The Role of Visualization Approach on Student's Conceptual Learning

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The aim of this study is to investigate the role of visualization approach on students' conceptual understanding. The results of this study, while there is no statistical difference between the control and experiment groups in terms of procedural learning, experimental group students were more succesful in conceptual learning statistically.

Key words: visualization, conceptual knowledge, procedural knowledge.

INTRODUCTION

Researches in mathematics education has changed especially over the last four decades. Mathematical knowledge is among the foremost subjects in the change process. About learning psychology, Skemp (1971) searched firstly mathematics knowledge mentioned two kinds of knowledge. The first one is to recognize a set of symbols, which is mechanical knowledge that does not include conceptual understanding, but includes the ability to make procedures. The second one is the knowledge that can symbolize mathematical concepts; relate each other, and the knowledge that based upon abilities of making procedures with mathematical concepts (Baki 1998). Baykul (1999) defines that procedural knowledge is symbols, rules and knowledge used in solving mathematical problems and on the other hand, Baykul (1999) states that conceptual knowledge is described as mathematical concepts and relationship to each other.

Although many researches recently have done in mathematics education in showing that there are an important difference between conceptual knowledge and procedural knowledge (Ma 1999), conceptual and procedural knowledge complete and dependent on each other even if these types of knowledge seem to be independent from each other (Baki 1998). This knowledge distinction has discussed mathematics educators and has been accepted as general, there is no consensus these type of knowledge and their relation. They frequently try to make a distinction between conceptual knowledge and procedural knowledge, though the difference between these knowledge is not clear (Isleyen &Isik 2003). There is a relation between these knowledge. But, note that, in mathematics education, functional and permanent learning can be possible only by balancing conceptual and procedural knowledge (Noss & Baki 1998).

Since traditional mathematics teaching mainly cultivates skills, neglecting conceptual understanding of the underlying domain (Kadijevic 1999). The students' learning difficulties in acquiring the concepts of mathematics is abstract nature of mathematics. Since mathematical concepts are abstract, students learns mathematics by memorizing. One of the most important problems associated with the teaching mathematics is risen from the students' understanding difficulties in establishing the relationship between their knowledge and intuition about concrete structures and abstract nature of mathematics. It is not easy to find concrete examples in mathematical concepts.

There is a special importance of geometrical structures called as semi-concrete on teaching mathematics. An important component of forming concrete or at least semi-concrete of our mental representation of a concept is an external or physical reference (Konyalioglu et al. 2003). It is suitable usage of semi-concrete structure pointed out as geometric system in teaching of the abstract concept in mathematics. Graph, diagram, pictures and geometrical shape or models are a tool for visualization of the abstract concept in mathematics. By means of these, human reason sets up a relation between physical or external world and the abstract concepts (Konyalioglu 2003). It can be considered concepts such as geometric structures and mathematical-physical models for meaningful teaching mathematics. Also, mathematical concepts are abstract that one needs highly cognitive achievements to assimilate them (Baki 2000). By using visualization approach many mathematical concepts can become concrete and clear for students to understand. The term visualization is used in different meaning between mathematics educators. It is used the paper as it was defined by Zazkis, Dubinsky and Dautermann(1996), that is, as an act in which an individual establishes a strong connection between an internal construct and something to which access is gained through the sense. Such a connection can be made in two direction. An act of visualization may consist of any mental construction of object or processes that an individual associates with objects or events perceived by an external source. Alternatively, it may consist of the construction, on some external medium such as paper of objects or events. Consequently, the act of visualization is translation from external to mental. Visualization can be alternative method and powerful resource for students doing mathematics, a resource that can upon the way to different ways of thinking about mathemathics than the linguistic and logico-propositional thinking of traditional and the symbol manipulation of traditional algebra (Konyalioglu et al. 2003). Use of the visualization approach provides students to look at mathematics course, which was seen as a cumulation of abstract structures and concepts from a different perspective.

METHOD

The students used in this study were 60 sophomores enrolled in linear algebra course designed for the profesional teaching of mathematics program. All students have had the same formal education in mathematics. They took calculus and set theory course in the first year. Those courses did not include linear algebra content.The students were divided randomly into two groups consist of 30 students. All students were given basis knowledges deal with vector concept required for linear algebra.They were taught vector space concept by one instructor for three one-hour lectures per week. In the process, vector space concept to experimental group was presented in the two hours geometrically and one hour algebraically. Control group was also presented in the two hours algebraically and one hour geometrically. At the end of four weeks, the two groups were given same test. The question in the test were chosen to be simple problems on the vector space concept, which can be solved directly by applying the vector space definition.

