A comparative analysis of mathematics curricula in Korea and England focusing on the content of the algebra domain

Hye Jeang Hwang
Chosun University, Korea
sh0502@chosun.ac.kr

I. The purpose of this study

Many Korean authorities in mathematics education have pointed to the level of difficulty and the sheer quantity of the content areas to be mastered in school. Therefore, the time has come to examine the appropriateness of this level of mathematics content. One meaningful approach to this task would be to compare the content of mathematics curricula in different countries in an attempt to perceive an international tendency in mathematics education. Like Korea, England administers a single national curriculum, so in this paper the English curriculum will be examined.

The purpose of this study is to examine the latest English mathematics curriculum (published in 1999) and compare it with the current Korean one (published in 1997). However, in order to make this comparison feasible, we will limit the focus to the content of the algebra domain although in England there is significant emphasis on the content of both 'shape, space, and measures' and 'handling with data' domains. Ultimately, this study will compare mathematics curricula in the two countries with respect to their organization and special characteristics, focusing on the content of algebra domain including numbers, operations, and functions. It will identify the content that receives greater or lesser emphasis in each country and whether the content is introduced earlier or later in each country.

II. Introduction to mathematics curricula in Korea and England

1. The structure of the curriculum

The Seventh National Mathematics Curriculum of Korea was published in the year 1997. Mathematics for the National Common Basic Education Period (grade 1 to 10) is organized and implemented in 20 steps (semesters), which are comprised of standard content and enriched content. During this Period (grade 1 to 10), all of the students have to take 'Mathematics' as a required subject.

From grades 11 and 12, students take elective courses. They can choose the subjects they want to take according to their interests and career goals. There are six mathematics-related elective subjects, namely

‘Mathematics I’, ‘Mathematics II’, ‘Applied Mathematics’, ‘Discrete Mathematics’, ‘Calculus’, ‘Probability and Statistics’. Students planning to take their college entrance examinations should take at least ‘Mathematics I’ and/or ‘Mathematics II’ subjects. (Refer to Table 1) Because of this structure, this study deals with the content of three subjects, ‘Mathematics’, ‘Mathematics I’, and ‘Mathematics II’.

Table 1. The Korean curriculum

School	Grade	Age	Course	Subject
elementary	1~6	8~14	required course	Mathematics
Middle	7~9	14~17		
High	10	17-18		
	11~12	18~20	Elective course	Mathematics Mathematics (Mathematics required) Calculus (Mathematics required) Applied Mathematics Discrete Mathematics Probability and Statistics

‘The programme of study: mathematics’ in the English mathematics curriculum, published in the year 1999, is divided into four key stages with each stage including two or three grades (or ages). This is shown in Table 2.

Table 2. The English curriculum

School	Stage	Grade	Age
Elementary	key stage 1	1~2	5~7
	key stage 2	3~6	7~11
Secondary	key stage 3	7~9	11~14
	key stage 4	10~11	14~16

As shown on Table 2, the English national curriculum is administered up to the age of 16. In England, mathematics is not required for all students beyond the age of 16 but it is optional for the students to choose to study mathematics for another 2 years. This is the case for about 10% of the cohort for A-level examination, and during this period topics such as calculus are dealt with. In effect, the key difference in math curricula of the two countries may be that in Korea the majority of students up to the age of 20 are required to take a mathematics subject whereas in England only a small percentage of students continue with their mathematical studies beyond the age of 16.

2. The main areas of mathematics

In Korea, the main areas of ‘Mathematics’ (for grade 1 to 10) are ‘number and operation’, ‘letters and expressions’, ‘patterns and functions’, ‘geometric figures’, ‘measurements’, ‘probability and statistics’. The main areas in ‘Mathematics I’ and ‘Mathematics II’ are comprised of ‘algebra’, ‘analysis’,

‘geometry’, and/or ‘probability and statistics’. This organization is shown on Table 3, which also shows the comparison to the content areas in the English curriculum.

