

# What is good college mathematics teaching?

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**Abstract:** This article attempts to answer the question “What is good college mathematics teaching?” by examining three sources of information: research, student course evaluations, and responses on the website RateMyProfessors.com. By intersecting the sources, the answer is given that good college mathematics teachers are fair, spend time with students outside of class, and explain every little detail of problem solutions. Also interesting is what does not end up in the intersection of these three sources. Examples include subject matter knowledge, enthusiasm, referring to recent applications, and making the subject matter interesting.

**Keywords:** good mathematics teaching, RateMyProfessors.com, course evaluations, students, college mathematics

The set of all college mathematics professors who are good at teaching is ill defined. Yet, it seems that there is often agreement on who is a good teacher and who is not. Some researchers even suggest that although it is difficult to define good mathematics teaching, one knows it when one experiences it (Cohen & Seaman, 1997). Other researchers have such general definitions that their usefulness is questionable. Cashin (1989) defines effective teaching as “all of those instructor behaviors which help students learn” (p. 4).

This paper attempts to give a more definitive answer through three sources of information about good mathematics teaching. The first source is how researchers have defined a good teacher. The second source is items on student course evaluations. Finally, an analysis of responses on the web site RateMyProfessors.com gives a partial answer.

## A Summary of the Research

Recent research suggests that all college teachers think they are good teachers (Price, 2006). Chances are good, however, that not all students think every college teacher is a good teacher. Attempting to define good teaching has gone on for years. Polya (1962)

gave ten commandments for good college mathematics teaching.

1. Be interested in your subject.
2. Know your subject.
3. Know about the ways of learning: The best way to learn anything is to discover it by yourself.
4. Try to read the faces of your students, try to see their expectations and difficulties, put yourself in their place.
5. Give them not only information, but “know-how” attitudes of mind, the habit of methodical work
6. Let them learn guessing.
7. Let them learn proving.
8. Look out for such features of the problem at hand as may be useful in solving problems to come—try to disclose the general pattern that lies behind the present concrete situation.
9. Do not give away your whole secret at once—let the students guess before you tell it – let them find out by themselves as much as feasible.
10. Suggest it; do not force it down their throats.

Rosenshine and Furst (1971) concluded that effective teaching involved such variables as clarity, variability, enthusiasm, task-oriented behavior, opportunity to learn, and involvement of students. Brophy (1982) states that effective teachers are smarter than non-effective teachers and extensively plan out their classroom tasks.

McKinney (1986) compared expert teachers to novice teachers and concluded that expert teachers were more organized, had more content knowledge, were more clear in their explanations, and “were more adept at explaining why, how, and when mathematical concepts are used” (Wilson, Cooney, & Stinson, 2005).

Chickering and Gamson (1987) compiled education research to form the seven principles for good practice.

1. Encourage student-faculty contact
2. Encourage cooperation among students
3. Encourage active learning
4. Give prompt feedback
5. Emphasize time on task
6. Communicate high expectations
7. Respect diverse talents and ways of learning.

Whitman and Lai (1990) conducted a study in which the beliefs about effective teaching of mathematics held by teachers from Japan and Hawaii were compared. A summary offered by the researchers follows:

There are teaching techniques considered effective by teachers in both socio-cultural settings. They included making encouraging remarks to students as they work, getting materials, equipment, and space ready before class, and reviewing tests with students shortly after the tests have been graded. Although there are similarities in beliefs of what constitutes effective teaching, the differences in beliefs are more notable. Six techniques were considered effective by both groups as compared to nineteen techniques that were viewed significantly differently as to their effectiveness. (p. 75)

The conclusion is that effective teaching may depend on the culture.

The Third International Mathematics and Science Study (TIMSS) defines effective teaching as a “complex endeavor requiring knowledge about the subject matter of mathematics, the ways students learn, and effective pedagogy in mathematics” (Beaton et al., 1996, p. 131). Interestingly, TIMSS data does not reveal a best methodology, as “teachers can adopt a variety of organizational and interactive approaches in mathematics class” (Beaton et al., 1996, p. 151) and many of them can be very efficient.

