

Seeds of Professional Growth Nurture Students' Deeper Mathematical Understanding

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Relatively speaking, today's teachers and students can more easily access a variety of instructional materials than ever before, partly due to the development of technology like the Internet and other electronic resources. It is good to have a quantitatively wide range of choices. However, on the other side of the same coin, having a wide range of choices implies that the teacher's ability to choose quality tasks and to guide students to deep mathematical understanding through these tasks becomes more critical.

The importance of quality mathematical tasks has been widely noted. Common features of quality tasks have been expressed in several ways, such as the demand for mathematical challenge (Jaworski, 1994) and uncertainty (Zaslavsky, 2005). These phrasings communicate the same idea: students' meaningful mathematical learning and understanding occurs when they experience difficulties in solving their tasks and resolve the difficulties through various measures. At the same time, quality tasks can enhance teachers' mathematical and pedagogical knowledge through reflecting upon their students' needs and difficulties (Cooney, 1994, 2001; Zaslavsky, 2005).

In this paper, we want to share the experiences we had with middle school students while we engaged in developing deep mathematical learning opportunities for students utilizing one of commonly accessible resources today, virtual manipulatives. Our aim in this paper is to describe the process that we, as mathematics teachers, used to refine our own learning and teaching alongside the middle school students. We hope that our own professional development encourages many mathematics teachers to consider a

reexamination of the tasks they currently use that ultimately leads to an expansion of their potential.

### **Looking for the Seed**

Our own professional growth has evolved from our university's annual event, called the "College Adventure Program." Sixth graders from a local upper-elementary school spend two days in May of each year learning about college campus life. Professors interact with groups of sixth graders to model various job responsibilities as teachers and researchers. We planned to use this opportunity as a mutual learning process for both ourselves and the students. Our original impetus was to gain a greater awareness of and increase our own competency with virtual manipulatives, which are the outcomes of the development of recent technology. A virtual manipulative is "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer, Bolyard, & Spikell, 2002, p. 373). The existing literature on virtual manipulatives predominantly reports attractive benefits of using them. Some benefits include the ease of access and management, immediate feedback, multiple representations, motivation for students and teachers (e.g., Clements & McMillen 1996; Crawford & Brown 2003; Dorward 2002; Forster, 2006; Heath 2002; Moyer & Bolyard 2002). We intended to investigate how middle school students perceive these advantages and benefit from them.

### **Planting our First Seed**

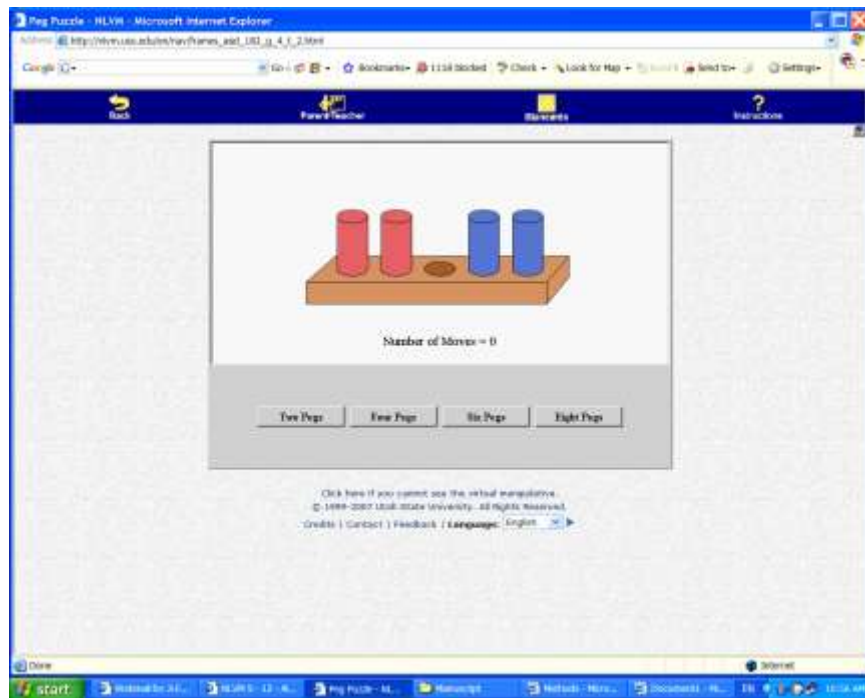
For our first year of interaction with sixth-graders, 2006, we designed exploratory sessions. We chose six virtual manipulatives from the National Library of Virtual Manipulatives [NLVM] site (<http://nlvm.usu.edu/>) for the students to explore: Number

