

ASSESSING THE MATHEMATICS ACHIEVEMENT OF COLLEGE FRESHMEN USING PIAGET'S LOGICAL OPERATIONS¹

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INTRODUCTION

An alarming observation of Filipino students reveals that they excel in knowledge acquisition but fare considerably low in lessons requiring higher order thinking skills. This sorry state is evident in the performance of students in national and international surveys on mathematics and science competencies. Separate studies on the mathematics performance of pre-service teachers (Philippine Daily Inquirer, 1988, p.14) and mathematics teachers on the 1993 and 1994 Professional Board Examination for Teachers (Ibe, 1995) reveal the same dismal picture of mathematics competencies of those who intend to teach it as well as those who have been teaching at the elementary and secondary levels.

Gathering evidences of the problem at the national level helps administrators find ways to remedy the situation. But interventions can also start at the classroom level. Teachers can also do their share of investigation and exploration of the problems students encounter in learning mathematics.

Since changing times require schools to develop critical, creative and independent thinkers, teachers can initially identify impediments to the attainment of these goals. Diagnosis

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of the process skills of students has to be undertaken by teachers to help them identify students' needs.

Teachers should maintain the development of cognitive skills as the central goal of their instruction. They should be able to design a curriculum that allows students to develop their process skills that lead to expertise in problem solving. They should be able to create an environment that nurtures the capabilities of students and develops learners' potentials to the fullest.

Expert problem solvers are thinking individuals who can observe, classify, measure, communicate, predict, interpret, analyze, synthesize, deduce, and infer. They can see, organize, and make sense of every information given in a problem situation through reflective abstraction (Limjap, 2002, pp.57-58). They possess the process skills needed to systematically engage in any mathematical task. College freshmen are expected to possess these process skills as they tackle the course in college algebra.

PIAGET'S THEORY OF COGNITIVE DEVELOPMENT

Several theories have been developed to explain the cognitive development of children. Jean Piaget, a Swiss psychologist and biologist formulated one of the most widely used theories of cognitive development. Piaget's theory continues to be an adequate theory of intellectual development for educational research in spite of the various neo-Piagetian theories that evolved.

The potentials of formal operational thought according to this theory develop during the middle school years. According to the theory, these potentials can be actualized by ages 14, 15, or 16 years, if proper learning experiences are encountered by the individual. These experiences consist of both the individual and social aspects of learning. As the students attain formal thought, they are able to apply mental operations not only to concrete objects, but to objects, situations, ideas, and concepts that are not directly perceived.

Apparently, learning mathematics involves formal operational thought. The research of Piaget shows that individuals are formal operational thinkers by ages 15 or 16, the usual ages of college freshmen in the Philippines. Consequently, mathematics teachers expect college freshmen to perform at the formal level. So, the curricula and syllabi for mathematics courses in the Philippines are designed with the goal of engaging students in formal level learning activities. This study was conducted using Piaget's theory to investigate whether a group of college freshmen performs at the expected level of formal thought.

Central among Piaget's conceptions is the idea of sequential stages of cognitively different levels of intellectual development. There are four stages: the Sensory-motor stage (age 0-2 yrs.), the Pre-operational stage (age 2-7 yrs.), the Concrete Operational stage (age 7-11 yrs.), and the Formal Operational stage (ages 11-16). For our purpose, let us focus the discussion on the concrete operational and formal operational stages and limit the description of these stages to the logical operations that Piaget and Inhelder gave (1964). Since the subjects of this study are freshmen of a polytechnic college, emphasis is given to the description of the possible thought processes at their age level.

The Concrete Operational stage is the initial period of logical thought. According to Piaget, at this stage, the mental activities of the learners begin to change from notions to intellectual perceptions. Learners begin to develop intellectual perceptions of concrete objects. As they manipulate data, they are able to deal with several variables, which appear in the form of the properties of data. They demonstrate logical thinking in relation to physical objects as they develop such mental abilities as the conservation of properties of objects. They are able to associate objects, mentally reverse actions and combine information about certain data. Their mental activities include holding two or more variables at a time when studying objects, conserving their numerical properties, classifying and ordering them. According to Piaget, the concrete-operational stage is marked by the development of operations such as classification and seriation. He defined concrete operations as mentally internalized and reversible systems of

thought based on manipulations of classes, relations and quantities of objects. At this level, the major restrictions are that mental operations can be applied only to first-hand situations. This makes thinking and understanding limited to objects that are concrete or physically present. The understanding of abstract concepts at this level is very limited. The learners may be able to classify objects according to their properties, order objects and events, group according to categories and balance series of relations of the systems. This allows them to engage in the logical operations of classification, seriation, logical multiplication and compensation.