As mentioned before, this study will limit its focus to the content of the algebra domain, and therefore it will only deal with the main areas of ‘number and operation’, ‘letters and expressions’, and ‘patterns and functions’ for ‘Mathematics’, and of ‘algebra’ and ‘analysis’ for ‘Mathematics I’ and ‘Mathematics II’ in the case of Korea. It will also deal with the main area of ‘number’ (or ‘number and algebra’) for the key-stages 1 to 4 in the case of England. (Refer to Table 3)

Table 3. The main areas of the algebra domain in the Korean and English curricula

<table>
<tr><th colspan="3">Korea</th><th colspan="3">England</th></tr>
<tr><th>Mathematics</th><th>Mathematics</th><th>Mathematics</th><th>Key stage 1</th><th>Key stage 2</th><th>Key stage 3 ~ 4</th></tr>
<tr><td>number and operation</td><td rowspan="2">Algebra</td><td rowspan="2">Algebra</td><td rowspan="3">Number</td><td rowspan="3">number</td><td rowspan="3">number and algebra</td></tr>
<tr><td>letters and expressions</td></tr>
<tr><td>patterns and functions</td><td>Analysis</td><td>Analysis</td></tr>
<tr><td>geometric figures</td><td rowspan="2"></td><td rowspan="2">geometry</td><td rowspan="2">shape, space, and measures</td><td rowspan="2">shape, space, and measures</td><td rowspan="2">shape, space, and measures</td></tr>
<tr><td>measurements</td></tr>
<tr><td>probability and statistics</td><td>probability and statistics</td><td></td><td></td><td>handing data</td><td>Handing data</td></tr>
</table>

Unlike the Korean curriculum, the English curriculum has the same parts (namely, ‘using and applying number’, ‘numbers and number system’, ‘calculations’, ‘solving numerical problems’, ‘equations’, ‘formulae and identities’, ‘sequences, functions and graphs’) under the main area of ‘number’ (or ‘number and algebra’) and each part is divided into several sub-areas, which are shown on Table 4. The detailed content areas can be easily understood by reading *The National Mathematics Curriculum for England* (http://www.nc.uk.net).

However, the Korean curriculum has not been formally translated into an English version, and so such comprehensive information about the Korean curriculum in English does not exist. But a rough grasp of the content of the Korean mathematics curriculum is available through the Appendix presented in this paper.

III. The procedure of this study

The procedure of this comparative analysis will be as follows:

First, the essential and core content areas in the main areas of ‘number and operation’, ‘letters and expressions’, and ‘patterns and functions’ in the Korean curriculum will be selected and arranged (ordered) in a table according to grade level. (Refer to Appendix)

Second, the content areas of the main area of ‘number’ (or ‘number and algebra’) in the English curriculum will be selected and displayed according to the sub-areas. (Refer to the lists of sub-areas shown in Table 4) Specifically in order to grasp the hierarchical nature of the content of each sub-area according to the progress of key-stage, content areas that are similar or identical will be displayed in a chart for every key stage. In addition, the content areas of the English curriculum are matched with the numbered content areas of the Korean curriculum in the chart. (Refer to Table 5 8) Based on the results of those tables (charts), a comparative analysis of mathematics curricula in two countries might be accomplished. But because of limited space of this paper, only some parts of the tables (charts) are shown.

Table 4. The sub-areas under the main area of ‘number’ (or ‘number and algebra’) in England

Part	Sub-area				
	Key stage 1	**Key stage 2**	**Key stage 3**	**Key stage 4 (foundation)**	**Key stage 4 (higher)**
using and applying number(and algebra)	problem solving	Problem solving	Problem solving	problem solving	problem solving
	Communicating	communicating	communicating	communicating	communicating
	Reasoning	reasoning	reasoning	reasoning	reasoning
numbers and number system	counting	Counting			
	number patterns and sequences	number patterns and sequences			
	the number system				
		Integers	integers	integers	integers
			powers and roots	powers and roots	powers and roots
		fractions, percentages and ratio	fractions	fractions	fractions
			percentages	percentages	percentages
			ratio and proportion	ratio	ratio
		decimals	decimals	decimals	
calculations	number operations and the relationships between them	number operations and the relationships between them	number operations and the relationships between them	number operations and the relationships between them	number operations and the relationships between them
	mental methods	mental methods	mental methods	mental methods	mental methods
		written methods	written methods	written methods	written methods
		calculator methods	calculator methods	calculator methods	calculator methods
solving numerical problems					
equations, formulae and identities			use of symbols	use of symbols	use of symbols
			index notation	index notation	index notation
			equations		equations
			linear equations	linear equations	linear equations
			formulae	formulae	formulae
			direct proportion		direct and inverse proportion
			simultaneous linear equations		simultaneous linear equations
			inequalities	inequalities	
					quadratic equations
					simultaneous linear and quadratic equations
			numerical methods		numerical methods
sequences, functions and graphs			sequences	sequences	sequences
			functions	graphs of linear functions	graphs of linear functions
			gradients	gradients	
				Interpret graphical information	Interpret graphical information
					quadratic functions
					other functions
					transformation of functions