Problem 1: Which of the following sets is a subspace of vector space V ? Give your answer with explanition.

a) $W = \{ (x,y) \in \boldsymbol{R}^2 : y = x + 1 \}$
b) $W = \{ (x,y) \in \boldsymbol{R}^2 : y = x^2 \}$

Problem 2: W_1 and W_2 are non-trivial subspaces of a vector space V. Is $W_1 \cup W_2$ a subspace of vector space V ?

Problem 3: Let V be a vector space and w a fixed vector in V. If W is a set of all scalar multiplications of w, is W a subspace ?

The students were asked to answers these questions both algebraically and geometrically. It has been explanied V is equal to $\boldsymbol{R}^2$ or $\boldsymbol{R}^3$ for geometric descriptions. Geometric description is the students' answers involved only written geometric descriptions. Model answers for the three problems set in this study coursewerw given in Appendix.

The responses which the students handed to the questions have been submitted at following tables. On the end of four weekly course process, two groups were asked to answer three questions given above. Problem 2 and problem 3 require conceptual knowledge. The results analysed by SPSS packet program. The results are presented by percentages, frequencies and t-test is carried out. Significance level was taken as p=0.005.

FINDINGS AND DISCUSSION

It is allowed to their correct and incorrect answers without interesting in algebraic and geometric descriptions of the students at following table. Furthermore, it is clarified that the students could not reply to the question on others column.

. **Table:** The students'general responses

Problem	Group	Correct Answer	Incorrect Answer	No response	p-value
1-a	**A**	%86,6	%6,7	%6,7	0,497
	B	%80,0	%13,3	%6,7	
1-b	**A**	%80,0	%13,3	%6,7	0,549
	B	%73,3	%20,0	%6,7	
2	**A**	%63,3	%30,0	%6,7	0,004
	B	%26,7	%60,0	%13,3	
3	**A**	%86,6	%3,4	%10,0	0,004
	B	%53,3	%20,0	%26,7	

Test result show that the students who were exposed to experimental group students are ,on average, more successful than control group students at the 0.05 significance level. As seen from Table, it was found that the students in experimental group were more succesful than the students in control group without regarding conceptual and procedural learning. Most of the students in both group answered problem 1-a and problem 1-b correctly by using only subspace description. This high percentage of correct answers may be due to solving similar exercices in the teaching process in the classroom. Although a different settings of problem 1-a and problem 1-b were discussed during the instruction, other problems were not discussed at any time during the instruction. Students in experimental group were found to be more succesful than the students in control group in solving the problem 2 and problem 3 which required more conceptual understanding. When two groups were compared, students'responses in experimental group included less number of incorrect answers than control group students' responses. Although there is not a meaningful difference between procedural knowledge of students from both groups, there is a meaningful difference between conceptual knowledge. In addition, students responses to problem 2 and problem 3 included less number of correct answers than the responses given to the other problems. Although problems 2 and problem 3 required more conceptual understanding dealing with subspace concept, the success for these problems can be explained that students in experimental group than students in control group comprehend well intitutionally. The distinction between concept definition and concept image supports our conclusion that differences between the achievements of two groups can be attributed to the different instructional approaches used. Although these concepts are given under equal circumtances to two groups, students' performance in solving these problems are found to be different. An explanation to this could be that though these two groups received the same concept definition of vector space, they were exposed to different experiences which resulted in forming different concept image. Students from experimental group had accomplished these problems because they had well concept image deal with vector space.

CONCLUSION

It can be considered that geometrical structures supported algebraically on the teaching of subspaces increase in meaningful learning of the students and help student's conceptual learning. In visualization approach students can perceive relations between abstract concepts and semi-concrete structure and make sense of abstract concepts in mathematics, and thus this approach facilitates student's understanding abstract concepts. Including visualization into the teaching process increases the students' conceptual learning. It could be suggested that the teaching method applied in this study could be extended to teaching the other abstract concepts in mathematics. Visualization must be used both a tool and an aim in mathematics education.

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APPENDIX: Model answers for the three problems set.