Piaget's theory claims that learners at the concrete operational level begin to develop their mathematical thinking. They can remember images of physically absent objects but cannot deal with ideas. This mental ability grows and develops as they increase their awareness of the views of others. As they share their thinking with others, they are able to articulate their meanings and start to understand the meanings others put on the same situation. They negotiate meaning and share their expertise as they raise their thinking skills to higher levels.

At the formal operational stage, Piaget believes that the learners are able to deal with ideas and think beyond the concrete reality. These ideas are expressed in terms of statements and propositions, which they use in communicating their logical thought. They develop the ability to solve problems mentally through the use of representations and mental models. They are able to coordinate thought processes and deal with more logical operations aside from those operations at the concrete level. They are able to follow the scientific way of finding solutions to problems or derive conclusions from the given premises of an argument. They manifest higher order thinking skills by responding to inquiries logically and making effective decisions. Aside from the operations that they engage in at the concrete operational level, they are able to exhibit ratio and proportional thinking, probability and correlational thinking needed to perform well in mathematics.

Piaget did not indicate that there are transitional stages between the ones mentioned. The thinking processes described by Piaget, as concrete and formal operational are a major concern of

educators. It is worthwhile to investigate the thinking levels in terms of the performance of groups of students, to find out exactly what to expect from them. However, this paper adopts certain sublevels within the concrete and formal operational stages based on the answers that they give and scores that they got in the test of logical operations.

Piaget's theory suggests that college students should be capable of propositional thinking and should mature in their ability to engage in inquiry-based activities in science and mathematics. The thinking at this stage is the kind required for scientific reasoning. A high level of formal thought is basically hypothetico-deductive and is propositional, not empirical in nature.

Results of the studies by Karplus (1977) show that a large fraction of students use concrete reasoning patterns extensively. The realization that the typical class of college students consists of concrete thinkers and formal thinkers who employ different problem solving strategies should cause all college teachers to reevaluate what they are doing in the classroom.

Some educators believe that colleges and universities should adopt a system of tracking students in order to cater to the needs of different groups of students with different levels of thinking. Other educators believe that colleges and universities should make an institutional commitment to the development of formal reasoning, thereby requiring that all students graduating from college demonstrate a mastery of formal reasoning patterns. Whatever solutions we intend to adopt in the process, it is important that we devise ways of finding out the level of cognitive performance of our college students in mathematics. At the present time, we do not know whether teaching really can produce a formal thinker. However, the development of a successful technique for doing so poses an exciting challenge on mathematics teachers at all levels.

Having a large number of concrete-thinking college freshmen have implications for the development of science and mathematics courses in an institution. One relevant goal of teaching mathematics and the sciences is the development of citizens who are knowledgeable of the political implications of the practices of mathematicians and scientists and are well acquainted

with current national and international issues involved. Unfortunately, a concrete thinkers may not be equipped with the cognitive skills to make such discernment, nor pass sound judgments nor transfer their skills to real life situations. They may not be able to analyze errors, construct support and do objective investigation of a phenomenon. This situation has to be dealt with by the educators and curriculum makers.

Piaget's concept of logical thinking especially as presented in two of his books (Inhelder and Piaget, 1958 and 1964) has been extensively studied and popularly utilized for the purpose of science and mathematics teaching at all levels. Piaget emphasized the need to understand the concept of logical operations. He defined these operations in terms of the actions that can be carried out in thought as well as in actual execution. These operations are conserved, invariant, and reversible. He claimed that learners need to use these operations in order to get to the structure of knowledge and its transformation.

The logical operations namely: Classification, Seriation, Logical Multiplication, Compensation, Proportional Thinking or Ratio, Probability and Correlational Thinking can be used as cognitive tools in mathematical problem solving. According to Piaget's levels of intellectual development, these operations are developed at varying degrees when the learners reach the concrete operational and formal operational levels. A learner at the concrete level can engage in tasks involving classification, seriation, logical multiplication and compensation. A learner at the formal level can engage in tasks that require the above mentioned logical operations including proportional thinking or ratio, and probability and correlational thinking.