IV. An analysis of mathematics curricula in Korea and England

1. The content receiving greater emphasis in England than in Korea

(1) Approximation

In the ‘instructional guidelines’ section of the Korean curriculum, the guideline ‘estimation is done before calculation while using written methods’ is included (recommended) in the second, third, and fourth grade levels. But the content of approximation is formally introduced only in the fourth grade level, and the content areas are ‘knowing the meaning of rounding off, rounding up, and discarding’ and ‘knowing the meaning of estimation and applying it to the real life situation’.

In contrast, in the English curriculum, approximation is broadly emphasized in a variety of the sub-areas of ‘counting’, ‘integers’, ‘decimals’, ‘number operations and the relationships between them’, ‘mental methods’, ‘written methods’, ‘calculator methods’, and ‘solving numerical problems’ throughout key-stages 1 to 4. A couple of examples follow:

- round up or down after division, depending on the context (the sub-area of ‘number operations and the relationships between them’ at key-stage 2)
- understand the calculator display, knowing when to interpret the display, when the display has been rounded by the calculator, and not to round during the intermediate steps of a calculation (the sub-area of ‘calculator methods’ at key-stage 4 higher)

(2) Mental methods

Although in Korea, the content of mental methods is not formally introduced in the curriculum, the English curriculum emphasizes the content of mental methods, setting it as a separate sub-area. In this area, the curriculum deals with the appropriate use of mental methods and specifically focuses on the use of mental methods by recalling relevant mathematical knowledge (facts) in order to solve diverse problems including calculation. As an example, in the English curriculum, the content area of ‘deriving unknown facts from those they know (for example, estimate $\sqrt{85}$)’ is shown. This example seems to lead the students to calculate $\sqrt{85}$ as 9. , mentally recalling the fact that $9 \times 9 = 81$. In addition, the English curriculum extends the use of mental methods to decimal and exponential numbers, and further includes the content area of ‘develop a range of strategies for mental calculation’. The rest of the content areas of mental methods are shown in Table 5.

Table 5. The content of ‘ mental methods’ in England

The sub-area of ‘mental methods’				
Key stage 1	**Key stage 2**	**Key stage 3**	**Key stage 4 (foundation)**	**Key stage 4 (higher)**
• develop rapid recall of number facts				• recall integer squares from 2x2 to 15x15 and the corresponding square roots, the cubes of 2. 3. 4. 5 and 10, the fact that $n^0 = 1$ and $n^{-1} = \frac{1}{n}$ and for positive integers n [for example, ..], the corresponding rule for negative numbers [for example, ..], $n^{\frac{1}{2}} = \sqrt{n}$ and $n^{\frac{1}{3}} = \sqrt[3]{n}$ for any positive number n [for example, ..]
1.1.1.				
• know addition and subtraction facts to 10 and use these to derive facts with totals to 20	• recall all addition and subtraction facts for each number to 20			
1.1.3.	1.1.3.			
• carry out simple calculations of the form 40+30= , 40+ =100, 56- =10	• work out what they need to add to any two-digit number to make 100, then add or subtract any pair of two-digit whole numbers	• recall all positive integer complements to 100 [for example, 37+63=100]	• recall all positive integer complements to 100 [for example, 37+63=100]	
1.1.4., 1.2.1., 2.2.1.	2.1.2.			
• record calculations in a number sentence, using the symbols ...				
1.2.2., 2.2.3.				
• develop a variety of methods for adding and subtracting, ...	• handle particular cases of three-digit and four-digit additions and subtractions by using compensation of other methods [for example, 3000-1997, 4560+998]			
1.1.5.	2.1.5., 3.1.2., 3.1.3.			
• know multiplication facts for the x2 and x10 multiplication tables and derive corresponding division facts	• recall multiplication facts to 10x10 and use them to derive quickly the corresponding division facts	• recall all multiplication facts to 10x10, and use them to derive quickly the corresponding division facts	• recall all multiplication facts to 10x10, and use them to derive quickly the corresponding division facts	
2.3.4., 2.3.5., 3.1.4.	2.1.4., 2.3.4., 3.1.4.	2.1.4., 2.3.4., 3.1.4.	2.1.4., 2.3.4., 3.1.4.	
• know doubles of numbers to 10 and halves of even numbers to 20	• double and halve any two-digit number			
	• multiply and divide, at first in the range of 1 to 100 [fro example, 27x3, 65 ÷5], then for particular cases of larger numbers by using factors, distribution or other methods			
	3.1.7.			

