, and actually pour in water or some stuff, to compare the volume."

The last question is shown next:

4. Classify problems 1a, 2a, and 3a, as easy (E), of medium difficulty (M), or difficult (D).
Give reasons for your classification.

The results of Question 4 are shown in Table 4.

Table 4: Results of Question 4

Question #	Easy (E)	Medium (M)	Difficult (D)
1a	64 (100%)	0	0
Explanation	Very familiar		
2a	54 (84%)	8 (13%)	2 (3%)
Explanation	Familiar	Have to remember rule	Always had difficulties with fractions
3a	10 (16%)	45 (70%)	9 (14%)
Explanation	Same paper is used, so answer is obvious	Have to know the formula, then can do it	Always had difficulty with word problems

From Table 4, everyone believed the multiplication problem was easy, because they were so familiar with those types of problems. The division problem, on the other hand, was not unanimously believed to be easy: only about 84% found it easy. Those who did not find it easy attributed their difficulty to having forgotten the rule, or to having difficulties with fractions. For Question 3a, even though only about 5% of them got the correct answer, that $V_L > V_w$, 16% believed it was easy, because it was 'just common sense: the paper does not change in size, so the volume cannot change, as it--the cylinder--is made from the same amount of paper." Among the others who did not find it easy, it was because they had "forgotten the formula," or "always had difficulty with word problems."

Conclusion and implications

The findings of this study indicate that the majority of the 64 preservice teachers seemed to be in the *traditional* phase of subject matter knowledge, where they could do the computations in an algorithmic manner, but were not able to transform that knowledge to either the *pedagogical* phase, or to the reflective phase. This is best shown by their responses to the cylinder problem, when having a deep understanding of mathematics that goes with the reflective phase, would have allowed them to reason out the answer, even without remembering any formula for the volume of a cylinder. Indeed, this was a similar problem posed for middle school children (in <http://www.figurethis.org>), and these preservice teachers are supposed to be teaching kids from grades K-8. In their defense, however, it must be stated that they had only completed one math methods course, and were about to take the 2nd math methods course.

What is disconcerting, however, is that almost every one of the 64 preservice teachers had had some college mathematics, and so it is surprising that so many of them were having difficulty with the problem of dividing a whole number by a fraction. One would have expected them to have at least the learned-knowledge of what they had learned in school and college, or be in the traditional phase of subject matter knowledge, where they could at least do the procedures. Also, not being able to give a contextualized word problem associated with multiplication and division, is also an indication of a not having attained the reflective phase of subject matter knowledge.

One possible way to address this lack of subject matter knowledge, especially the reflective phase, is to give opportunities for the preservice teachers to reflect on the actual mathematics behind whatever math topics they are supposed to teach. As an example, if students are asked to find all possible (whole number) linear dimensions for rectangles that can be made out of the whole length of a rope of 36 inches long, they could be asked to reflect on the math behind this activity. If they can, for example, be led to see that it reduces to finding two addends whose sum is 18 (such as 1 & 17, 2 & 16, etc), then we are helping them in the reflective phase of subject matter knowledge.

Such knowledge should make them more flexible in their teaching, as they will not only know how to do the computations, but can also see the larger picture of the math on which that computation is based, and can therefore use developmentally appropriate approaches to teach the topic at hand.

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