Karplus (1977), describes classification as the systematic arrangement in groups or categories according to established criteria. It is one of the first logical operations that an individual is expected to develop. Seriation is the arrangement in a series or succession. Logical multiplication refers to operations of multiplication relating to, involving, or being in accordance with logic. Compensation is about counter balancing, making appropriate or supplying equivalence. This may refer to the additive compensation or the compensating effects of

variables that describe a physical system like the balance beam. Students in the late concrete stage are expected to have these process skills aside from classification and seriation. Ratio or Proportional Thinking is the establishment of relations of one part to another or of a whole with respect to magnitude, quantity or degree. This may refer to the understanding of such numerical relationships as 5:6 or of algebraic relationships of two variables such as $y = 2x$. Probability Thinking is the establishment of a logical relation statement such that evidence conforming to one conforms to the other to some degree. Students at the early formal stage are expected to have acquired all six process skills that have been mentioned. Correlational Thinking is the establishment of correlation or causal relationship. It may also refer to the presentation or setting forth so as to show relationships. Students at the late formal stage should have acquired this process skill together with the other six logical cognitive skills.

REASONING PATTERNS

To describe the level and progress of reasoning of learners, the concept of reasoning pattern can be used. Instead of dealing with the cognitive levels or logical operations of intellectual development, identifiable and reproducible thought processes directed at a kind of task can be observed. Thought processes called reasoning patterns include serial ordering or seriation, and correlation. Thus, serial reasoning is used in ordering a set of numbers, ordering minerals according to hardness, as well as ordering solid materials according to their density. Correlation is used to identify relationships of variables as either direct or positive, inverse or negative or no correlation.

Reasoning patterns may be used to describe the seven logical mathematical operations by Jean Piaget and to interpret the thought processes of the students. Some of these operations can be easily identified in a learner's words and actions, while the others require detailed analysis of extensive tasks. These reasoning patterns are intended to allow teachers to classify their

students' thought processes on the basis of classroom conversation, observations, written work or interview.

Thus the reasoning patterns analyzed in this study manifest the seven logical operations that Piaget used to describe the intellectual development of the learners. There are other reasoning patterns but this study dwells only with those patterns as they are used in mathematics, given in Table 1.

TABLE 1. Reasoning Patterns in Mathematics

Reasoning Pattern	Description
1. Classification	Makes reference to variables or numerical properties involved in groupings
2. Seriation	Reasons relative to a set of attributes or numerical properties for ordering
3. Logical Multiplication	Makes reference to categories, relations or functions as they are applied to multiplication.
4. Compensation	Refers to the reason property used for balancing.
5. Proportionality	Refers to relative magnitude of the increase and decrease of ratios.
6. Probability	Reasons in time of the likelihood of possible outcomes.
7. Correlation	Can reason with relationships of variables or symbols.

An important characteristic of all reasoning patterns is that they do not refer to specific facts, experiences, or relationship, but that they are concerned with certain recurring relationships. The reasoning patterns listed above are not completely independent. Thus, in certain tasks, the ability to classify is necessary to be able to order objects or mathematical ideas.

Educators believe that teachers' awareness of the reasoning patterns of their students will lead to effective teaching and active learning. They insisted that curriculum content and activities should match the students' reasoning abilities. With the proper learning experience, instruction can develop the higher order thinking skills of students and help lift the intellectual development of students to higher levels.

The Test of Logical Operations (TLO) in Mathematics, which is the main instrument of this study was designed and constructed on the basis of Piaget's seven logical operations. It is believed that an individual student has to utilize these operations while learning concepts in mathematics, and the sciences.

The reasoning patterns are manifestations of the thought processes using the logical operations and the higher order thinking skills used in problem solving and in hypothetico-deductive reasoning. Students exhibited their reasoning patterns as they answered the TLO and during the follow up interview of their answers on the TLO.

With these descriptions of the reasoning patterns as a guide, the study categorized the adequacy of the reasoning of the subjects using the rubric.

METHOD

This study investigated the cognitive skill achievement in mathematics of fifty-nine (59) college freshmen of Bataan Polytechnic State College, Balanga Campus in Balanga City, Philippines. They were in the Bachelor of Science in Education and associate course in Computer Technology programs. They were taking up a one semester course on College Algebra. A teacher made instrument, called the Test of Logical Operations in Mathematics (TLO), was designed and constructed primarily with the purpose of identifying the students' level of cognitive skills achievement in mathematics using Piaget's seven logical operations.

This study made use of a descriptive empirical research design. Data gathered were analyzed qualitatively and quantitatively. Scores in the TLO were used to describe the students' level of cognitive achievement in mathematics based on the reasoning patterns that they exhibit. The rubric was used to determine the adequacy of their reasoning patterns.