The sub-area of 'mental methods' (continued)				
Key stage 1	**Key stage 2**	**Key stage 3**	**Key stage 4 (foundation)**	**Key stage 4 (higher)**
		• recall the cubes of 2, 3, 4, 5, and 10, and the fraction-to-decimal conversion of familiar simple fractions [for example, 1/2, 1/4, 1/5, 1/10, 1/100, 1/3, 2/3, 1/8] • round to the nearest integer and to one significant figure	• recall the cubes of 2, 3, 4, 5, and 10, and the fraction-to-decimal conversion of familiar simple fractions [for example, 1/2, 1/4, 1/5, 1/10, 1/100, 1/3, 2/3, 1/8] • round to the nearest integer and to one significant figure	• round a given number of significant figures
		• estimate answers to problems involving decimals • add and subtract mentally numbers with up to two decimal places [for example, 13.76-5.21, 20.08+12.4]	• estimate answers to problems involving decimals • add and subtract mentally numbers with up to two decimal places [for example, 13.76-5.21, 20.08+12.4]	• convert between ordinary and standard index form representations [for example, $0.1234 = 1.234 \times 10^{-1}$], converting to standard index form to make sensible estimates for calculations involving multiplication and/or division
		• multiply and divide numbers with no more than one decimal digit [for example, 14.3x4, 56.7 ÷7]	• multiply and divide numbers with no more than one decimal digit [for example, 14.3x4, 56.7 ÷7]	
		5.1.6.	5.1.6.	8th grade (the area of measurements)
• develop a range of mental methods for finding, from known facts, those that they can not recall, including adding 10 to any single-digit number, then adding and subtracting a multiple of 10 to or from a two-digit numbers		• develop a range of strategies for mental calculation • derive unknown facts from those they know [for example, estimate $\sqrt{85}$]	• develop a range of strategies for mental calculation • derive unknown facts from those they know [for example, estimate $\sqrt{85}$]	• develop a range of strategies for mental calculation • derive unknown facts from those they know
1.1.5., 2.1.7.				

(3) Calculator and computer use

The Korean curriculum does not include calculator and computer use, except some general recommendations in the ' instructional guidelines' section as follows:

- the use of table of square roots or calculators is recommended when an approximated value of square root is needed, but the method of finding the values of square roots should not be used in mathematics class (this recommendation is for the ninth grade level)

- the active utilization of various forms of educational technology is recommended (for the first to tenth grade levels)
- with the exception of the content required to enhance computational skills in the primary school level, the active utilization of calculators and computers is recommended in order to improve the understanding of concepts, principles, and rules, and to enhance problem solving abilities (for the first to tenth grade levels)
- the use of calculators in logarithm calculation problems is recommended (for the elective course 'Mathematics I')

However, the English curriculum introduces calculator use from key-stage 2 by setting it as a separate sub-area. At key-stage 2, the basic methods for using calculators, namely knowing of how to use calculators and to select the correct key sequence for calculations, are introduced. Starting from key-stages 3 to 4 (higher), more effective and efficient use of calculators in solving complicated calculation problems (for example, large numbers, fractions, decimals, calculation on money, reciprocals, trigonometric functions, statistics) is dealt with. In addition, the English curriculum deals with content areas such as 'understand the calculator display, interpreting it correctly ..., and knowing not to round during the intermediate steps of a calculation'.

On the other hand, the English curriculum does not have a separate sub-area of computer use, illustrating that computer use is less emphasized than use of the calculator. However, it is impressive that the content area of computer use in the English curriculum is presented in such a concrete way.