Cohen and Seaman (1997) examined good teachers (these were teachers who faculty, administration, students, and parents agreed were good teachers) for common

characteristics. They concluded that good teachers have the following traits:

1. Confidence in knowledge of subject matter
2. High-quality explanations
3. Attention to individual differences
4. Sense of humor
5. Management through high awareness
6. Students engaged in active learning

Some researchers argue that good teaching is a matter of engaging students and thus, while content knowledge is important, pedagogical content knowledge is crucial (Mapolelo, 1998) and good teachers reflect on both their own understandings and students' understandings (Schifter, 1998). Ma (1999) finds that effective teachers have profound understanding of fundamental mathematics. That is, they understand the concepts behind the mathematical procedures.

Reynolds and Muijs (1999) summarize United States research on effective mathematics teaching by saying it contains the following elements.

1. Students have many opportunities to learn; that is, the amount of time children are actively engaged is large.
2. Teachers are academically orientated (versus socially orientated or other orientations).
3. Teachers manage the classroom well.
4. Teachers have high expectations.
5. Students do not spend much time on their own (without the teacher leading).
6. The teaching is heavily interactive.

Wilson, Cooney, and Stinson (2005) interviewed nine teachers to gain their views of good mathematics teaching. The teachers thought that a good teacher had knowledge of the subject, is able to engage and motivate students, has effective management skills, and emphasizes understanding over rote procedures.

Finally, the National Council of Mathematics (NCTM) has defined good mathematics teaching in numerous documents (1989, 1991, 1995, 2000). In the latest document (2000), they have a Teaching Principle, which states: "Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well" (p. 16).

#### Student Evaluations of College Mathematics Teaching

Nearly every college has some type of student evaluation form for teaching and such forms are the most common method for evaluating effective teaching (Seldin, 1993). Most do not have separate ones for mathematics teaching. Regardless, if we assume that the evaluation forms give a measure of good teaching, then common items on these forms should reveal a definition of good teaching.

It is beyond the scope of this paper to discuss in any detail the validity of this previous assumption. In the year 1988, Cashin reported that over 1300 articles and books had been published on research about student evaluations. Some researchers believe that the items on course evaluations are simply "what someone decides to ask the students at a given time" (Achtemeier, Morris, & Finnegan, 2003, p. 10). Others argue that student evaluations are both reliable and a valid measure of good teaching (Cashin, 1990; Davis, 1993). Some research concludes that students' ratings correlate positively with how much they learned in the course (Abrami, d'Apollonia, & Rosenfield, 1996).

Pallet (2006) reports on seven abuses and misuses of student evaluations, citing 1) over-reliance on student ratings in the evaluation of teaching; 2) making too much of too little; 3) not enough information to make an accurate judgment; 4) questionable administrative procedures; 5) using the instrument inappropriately; 6) insufficient attention to selecting/developing an instrument; and 7) failure to conduct research to support the validity and reliability of student ratings tool. Only the last two are of concern to the current study. This study is not concerned over how students actually fill out student evaluations, but only the items that are found on student evaluations. Thus, it is a concern if insufficient

attention has been given to developing the forms.

In terms of the content validity of the items on student evaluations, there is considerable disagreement among scholars. Cashin (1989) suggests that college teaching involves seven different areas: subject matter mastery, curriculum development, course design, delivery of instruction, assessment, availability to students, and administrative requirements. Further, Cashin suggests that student evaluations only contain items on delivery of instruction, assessment, availability to students, and administrative requirements. Although some student evaluations do have items assessing the subject matter mastery of the instructors, some researchers argue that students are not capable of making such a judgment (Pallet, 2006).