Students used in this study were on the average age of 17 years old, who according to Piaget's theory on cognitive development should be on their formal operational stage.

INSTRUMENT

The Test of Logical Operations in Mathematics was designed, constructed and developed on the basis of seven of Piaget's Logical Operations. This test was validated and its reliability was established before it was administered to the subjects at the end of the semester. The test required respondents to solve single-step problems in mathematics for one and a half- hour. There were 5 items on each logical operation, giving the test a total of 35 items.

The items are limited to topics covered in most high school mathematics courses. This includes selected topics in geometry, arithmetic, statistics and algebra. Items in geometry consisted of problems on angle measures, angle relation, and area. The problem- solving test in algebra consisted of different types of routine problems using linear equations, and progressions. In arithmetic, topics include percentage, ratio and proportion, fraction, decimals and conversion techniques. Statistics problems using probability, and correlation are included.

The items in the instrument show that the reasoning patterns are not completely independent. As an example, while the following items on classification assess the learner's ability to group variables according to certain numerical properties, these items may deal with basic concept of ordering found in the following:

Classification item no. 5. *Arrange the following numbers from highest to lowest after rounding off to the nearest tens: 411.7; 495; 396.75; 390.8; 464.75.*

Similarly, the following difficult item on classification also requires the use of ratio and proportionality to identify equivalent fractions, and ordering:

Classification item no. 1. *Arrange the following fractions from biggest to smallest:*

$$\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{2}{3}, \frac{1}{2}.$$

On the other hand, while the students are required to use seriation in the following item, they also need to classify the numbers according to the property of exponentiation.

Seriation item no. 2. *Determine the next number in the series.1, -2, 4, -8, __.*

Another item on seriation which needs the ability to classify the increasing increment into odd numbers to be able to identify the series is the following:

Seriation item no. *What is the next number in the sequence: 1, 2, 5, 10, ___?*

In these items, the dominant reasoning patterns in the problem solving schema are classification for the first two examples, and seriation for the last two examples.

Logical multiplication items deal with problems involving parts that are factors of a whole. The problem may ask for the whole amount, or the amount of the unknown part. The following problems ask for the parts which are factors of a whole:

Logical multiplication item no.3. *Peggy earns 45 pesos per hour. How many hours will she need to work to earn 945 pesos?*

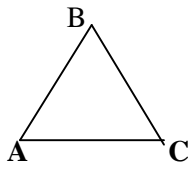
Logical multiplication item no. 4. *Mr. Regalado subdivided his 46,000 sq.m. land into lots, each of which has an area of 250 sq. m. How many lots were produced?*

The items on compensation deal with the ability to determine equivalent numerical values. As such, the thought processes on proportionality also come in the following items:

Compensation item no. 1. *Give the reduced equivalent of $\frac{25}{125}$.*

The item may also deal with a simple act of balancing values such as the following:

Compensation item no. 4.



Triangle ABC is an isosceles triangle whose base angles A and C measure 60° , determine the degree measure of angle B.

However, all the items on Ratio or Proportional Thinking involve thought processes on compensation. The student has to work within the schema of proportional reasoning in the following item:

Proportional Thinking item no. 2. *A committee has 15 members. The ratio of women to men in the committee is 2:1. How many men and how many women are on the committee?*

There are items that make no direct mention of the concept of ratio and proportion but require its use as in the following:

Proportional Thinking item no. 5. *If an automobile consumes 5 liters of gasoline for 14 kms., how far can it go on 20 liters?*

There are items that require the recognition of all possible outcomes using thought processes such as logical multiplication and classification, aside from probability thinking. These are as follows:

Probability Thinking item no. 2. *A card is chosen at random from a well-shuffled deck of 52 cards. Find the probability of choosing a king of hearts?*

Probability Thinking item no. 3. *A bag contains 2 red, 1 green and 3 blue marbles. If you choose a marble at random, find the probability of choosing a green marble.*

The test items on correlation require students to measure the degree of association or strength of the relation between two given variables. This involves classification of the relationship into positive or direct correlation, negative or inverse correlation and zero correlation. The following instruction tells the students to compare the given variables:

Tell the kind of correlation you would expect to exist between the following pairs of variables:

Correlational Thinking item no. 3. *The trade-in value of a car and the age of a car.*

Correlational Thinking item no. 4. *Height and age.*

The students were asked to justify their answers. Criteria were then set to determine the score that the student gets for each item.