- use systematic trial and improvement methods with ICT tools to find approximate solutions of equations where there is no simple analytical method (for example, $x^3 + x = 100$) (the sub-area of 'numerical methods' at key-stage 3)
- use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them [for example, $x^3 - x = 900$] (the sub-area of 'numerical methods' at key-stage 4 higher)
- plot graphs of : simple cubic functions [for example, $y = x^3$], the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, the exponential function $y = k^x$ ... [for example, $y = 2^x$], the circular function $y = \sin x$ and $y = \cos x$, using a spreadsheet or graph plotter as well as pencil and paper; recognize the characteristic shapes of all these functions (the sub-area of 'other functions' at key-stage 4 higher)

(4) Interpreting graphical information

In Korea, the content of functions, first introduced in middle school, focuses excessively on drawing graphs of diverse functions (for example, linear and quadratic functions, trigonometric functions, and logarithmic and exponential functions, etc.) and knowing their properties. But the English curriculum emphasizes the interpretation of functions, specifically connected with real-life situations, setting it as a separate sub-area. The English curriculum deals with content areas such as ‘interpreting information presented in a range of linear and non-linear graphs’ at key-stage 4 (foundation) and ‘discussing and interpret graphs modeling real situations’ at key-stage 4 (higher), both stages including concrete and a variety of examples such as graphs describing trends, distance-time graph, and velocity-time graph.

2. The content being introduced later in England than in Korea

(1) Written methods

When comparing the new Korean curriculum with the old one, we notice that the new curriculum puts emphasis on understanding the principles of number operations rather than on doing computations by drill and practice. In spite of this, in Korea, acquisition of fluent computation skills is still considered very important.

On the other hand, since the English curriculum deals with computation through the emphasis on approximation, mental methods, and calculator use, the curriculum seems to be put less importance or pressure in acquiring written methods. Rather, the curriculum tends to introduce the content of written methods a little later than the Korean curriculum. As an example, in Korea, the content of rationalization of a denominator is dealt with at the ninth grade level, whereas it is dealt with at key-stage 4 (higher) in England. (Refer to Table 6). In any case, the English curriculum seems to put an emphasis on the content of written methods that is connected to problems in the context of realistic situations, which are often complicated.

Table 6. The content of written methods in England

The sub-area of 'written methods' (partially shown)				
Key stage 1	**Key stage 2**	**Key stage 3**	**Key stage 4 (foundation)**	**Key stage 4 (higher)**
		• use standard column procedures for addition and subtraction of integers and decimals	• use standard column procedures for addition and subtraction of integers and decimals	
		4.1.11.	4.1.11	
		• use standard column procedures for multiplication of integers and decimals, ...	• use standard column procedures for multiplication of integers and decimals, ...	
		5.1.6.	5.1.6.	
		• solve a problem involving division by a decimal by transforming it to a problem involving division by an integer	• .solve a problem involving division by a decimal (up to two places of decimals) by transforming it to a problem involving division by an integer	
		5.1.8.	6.1.4.	
		• solve simple percentage problems, • solve word problems about ratio and proportion, ...	• solve percentage problems, • solve word problems about ratio and proportion, ...	• solve percentage problems,
		6.3.2.	6.3.2.	
				• represent repeated proportional change using a multiplier raised to a power
				11.1.8
				• calculate an unknown quantity from quantities that vary in direct or inverse proportion
				7.3.1.
				• calculate with standard index form [for example, $(2.4\times10^{7})\div(5\times10^{3})$ $= 4.8\times10^{3}$]
				8.2.2.
				• use surds and π in exact calculation, without a calculator
				7th grade (area of measurements), 9.1.1.
				• rationalize a denominator such as $\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$
				9.1.4.

(2) Number system

In the Korean curriculum, terminating decimals, nonterminating decimals, and recurring decimals are dealt with at the eighth grade level, and furthermore the content of irrational numbers is introduced at the ninth grade level. But the English curriculum introduces terminating decimals at key-stage 4 (higher).

In addition, the English curriculum deals with addition, subtraction, multiplication, and division of integers and ‘any number’ at key-stages 3 and 4 (foundation) and it deals with multiplication and division of ‘any number’ at key-stage 4 (higher). Then, at key-stages 3 to 4 (higher), the curriculum includes the sub-area of ‘power and roots’, illustrating that ‘any number’ dealt with for operations implies to numbers up to real numbers including irrational numbers. Namely, the English curriculum seems to deal with a wide range of numbers for operations by using the term of ‘any number’.