The decision was made to gather as many student course evaluations as could be gathered off the web, by entering key words such as "student course evaluations" and downloading the forms. Universities were restricted to post-secondary institutions in the United States. Obviously, this constitutes a convenience sample, and no claim is made that we succeeded in gathering a random or representative sample. It is possible that these evaluations differ from others. Perhaps they are even better than others (since they are so readily available). Regardless, our goal was not to evaluate evaluations, but only to use the items found on them to contribute to a definition of good teaching. Thirty evaluations were found this way.

Many of the forms contained items that do not contribute to a definition of good teaching. For example, how would you rate the instructor's overall teaching ability? This study is attempting to define good teaching and not give an accurate analysis of student evaluation forms. For these same reasons, any open-ended items (such as "what are the instructor's best and worst qualities") were ignored.

On 76% of the evaluations gathered, there was an item regarding how available the instructor was outside of class. On 73% of the evaluations was an item concerning whether the instructor treated the students with respect and/or "appropriate concern." Seventy-two

percent of the evaluations contained an item regarding the fairness of grading policies. Also at 72% was an item regarding how helpful and/or timely was the feedback received from the instructor.

A bit lower, at 66% was an item about how well the instructor explained material. Again lower was an item regarding how organized and prepared the instructor was (62%). Also at 62% was an item about the quality of assignments that were given; that is, were they helpful for the student's learning of the material. At 55% was an item about the instructor making expectations clear. Finally, at 52% was an item about the instructor handling questions appropriately.

A few other items were also fairly common. "The instructor made the subject matter interesting" was on 48% of the evaluations. "The instructor communicated enthusiasm for the subject area" was on 41% of the evaluations. Items about encouraging student engagement, the pace of the course and knowing the subject matter were on nearly a third of the examined evaluations. Three additional items were each on approximately 24% of the examined evaluations. They were "the instructor motivated the students", "the instructor referred to real-life applications", and "the instructor spoke clearly and audibly".

There were several other items that only showed up on one or two evaluations. On one of the examined evaluation there was an item that asked if the instructor used a variety of teaching techniques. A couple of evaluations had an item about the quality of the syllabus.

#### RateMyProfessors.Com

RateMyProfessors.com is a web site where students at post-secondary institutions can "rate" their professors. A scale of 1 (lowest rating) to 5 (highest rating) is given to professors on three scales: clarity, helpfulness, and easiness. In addition, students can mark the professor "hot" or not (this is apparently a comment on their level of sex appeal) and also give written comments. Students' posts are anonymous. The site opened in 1999 and its one-millionth post occurred on August 26, 2003 (Felton, Mitchell, & Stinson, 2004).

These ratings come from students who are motivated to leave a rating. There is nothing mandatory about leaving a rating at RateMyProfssors.com. Thus, one might assume that students with strong positive or negative opinions are more likely to enter ratings and comments on the site. A study done by Felton et al. (2003) analyzed postings for over 3190 professors from over 65,000 students. They conclude that students give higher ratings to professors that they view as easy and/or sexy.

In this study, we are not considering the numeric ratings of the professors, nor are we even interested in the reliability or validity of such numeric ratings. We are only interested in the student comments. Although certainly these comments might suffer from bias based on easiness or sexiness or other factors, the comments are still the comments of students. These students are not necessarily typical (since they were motivated to comment on RateMyProfessors.com), but they are still students with opinions. Thus, we take students' comments from the site and ascertain what these students think good mathematics teaching is.

To select the sample, a random number generator was used to select 10 states. The ten selected states were Arkansas, Illinois, Minnesota, Missouri, Nevada, Oklahoma, Pennsylvania, South Dakota, Utah, and West Virginia. For each of the 10 states, a random number generator was used to select 10 schools. For each school, the first response was recorded for each mathematics professor, as long as the response was given later than December 31, 2005. In addition, from each school the first response was recorded for an equal number of non-mathematics professors who were randomly selected. This resulted in nearly 2000 entries; half of mathematics professors and half of non-mathematics.