DATA ANALYSIS

The problem solving test items were credited on the merit of how the students arrived at the final answer and not on the final answer alone. The multiple count-scoring scheme proposed and used by Schoenfeld (1982) to measure problem solving performance was adapted with some modification in Table 2.

TABLE 2. Scoring Continuum for Test of Logical Operations in Mathematics

0	1	2	3	4
Made no attempt to solve the problem.	Made little attempt in the form of sketches; jotting down needed relationships, jotting down needed data; or overtly explaining how to solve the problem.	Showed understanding of the problem by the representations made and early attempts to solve the problem; problem solved about halfway.	Made a great progress in the solution. Problem is nearly solved, solution is correct but minor errors are committed.	Problem is fully and correctly solved.

The students under this study were categorized based on the results of the TLO test in mathematics. Using the scores as basis and looking at the level of understanding that they demonstrated in answering the test, they were classified as follows:

1. Concrete 2A or Early Concrete: (0-35) points

These are students who make no attempt to solve the problem or make attempt in the form of sketches; jotting down needed relationships; jotting down needed data; or overtly explaining how to solve all the problems of the test. They perform initial actions on the problem but are not able to progress to a higher level of performance. They may be able to solve some problems, but these are those that require the use of logical operations on very familiar mathematical concepts only. There are many inconsistencies in the solutions and thus the scores are kept at the minimum. There is very little evidence of attainment of the reasoning patterns in mathematics at the formal level.

Students under this category show poor comprehension of the logical operational problems by not being able to determine the problem goal correctly. More so, they are unable to see the correct relationships among the pieces of information in the problem as manifested by their incorrect representation of the problems.

2. Concrete 2B or Late Concrete: (36-70) points

These are students who show understanding of the problems given on each logical operation by the representations and early attempts that they make to solve all the problems.

They act on the problem and may be able to exhibit reasoning patterns successfully on a few items. They may be able to solve more problems that require the use of logical operations on the basic mathematical concepts, thus the scores gained are better. There are major inconsistencies though in the solutions, just like those who belong to the preceding category. There is no strong evidence of attainment of the reasoning patterns in mathematics at the formal level.

Other reason that may lead to the low score is the incomplete solution given by the students. While the solution may be logical, essential errors are committed. Thus, they are able to pursue the subgoal but fail to arrive at the main goal of the problem.

3. Formal 3A or Early Formal: (71-105) points

These are students who make a great progress in the solutions, where problems on all logical operations are nearly solved, and whose solutions are correct but minor errors are committed on all of the items of the test. There are some evidences of the attainment of reasoning patterns in mathematics at the formal level. Their thinking processes are clearly manifested in the problem solution that they gave.

Students under this category are able to show a connection between steps or ideas that show how the problem is solved but minor errors in representations, strategies and relationships are committed. There is success in solving problems in almost all logical operations.

4. Formal 3B or Late Formal : (105-140) points

These are students who fully and correctly solve almost all the logical operational problems. There is strong evidence of the attainment of reasoning patterns in mathematics at the formal level. Students clearly articulated their thought processes by giving correct solutions most of the time.

Students under this category are able to demonstrate a global understanding of the problem, employ a cognitive schema which facilitates the understanding of problem structure, choose and utilize the appropriate solution strategies, and successfully attain the correct answer.

To analyze the performance of the students per problem type, the scores were interpreted qualitatively using Schoenfeld's Scoring Continuum as guide.

TABLE 3. Categories of Student Performance in One Problem Type

Mean Score	Qualitative Description
0 - 5.4	Low Understanding
5.5 - 10.4	Insufficient Understanding
10.5 - 15.4	Sufficient Understanding
15.5 - 20	Complete Understanding

Students exhibit low understanding of the problems in a logical operation if they failed to grasp the problems and made little attempt to solve them.

They exhibit insufficient understanding of the problems if the most they can do is interpret the problem correctly and make an overt explanation of the solution of the problem.

They show sufficient understanding of the problems if they are able to demonstrate or perform the usual standard logical operations involved. They are able to progress in the solution and errors stem from lapses in arithmetic or algebraic computations.

Students show complete understanding in logical operation if they manifest comprehension of the essence and structural relations of data in the problem. Students exhibit this characteristic if they have the ability to understand and grasp fully the mathematical task presented to them and solve with accuracy and deep insights. More so, students in this category can deal with more difficult problems. Mistakes committed are minor.