Line Bounce, Percentages, Diffy, Peg Puzzles, Mastermind, and Tangrams. We met three groups (N=28, 26, 22) of sixth graders in a university computer lab for about 50 minutes per group. Students worked in pairs, and each pair of students explored two randomly-selected applets for 15 minutes per applet. Student pairs shared a computer, talked, and discussed with each other. Other than where the applet was located, we gave no more information or instruction, and students were told that neither their teachers nor the university professors could answer any questions. This let the students know that it was a completely exploratory opportunity. After a 15-minute exploration, each pair of students was asked to jointly complete a short 5-minute survey asking students' learning process and their satisfaction levels. We also randomly selected two pairs of students and personally observed their discussion while they were exploring their two applets.

We noted several implications from our first-year interactions with students (see Authors' forthcoming article in *ON-Math* for more detailed survey results and implications). One aspect of students' self-reported responses indicated that students perceived that math-game-like virtual manipulatives (e.g., Peg Puzzle, Tangrams, Mastermind), which did not explicitly show numbers or operations, did not seem to increase their learning of mathematics content (see Figure 1 for the initial screen of Peg Puzzle and the directions from the NLVM site). For example, for the Peg Puzzle, about 44% of the students surveyed replied that after they explored the Peg Puzzle, they believed that they had practiced or learned mathematics. Our observation also indicated that students depended on trial-and-error strategy to solve questions in these math-game-like contexts. The excerpt shown in Table 1, which was taken from our observation notes, provides a brief description of how a pair of sixth-graders spent their 15 minutes of

exploration time on Peg Puzzle. This excerpt confirms that students relied on trial-and-error strategies and that they were only mildly cognizant of a pattern strategy. They were not able to apply it to a more difficult peg level.

Figure 1. Peg Puzzle Initial Screen and instructions from National Library of Virtual Manipulatives



#### Instructions:

This virtual manipulative is the classic logic puzzle.

The goal of the puzzle is to switch the pegs on the left with the pegs on the right by moving one peg at a time.

Move pegs by clicking and dragging them to open slots. A peg may only be moved to an open slot directly in front of it or by jumping over a peg to an open slot on the other side of it. You may not move backwards. The game ends when you win or get stuck.

Play versions of this game with different number of pegs by clicking on the buttons below.

Table 1. One student pair's discussion during free exploration time in 2006

- Students did not read the instruction, but figured out how Peg Puzzle works through several trials.
- Student A: Played 4-peg puzzle then “stuck”
- Student B: Retried 4-peg puzzle then “stuck”
- Students A and B took turn 4 times to solve 4-peg puzzle, which was what appeared at the initial screen. They were not successful. “Stuck” signs kept appearing.
- Student A: Noticed the choices of different peg levels at the lower edge of the applet and tried the 2-peg puzzle. Succeeded.
- Student B: “We can move the same peg only twice.”
- Student B: kept trying
- Student A: Distracted.
- Students A and B took turns and tried.
- Student B: “Is there a ‘cross-slide’ pattern?”
- Student A: “Maybe a pink-blue pattern?”
- Student B: “Look at the standards. Maybe, we can get some hints from there.”
- Student A: Did not try.
- Student A: Solved 4-peg puzzle.
- Student B: “How did you do it?”
- Student A: “I don’t know. I just guessed.”