Many of the responses are phrases that do not contribute toward a definition of good teaching. For example, "great teacher" and "really can't teach" are common comments, but not informative of what it takes to be a good mathematics teacher. It is also very common to comment on how "nice" the professor is. However, even a professor who the student feels is a poor teacher is often described as a "nice guy" or a "really nice person." For

example, one student wrote “[Professor’s name] is a nice lady, but she is possibly the worst teacher I have ever had.” Students also complain about mathematics itself (it is difficult, it is boring), but this does not seem to affect the other comments that they write about the professor. Here is a sample comment: “The subject is a bit dull (hello, it’s MATH), but [professor’s name] was a great teacher!”

The most common reasons given for good mathematics teaching are the professor is fair, the professor is willing to help outside of class, and the professor is good at explaining every little detail. These three reasons are also the most common reasons given for good non-mathematics teaching, as well. Fairness and helping outside of class are indistinguishable between mathematics and non-mathematics teaching. In non-mathematics teaching, students tend not to use the word “step” as often, but they still describe that the professor explains the details.

Fairness is mostly in grading, how the tests are written, and general policies (such as make-up exams). Sample responses follow.

- She will help anyone who actually tries.
- [Professor’s name here] explains everything really really well, every single little tiny step of it so that everyone can understand it. He tells you what is going to be on his tests ahead of time and will help you anytime, not just in class.

Most of the reasons given for poor mathematics teaching are simply the opposite of those given for good mathematics teaching. That is, the professor is not fair, the professor is not willing to help outside of class, and the professor is unclear and skips steps. Also under poor teaching, students do considerable complaining about accents and the resulting difficulty students find in understanding what the professor is saying. Here is a sample comment: “This teacher makes addition seem hard.”

Somewhat surprisingly, it was rare for a student to comment on a professor’s subject matter knowledge. Perhaps students do not know whether professors know the subject or

not. Or perhaps students feel at the undergraduate level all professors understand the topic they teach. A few times students thought that a professor was brilliant, but not able to teach (e.g., "he knows the math, but doesn't know how to teach it."). Enthusiasm was also a trait not often mentioned by students. In fact, too much enthusiasm struck students as a negative. "X is waaay too in love with math."

### Concluding Thoughts

What constitutes good mathematics teaching? If we intersect the answers given by our three sources (research, student evaluations, comments on RateMyProfessors.com), we come up with very little that intersects all three. The professor must be available to the students and even encourage student-faculty contact. The professor must explain things well. The professor must be fair in grading and prompt in feedback.

Knowledge of the subject matter does not make it into our definition. This might be a limitation of the sources. Knowledge of the subject matter is often mentioned in research, but quite a bit of research suggests that subject matter knowledge does not matter that much (Mapolelo, 1999; Fennema & Franke, 1992). Subject matter knowledge is on some student evaluations. However, the reason it is not on many student evaluation forms might be because it is assumed that students are not capable of measuring such a thing. In addition, students might not mention subject matter knowledge on RateMyProfessors.com because they assume it as a given.

It does seem clear from all three sources that a certain pedagogical approach is not necessary to ensure good teaching. That the professor is enthusiastic is a common question on evaluations but seems of little interest to students on RateMyProfessors.com, and it does not seem to be a major variable discussed in research. In addition, making the topic interesting and motivating the student are not variables that students include in good teaching. Some students express that motivation should come from them (e.g., Professor X will help anyone who tries) and that the topic of mathematics is not interesting!

By their very nature, the sources used in this study result in limitations. We cannot

and do not claim that our definition of good mathematics teaching is complete. However, we can say with some assurance that if one wants to be a good mathematics teacher, one is going to have to give of one's time. One must give time to students (be willing to help) and to grading (giving prompt feedback). In addition, professors need to work at explaining, even "every little step."

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