The reliability and validity of the instrument was established using inter-rater reliability quotient and face validity and the Product Moment Correlation Coefficient. The instrument showed a high degree of consistency, index of discrimination, face and concurrent validity.

To supplement the evidences of cognitive achievement of the students, an interview was conducted at the end of the semester to look deeper into the reasoning patterns of students who manifest different levels of cognitive achievement.

Students were interviewed individually at an average of thirteen minutes each. They were all advised to leave the interview room immediately and refrain from talking to the next interviewees. This was done to avoid interview interaction effect. The following set of categories adapted from Raven (1973) was used to interpret the result of the interview.

TABLE 4. Raven' s Reasoning Categories

Logical Operation	Adequate	Inadequate	Ambiguous
Classification	Exhibits the reasoning pattern as described in Table 1.	Refers to only one of the variables involved. Incomplete inference	Expresses the reason ambiguously. Does not refer to the variables involved in the operations.
Seriation			
Logical			
Multiplication			
Compensation			
Proportionality			
Probability			
Correlation			

Evidences of the adequacy of the reasoning patterns were sought during the interview. The protocol was based on the answers that the students gave in the Test of Logical Operations.

RESULTS

An analysis of the data reveals that out of 59 student respondents, 6 or 10.17% of the college freshmen are at the Concrete 2A level, 30 or 50.85% performed at the Concrete 2B level. 17 or 28.81% of the students performed at the Formal 3A level while 6 or 10.17 % performed at the Formal 3B level.

It is alarming to find out that 36 students out of 59 or 61.02% college freshmen in the study, mostly at the age of 17 who are expected to perform at the formal operational level were barely at the concrete operational level. At the age of 17, they are expected to be capable of propositional thinking and should have developed their ability to solve problems using the identified reasoning patterns as their tool. They should have acquired the problem solving schema using the thought processes involved in Piaget' s logical operations. This study shows otherwise. Table 5 summarizes this result as follows.

TABLE 5. Cognitive Skill Achievement of Students

Sublevel	Frequency (f)	Percentage(%)
1. Concrete Operational		
1.1 Early Concrete (Concrete 2A)	6	10.17%
1.2 Late Concrete (Concrete 2B)	30	50.85%
2. Formal Operational		
2.1 Early Formal (Formal 3A)	17	28.81%
2.2 Late Formal (Formal 3B)	6	10.17%
TOTAL	N=59	100%

Mean scores on each of the seven logical operations were determined to facilitate the performance evaluation. Performances of the students were ranked accordingly from the highest mean score to the lowest mean score. This shows us the details about the performance of students on particular logical operations. They are ranked respectively as follows: Logical Multiplication, Classification, Compensation, Seriation, Probability Thinking, Correlational Thinking and Ratio or Proportional Thinking. Table 6 presents the mean scores of the students in each of the seven logical operations and the qualitative descriptions. The ranking of the logical operations were also noted in the table.

Table 6 further shows that students performed with sufficient understanding on problems in Logical Multiplication but not on problems requiring the other six logical operations.

TABLE 6. Mean Scores of Students on each of Piaget’s Seven Logical Operations

Logical Operations	Mean Score	Rank	Level of Understanding
1. Classification (CL)	10.24	2	Insufficient
2. Seriation (S)	9.25	4	Insufficient
3. Logical Multiplication (LM)	13.22	1	Sufficient
4. Compensation (CM)	10.19	3	Insufficient
5. Ratio or Proportional Thinking (R)	7.56	7	Insufficient
6. Probability Thinking (P)	9.76	5	Insufficient
7. Correlational Thinking	8.76	6	Insufficient
GLOBAL MEAN	9.85		Insufficient

The highest score in each logical operation is 20

This supports the result in Table 5 showing that 61% of the students are at the Concrete Level of performance.

The table showing the insufficient understanding on Ratio or Proportional Thinking, Probability Thinking and Correlational Thinking supports Table 5 showing that only 10% of the students are at the late formal level.

From the foregoing discussion, it only suggests that formal thought, which is required for success in the mathematics lessons beyond basic arithmetic is not manifested by majority of the college students. This gives considerable importance to the question of what effect formal thought has on learning various topics in Mathematics.

It is quite alarming to note that students had sufficient understanding of problems on Logical Multiplication only. They have insufficient understanding of the problems using the other six logical operations.

To describe the performance of the subjects further, Table 7 was used. In this table, only the logical operations expected to be fully developed as described by Karplus et al (1983) at the 4 sublevels were investigated.