However, unlike the Korean curriculum, the English curriculum does not put an emphasis on the concepts themselves of real numbers including rational and irrational numbers. Additionally, the English curriculum does not deal with complex numbers even in the final stage, key-stage 4 (higher). But the Korean curriculum includes the concept of complex numbers (based on the concept of real numbers) at the tenth grade level, and it deals with the number system, namely the properties of the set of various numbers including complex numbers. Compared to the Korean curriculum, the English curriculum seems to put less emphasis on the content of number system.

Table 7. The content of number system in England

The sub-area of 'number operations and relationships between them' (partially shown)				
Key stage 1	**Key stage 2**	**Key stage 3**	**Key stage 4 (foundation)**	**Key stage 4 (higher)**
• understand addition and use the related vocabulary • understand subtraction as both 'take away' and 'difference and use the related vocabulary • recognize that subtraction is the inverse of addition	• develop further their understanding of the four number operations and the relationships between them including inverses • use the related vocabulary	• add, subtract, multiply and divide integers and then any number	• add, subtract, multiply and divide integers and then any number	
1.1.3.	1.1.3., 2.1.3., 3.1.4.	4.1.3., 6.1.5., 7.1.8.	4.1.3., 6.1.5., 7.1.8.	
• understand multiplication as repeated addition • begin to understand division as grouping (repeated subtraction) • use vocabulary associated with multiplication and division	• find remainders after division, then express a quotient as a fraction or decimal	multiply or divide any number by powers of 10, and any positive number by a number between 0 and 1	multiply or divide any number by powers of 10, and any positive number by a number between 0 and 1	• multiply or divide any number by powers of 10, and any positive number by a number between 0 and 1
		5.1.6., 5.1.7., 6.1.4.	5.1.6., 5.1.7., 6.1.4.	5.1.6., 5.1.7., 6.1.4.
				• multiply and divide by a negative number
2.1.3., 3.1.4.	3.1.6., 4.1.8., 5.1.7.			7.1.8.
	• choose suitable number operations to solve a given problem, and recognize similar problems to which they apply			
	4.2.3.			

(3) Direct and inverse proportion

In the Korean curriculum, both concepts of direct and inverse proportions are simultaneously dealt with for the purpose of introducing the function concept in the seventh grade level. In contrast, the English curriculum deals with direct proportion in the sub-area of 'direct proportion' at key-stage 3, and also it deals with inverse proportion in the sub-areas of 'direct and inverse proportion' (at key-stage 4 higher), 'written methods' (at key-stage 4 higher), and 'solving numerical problems' (at key-stages 3 to 4 higher). In particular, although the Korean curriculum deals with inverse proportions comprised of $y \propto x$ and $y \propto \frac{1}{x}$, the English curriculum deals with the content that additionally includes $y \propto x^2$ and $y \propto \frac{1}{x^2}$. This reason might be that direct proportion such as $y \propto x^2$ is a very important concept that can be applied in the field of physics. This is a good example that shows the connection between mathematics

and non-mathematical subjects.

3. Content introduced earlier in England than in Korea

(1) Even and odd numbers

The content of even and odd numbers is not introduced in the Korean curriculum, although the concept of even numbers is introduced as being multiples of 2 after working with the concepts of divisor and multiple in the textbook for the fifth graders. But the English curriculum deals with the content area of 'recognizing sequences, including odd and even numbers to 30 then beyond' in the sub-area of 'counting' at key-stage 1. It seems that the English curriculum includes the concepts (terms) of even and odd numbers from an early key-stage because those numbers are quite often used in a wide range of real-life situations.

(2) Negative integers

In the Korean curriculum, the content of integers and four operations of integers including negative integers are introduced only at the seventh grade level. But in the English curriculum, as shown in Table 8, the content of negative integers is repeatedly dealt with and gradually developed until key-stage 4 (higher). In particular, the content of operations by negative numbers is introduced at key-stage 4 (higher).

Table 8. The content of negative numbers in England

The sub-areas of 'counting', 'integers', 'number operations and relationships between them'				
Key stage 1	**Key stage 2**	**Key stage 3**	**Key stage 4 (foundation)**	**Key stage 4 (higher)**
	• recognize and continue number sequences formed by counting on or back in steps of constant size from any integer, extending to negative integers when counting back			
	1.1.2., 7.1.7.			
	• order a set of negative integers, explaining methods and reasoning	• understand and use negative integers, both as positions and translations on a number line	• understand and use positive integers, both as positions and translations on a number line	• understand and use negative integers, both as positions and translations on a number line
7.1.7.	7.1.7.	7.1.7.	7.1.7.	
				• multiply and divide by a negative number
				7.1.8.