Problem 1: It has been explanied V is equal to $\boldsymbol{R}^2$ or $\boldsymbol{R}^3$ for geometric descriptions. Geometric description (GD) is the students' answers involved only written geometric descriptions, such as: " The set of W in problem 1-a is not a subspace of $\boldsymbol{R}^2$ " has been geometrically shown in Figure-1.

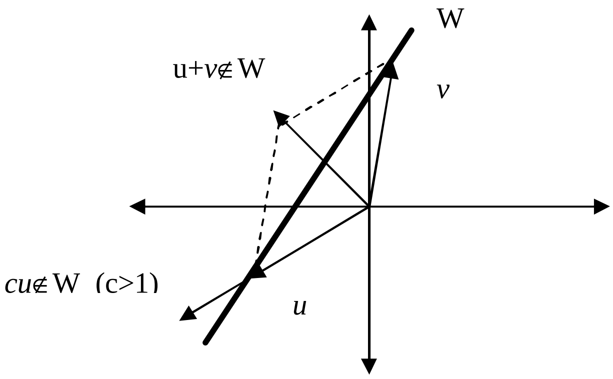


Figure 1

Any element of W set in problem 1-a is an ordered pair of the form (x, x+1). The set of W is not a subspace of $\boldsymbol{R}^2$ since it cannot obtain an ordered pair of the form (0,0) for no value of x variable.

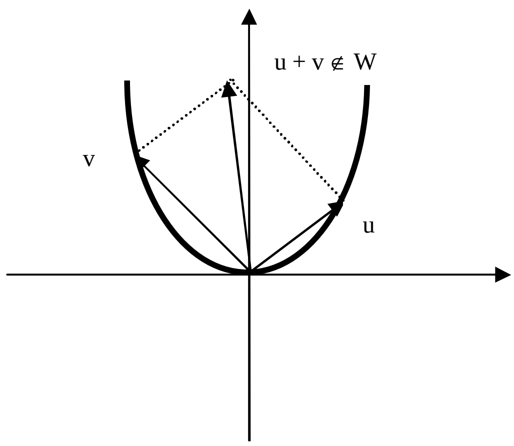


Figure 2

The set W which is the set of ordered pair of the form (x, x^2) in Problem 1-b as a different in Problem 1-a contains the point (0,0) for x=0. Because the set W is a subspace of $\boldsymbol{R}^2$, though it contains the point (0,0) is necessary, it is not adequate . The sum of any two elements in W is not in W. So W cannot a subspace of $\boldsymbol{R}^2$.

Problem 2: W_1 and W_2 are two subspaces of V. By chosening V=$\boldsymbol{R}^3$, it can be considered as W_1 is the set of the points on the axis-x and W_2 is the set of the points on the plane-xy. Namely,

$$W_1 = \{(x, y, z) \in \boldsymbol{R}^3 : x=0, y=0, z \in \boldsymbol{R}\}$$

$$W_2 = \{(x, y, z) \in \boldsymbol{R}^3 : x, y \in \boldsymbol{R}, z=0\}$$

That W_1 is a subspace of V=$\boldsymbol{R}^3$ is shown in Figure 3 and that W_2 is a subspace of V=$\boldsymbol{R}^3$ is shown Figure 4.