TABLE 7. Mean Scores on Logical Operations

Logical Operations	Sublevels Of Cognitive Skills Achievement and Their Mean Scores			
	Concrete 2A	Concrete 2B	Formal 3A	Formal 3B
1. Classification	7.17	9.33	10.82	14.67
2. Seriation	4.97	8.07	10.88	14.0
3. Logical Mult	*	12.23	15.17	19.0
4. Compensation	*	7.63	14.06	16.33
5. Proportional Thinking	*	*	10.0	16.17
6. Probability Thinking	*	*	14.29	19.5
7. Correlational Thinking	*	*	*	18.5
GLOBAL MEAN	6.07	9.32	12.54	16.88

* logical operations not fully developed among subjects in the respective sublevel

Data from this table reveals that Concrete 2A students answered the problems on classification and seriation with insufficient understanding as revealed by the mean of 6.07. They answered the problems on seriation with low understanding.

Concrete 2B students answered problems on Logical Multiplication with sufficient understanding. They answered the problems on Classification, Seriation and Compensation with insufficient understanding. A global mean of 9.32 suggests that students of this category performed all the logical operations expected of them with insufficient understanding.

Formal 3A students answered the problems on Classification, Seriation, Logical Multiplication, Compensation, and Probability Thinking with sufficient understanding. They had insufficient understanding of the problems on Ratio or Proportional Thinking. The global mean of 12.54 suggests that this group of students were able to answer the problems on all the logical operations expected of them with sufficient understanding.

An exemplary performance of Formal 3B students is revealed by their global mean score of 16.88. Students in this group answered the problems on Logical Multiplication, Compensation, Ratio or Proportional Thinking, Probability Thinking, and Correlational Thinking with complete understanding. They answered the problems on Classification and Seriation, with sufficient understanding.

It can also be noted from the table that the mean score value for each logical operation increases as the cognitive level moves higher. As an example, mean scores on logical operation Classification under the four sublevels respectively are: 7.17, 9.33, 10.82, and 14.67.

The foregoing discussion only explains that as an individual goes through the four successive stages of cognitive development, logical operations also develop progressively. This also implies that individuals with higher level of cognitive performance exhibit expertise in solving problems that require varied logical process skills, while those with lower level of cognitive performance can solve limited types of problems only.

The results further imply that expertise on each logical operation develops progressively as the students are able to raise their cognitive skill achievement at a higher level.

FOLLOW UP INTERVIEW

Students were randomly selected for a follow up interview to gather more evidence of their reasoning patterns as they answer the TLO. The in-depth interview reveals the students' rationale for their answers. Out of 16 students who were selected, 1 belongs to the Late Formal Operational Group, 7 are in Early Formal Operational Group, 5 are in Late Concrete Operational Group and 3 are in the Early Concrete Operational Group. These students have a mean score of 70 points in the Test of Logical Operations in Mathematics and have a mean age of 17 years old.

To facilitate the qualitative profiling of data, this study used Raven's criteria for adequacy of the student's reasoning and gave corresponding numerical rates of 1 for adequate reasoning, 2 for inadequate reasoning, and 3 for ambiguous reasoning. Since all answers to the 35 items were verified, the range of values of the mean scores for each category had to be described in the table that follows.

With the use of the categories in Table 8, this study determined the adequacy of the reasoning patterns of students from different cognitive levels of performance.

TABLE 8. Categories for the Adequacy of Reasoning

Range of Values	Reasoning Pattern
1.0 – 1.5	Adequate
1.6 – 2.5	Inadequate
2.6 – 3.0	Ambiguous

The results of the interview given in Table 9 show that the reasoning patterns of Concrete 2A and Concrete 2B groups on the logical operations expected of them are inadequate. On the other hand, the reasoning patterns of the Formal 3A and Formal 3B groups of students on the logical operations expected of them are adequate.

TABLE 9. Mean Score on Reasoning Patterns of Students

Sublevel	Mean Score	Reasoning Pattern
Early Concrete (Concrete 2A)	2.33	Inadequate
Late Concrete (Concrete 2B)	1.71	Inadequate
Early Formal (Formal 3A)	1.29	Adequate
Late Formal (Formal 3B)	1.14	Adequate

Table 9 also reveals the relation of the cognitive achievement of the student with their reasoning pattern in mathematics. This result shows the deficiency in reasoning ability of students who perform at the concrete operational level. Science educators believe that the hypothetico deductive reasoning ability of the student is an excellent predictor of achievement, if that achievement is based on the students' reasoning patterns like the ability to generate and test hypothesis while investigating natural phenomena (Lawson, McElrath, Burton, James, Doyle, Woodward, Kellerman, Snyder, 1991, p.969). While this study did not look into the academic achievement of students in College Algebra, their cognitive performance in mathematics is reflected in the TLO, which they took right after they finished the course. In that light, this study supports the claims of Lawson et al (1991) regarding the promise of the reasoning pattern abilities to predict the capability of students to acquire theoretical concepts in mathematics and the sciences.