The students’ perceptions and actual performance gave us an opportunity to think about the mathematical knowledge involved and pedagogical measures that guide students to deep mathematical understanding.

### **Germinating**

After our first interaction with sixth graders in 2006, we reserved time for ourselves to explore the Peg Puzzle applet. Previously, we *assumed* that there must have been patterns in this applet activity. However, we, ourselves, did not explicitly investigate what they were. The students’ reactions in the 2006 study led us to articulate the patterns involved in a more organized way. Two of the students we observed mentioned patterns in movements and colors. However, they could not proceed from this observed pattern to predicting a future pattern, and just kept depending on the trial-and-



Figure 3. Peg Puzzle Solutions Predicted After Using Virtual Manipulatives

# of pegs (one side)	# of pegs (total)	# of holes	# of moves	Pattern (Slide-Jump and Blue-Red)
5	10	11	35	S J S J J S J J J S J J J S J J J S J J J S J J J S J J S J J S J J S J S
n	2n	2n+1	$n^2 + 2n = n(n+2)$	

Note. *Slide* means to drag the peg toward the opposite side in which it began. *Jump* means to jump over one peg of the opposite color toward the opposite side in which it began.

We experienced various stages of problem solving in the content of algebra while recording our findings in an organized manner, analyzing our findings, making a prediction, testing a prediction (10 pegs), developing an algebraic equation that describes the conditions of the game board and that would predict the total number of moves for any given game condition.

We wondered how much students could learn from this applet and how we could support their learning process in a similar manner that we experienced. Our own exploration evoked a question: “How can we guide students to the deep understanding involved in the Peg Puzzle that we had just experienced ourselves?” Thus, in the 2007 event, we provided brief instructions (about 20 minutes) that included various explicit prompts on the pattern recognition and the content-related cues in the NLVM site as well as provided a blank Figure 2 to act as a record sheet. The student pairs then had 15 minutes of free exploration time and completed a 5-minute survey. Randomly-selected pairs’ discussions were audio-taped.

The interaction between students and the instructor during the 20 minute content instruction is shown in Table 2. Navigation skills instruction include reviewing and

rephrasing the given directions, reviewing NCTM content and process standards and discussing which standards were related to Peg Puzzle.

Table 2. Teacher-Student interaction during content instruction in 2007

<ul style="list-style-type: none"> <li>• Teacher asked students to try 4-peg puzzle for 90 seconds.</li> <li>• Teacher asked, “Did anyone discover patterns?” <ul style="list-style-type: none"> <li>○ RBRB (Red, Blue, Red, Blue).....</li> <li>○ There must be a space between...</li> <li>○ We have to jump over different colors</li> </ul> <p><b><u>Students initially paid attention to the color pattern.</u></b></p> </li> <li>• Teacher moved the pegs as students directed: <ul style="list-style-type: none"> <li>○ RBRBR ? (stuck)</li> <li>○ RBRBR ? (tried one more time, but stuck)</li> <li>○ Students suggested starting with a Blue peg.</li> <li>○ BRBRB ? (stuck)</li> <li>○ One student provided a solution.</li> <li>○ BRRBBRRB ! (correct)</li> <li>○ One student suggested starting with a Red peg.</li> <li>○ Teacher asked students to predict the pattern.</li> <li>○ Student predicted “RBBRRBBR” pattern.</li> <li>○ Teacher demonstrated: RBBRRBBR ! (correct)</li> </ul> <p><b><u>Students kept on focusing on the color patterns.</u></b></p> </li> <li>• Teacher asked one more time: “Did anyone discover any other patterns?” <ul style="list-style-type: none"> <li>○ We have to alternate colors.</li> <li>○ Teacher: “What <i>action</i> do you do with colored pegs?” “Movement?”</li> <li>○ Several students said, “Jump!”</li> <li>○ Teacher took a motion. Then several students added, “Slide.”</li> <li>○ Teacher asked students to think about “moving patterns.”</li> <li>○ Students tried for 90 seconds.</li> <li>○ Students verbally presented their solution. Teacher demonstrated according to their directions and recorded their movement pattern under the color pattern: <p style="margin-left: 40px;">BRRBBRRB S J S J J S J S</p> </li> <li>○ Teacher questioned about the above patterns. <ul style="list-style-type: none"> <li>▪ One student said, “It starts with SJS and ended with SJS.”</li> <li>▪ Teacher asked, “What about in the middle?”</li> <li>▪ One student mentioned, “double”.... Another student suggested drawing a line between two J’s mentioning a mirror image.</li> </ul> </li> </ul> </li> </ul>
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### Signs of Growth