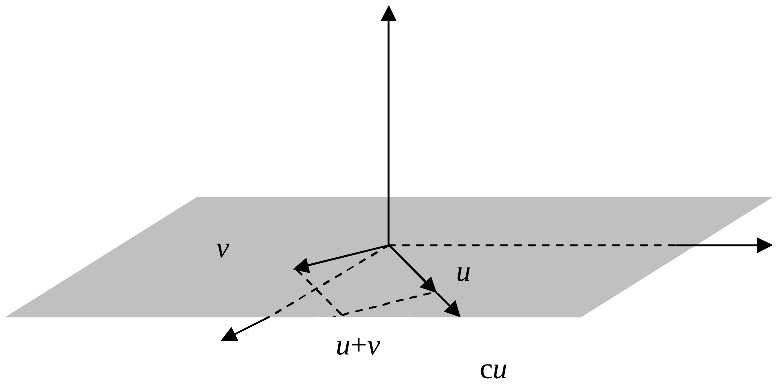


Figure 3

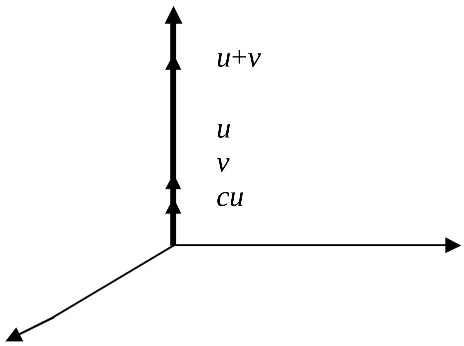


Figure 4

The addition of two elements in W_1 and the multiplication of an arbitrary element by scalar *c* are in the set. It can be shown in Figure 3. So, W_1 is a subspace of V=$\boldsymbol{R}^3$. The same situation is valid for W_2 (Figure 4). Because $W_1 \cup W_2 \subset$ V=$\boldsymbol{R}^3$, $W_1 \cup W_2$ is formed from points on the axis-z and the plane-xy (Figure 5).

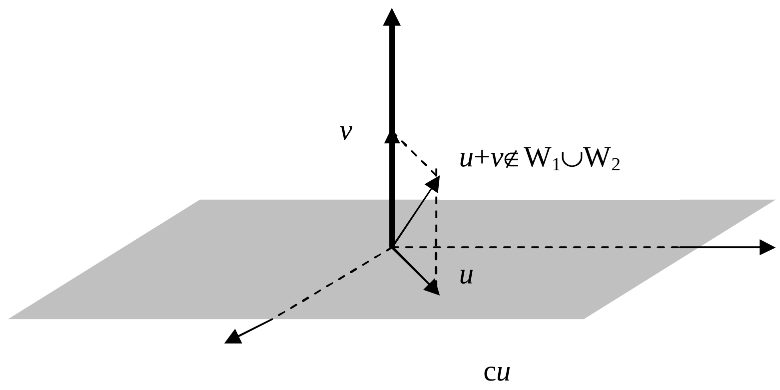
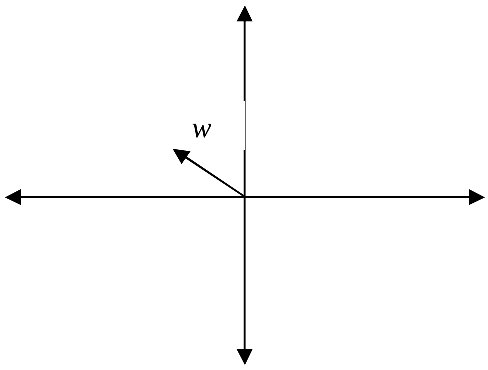


Figure 5

Consider any two elements in $W_1 \cup W_2$. $u, v \in W_1 \cup W_2$ for $u \in W_1$ and $w \in W_2$. As shown in Figure 5, $u + v \notin W_1 \cup W_2$ for $u, v \in W_1 \cup W_2$. So V is not a subspace of $\boldsymbol{R}^3$.

Problem 3: Let W be equal to $\boldsymbol{R}^2$. If w is any vector in V=$\boldsymbol{R}^2$, then it can be shown in Figure 7.



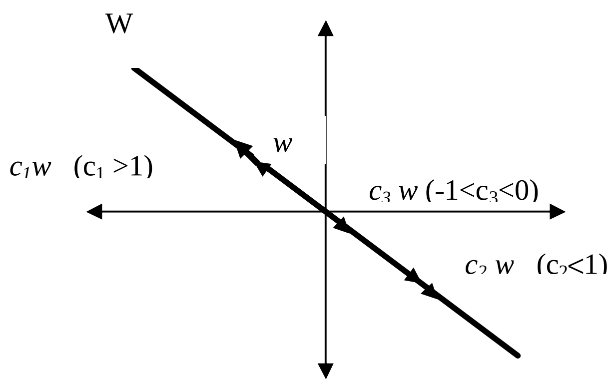


Figure 6 **Figure 7**

$$W = \{c_i w : w = (x,y),\ x,y,c_i \in \boldsymbol{R},\ i=1,2,3,\dots\}$$

The all scalar multiplication of W changes its' orientation and magnitude, but not change its' direction. The scalar multiplication of w by $c_i \in \boldsymbol{R}$ is visualized in Figure 7. The explanation of Problem 3 is similar to other problems.