A closer look at the results of the interview reveal the following:

TABLE 10. Student' s Reasoning Patterns Manifested in the Interview

Logical Operations	Adequate	Inadequate	Ambiguous
1. Classification	50%	50%	0%
2. Seriation	*56.25%	12.5%	31.25%
3. Logical Multiplication	*93.75%	0%	6.25%
4. Compensation	*73.33%	13.33%	13.33%
5. Ratio or Proportional Thinking	41.67%	7.14%	*51.19%
6. Probability Thinking	*54.54%	27.27%	18.18%
7. Correlational Thinking	*60%	6.67%	33.33%

* Highest value at the reasoning category

This table shows that 93.7% of the student respondents are able to exhibit adequate reasoning patterns on problems requiring Logical Multiplication. But only 41.67% of these students are able to reason out adequately on problems requiring Ratio and Proportional Thinking. The results are similar to the studies cited by Karplus et al (1983). This suggests the complexity of the nature of proportional thinking and should therefore be investigated further.

CONCLUSION

The learners' ability to use Piaget's logical operations in mathematics is manifested in their ability to solve word problems involving those logical operations. Evidence of attainment of thought processes at Piaget's levels of intellectual development can be gathered through investigation of their problem solving skills. An indicator of the acquisition of the problem solving skill is the ability to articulate one's problem solving solutions and reason out adequately. This study investigated the levels of cognitive achievement of college freshmen using the Test of Logical Operations. Interviews were conducted to investigate the adequacy of their reasoning patterns.

This study showed the relation between the levels of cognitive skills achievement of college freshmen and their formal reasoning patterns. Significant relationship was drawn between reasoning abilities and cognitive skill achievement in mathematics. This study showed that 61% of the college freshmen were at the concrete level. This study also reveals that as an individual goes through the four successive cognitive levels of performance, expertise on reasoning develops progressively.

This study further provided evidence that there are certain logical operations that are not fully developed even at the college level. One example is the Ratio or Proportional Thinking. A deeper investigation in this study showed that more than 50% of the college students have inadequate understanding of the concept of ratio and proportion as they exhibited ambiguous reasoning patterns during the interview.

PEDAGOGICAL IMPLICATIONS

Mathematics educators should take these findings very seriously and conduct further studies on learner's conceptual understanding of ratio and proportion which may extend to the concepts of rational numbers and fractions. The college mathematics teacher should probe into the thinking skills of college students and find out the adequacy of the reasoning patterns that their students employ in their problem solving activities.

Ideally, all or at least a great majority of college students should be at the formal level of performance in the test requiring Piaget's Logical Operations. This study shows that it is not necessarily the case. Educational planners should therefore recognize this reality when making curricular and other pedagogical decisions. They should look closely into the needs of the students and design their curriculum with those needs in mind. The goal of mathematics education at the college level should be the development of higher order thinking skills. But if the students are at the concrete level, it is impossible to achieve this goal without the proper remediation, interventions and scaffolding. This shows that even at the college level, teachers should employ learner centered teaching strategies that focus on how the students think.

Teachers should be encouraged to probe into the thinking skills of their students using alternative assessments aside from the pencil and paper test. Students can be made to articulate their ideas through interview and journal writings. Content and teaching strategies can be made to match the cognitive level of the students. Proper assessment tools should be used to diagnose learning. While responsibility in learning at the college level rests on the students, it is still the duty of the teacher to empower both the concrete and formal thinkers among these students so that they can experience progress in learning. It becomes more liberating for students if the teacher is able to raise the cognitive achievement of the students in Mathematics to the formal level with appropriate teaching materials and strategies.

One way of helping students gain expertise in problem solving is to be aware of the various logical operation processes that should be developed among the students through

mathematics instruction. Mathematics teachers should be able to create learning environments that develop these logical mathematical processes and the formal reasoning skills. They should be able to align the cognitive demands of their instruction with the cognitive levels of their students. This starts with the proper use for instruction of assessment instruments and interview protocols such as the Test of Logical Operations developed in this study.

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