There were several indications that students saw this exploration as an opportunity to learn important mathematics concepts. First, about 77% of students surveyed replied that they practiced or learned mathematics (About 44% of the 2006 group of students reported that they practiced or learned any mathematics when exploring Peg Puzzle.). Second, students analyzed and recorded their work in an organized way because we had provided that instructional support (e.g., a blank Figure 2 and Figure 3 as a record sheet). This led to the decreased dependency on the random trial-and-error strategy. Subsequently, students' discussion was more focused and specific compared to the previous year's free exploration. A student-pair's discussion excerpts shown in Table 3 apparently indicated that they tried to predict the results and to attempt to test their prediction through recognizing patterns.

Table 3. One student pair's discussion during exploration time in 2007

<p>Student A: "Two-peg puzzle."  Student B: "That was easy."  Student A: "Yeah. Let's do that on the sheet. Number of pegs...and we need to make a pattern. There is a blue red blue red. We need blue and red. Let's see. Are we starting? Blue-red-blue. 2-1-3-and it's a blue-red-blue. We're supposed to make X's, right? Yeah, just do X's. Now we can also put, or red-blue-red, either one will work."  Student B: "Alright. 4-peg puzzle. Red-red. I think it takes 8 moves. Yeah, it takes 8 moves".  Student A: "Are you positive?"  Student B: "Yeah. It's red-red-red-.... Wait, that was wrong. Yeah, it is. Oh no, I know it now. Red-blue-blue-red-red-blue-blue-red.... How many moves do we have so far?.... Solved 4-peg puzzle.... I'll write it down. (recording and trial continued)"  Student A: "What combination are we now starting with? Let's try a red-red-blue-red-....I think I know how to do. Blue-red over that ... Wait, blue or red first. Blue-red-red-blue-blue-red- here...here...it has to go blue-red-red-blue-blue-red-red-red-blue...No, that wouldn't work...blue-red-red-blue-....I know what I have to do, but...I've learned one thing so far. We cannot jump over something of your own color. I've learned that."  Student B: "You can't have two of the same colors in the same row. You can't have two of the same color next to you or that ruins your chances right there. Do we need to calculate how many moves so far? I think there is something like 16 moves. Let's try it. Red-blue-blue-red-red-blue-red-blue-red... (recording and trial continued)"</p>
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### **Looking Forward to More Fruit**

Although we observed visible changes in the students' performance and understanding between two years' events, we believe that our work is still in progress. Our belief is that the way teachers use instructional materials is far more important than what materials teachers use. Our own professional development encourages us to continue to listen to our students as they encounter difficulties in creating mathematical meaning behind the mathematical task provided. In the case of virtual manipulative exploration, students seem to be busy doing something, because they are clicking a mouse or typing on a computer keyboard. However, this "hands-on" work cannot be a direct indication of students' mathematically intellectual involvement, or further, of their mathematical understanding. By requesting and prompting the class to share their collective knowledge more students benefited than if a single pair would have made the discoveries, or as happened in 2006, no single pair was able to articulate the discovery.

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