(3) Sequences

The Korean curriculum deals with the properties of natural numbers such as divisor, multiple, greatest common divisor, least common multiple, prime factorization at the seventh grade level, and it introduces the term of sequences in ‘Mathematics I’ for the first time, which is usually dealt with in the eleventh grade level. But in the English curriculum, the content of sequences is introduced at key-stage 3, involving the basic content areas of sequences such as ‘generating common integer sequences (including sequences of odd or even integers, squared integers, powers of 2, powers of 10, triangular numbers)’. Additional content areas of sequences are dealt with at key-stages 3 to 4 (higher).

4. Differences between the English and Korean curricula

(1) A base unit for dealing with number concept

In England, the first content of the sub-area of ‘counting’ is ‘counting reliably up to 20 objects at first’. Thus the English curriculum seems to deal with the concept of number based on the number 20, gradually extending to the number 100. But in Korea, the number concept is based on the number 10, extending to the number 100. The Korean curriculum never focuses on the number 20.

In Korea, the numbers are read as sets of 10. Namely, the numbers past 10 are built upon the number 10 and the numbers past twenty are built upon the number 20, and so forth.

1 (hana)	2 (dool)	3 (saet)	...	9 (ahob)	10 (youl)
11 (youl-hana)	12 (youl-dool)	13 (youl-saet)	...	19 (youl-ahob)	20 (smool)
21 (smool-hana)	22 (smool-dool)	23 (smool-saet)	...	29 (smool-ahob)	30 (sulhun)

But in England, numbers are read as a set of 20 and then sets of 10.

1 (one)	2 (two)	3 (three)	...	9 (nine)	10 (ten)
11 (eleven)	12 (twelve)	13 (thirteen)	...	19 (nineteen)	20 (twenty)
21 (twenty-one)	22 (twenty-two)	23 (twenty-three)	...	29 (twenty-nine)	30 (thirty)

In England, unlike Korea, the number 11 is not read as ‘ten-one’ but ‘eleven’ and also the number 12 is not read as ‘ten-two’ but ‘twelve’, and so forth. So, the reason for using different base units in the two curricula may stem from the differing number system of the two languages.

(2) Units

The English curriculum deals with the content area of ‘solving problems involving ratio and proportion, a range of measures and compound measures, metric units, and conversion between metric and common imperial units, set in a variety of contexts’ at key-stages 3 and 4 (foundation). But in Korea, metric units such as liters, grams, and meters are found in everyday life and so conversion between metric and common imperial units is not needed in the curriculum. However, in England, the content of conversion between metric and common imperial units seems to be meaningful in that imperial units seem to be used more often in everyday life.

(3) Other content areas

The English curriculum does not deal with the content of functions and equations in as much depth as the Korean curriculum. However, although the Korean curriculum deals with linear functions at the eighth grade level, in which y is given explicitly in terms of x, the English curriculum deals with the content of functions in which y is given explicitly in terms of x or ‘implicitly’ from key-stage 3.

On the other hand, the content areas of sets, binary systems, absolute/quadratic inequalities, rational/irrational functions, logarithms, matrices, complex numbers, sequence of differences, mathematical induction, fractional/irrational equations, differentiation and integration, and so forth are not dealt with in the English curriculum, whereas these content areas are all dealt with in the Korean curriculum.

. The summary of this study

In summary, the results of this study are as follows:

First, unlike Korea, where most of the content areas are dealt with only once, each of the content areas of the English curriculum tends to be repeated or developed further throughout each key-stage. For example, while the Korean curriculum handles the content of linear equations at the eighth grade level, the English curriculum deals with the content of linear equations with integer coefficients at key-stages 3 and 4 (foundation) and it gradually deals with the content of linear equations with integer coefficients or fractional coefficients at key-stage 4 (higher). In addition, while the Korean curriculum deals with the content areas of equivalent fractions and representing fractions with a common denominator, and so forth only at the elementary school level, the English curriculum deals with such content areas continuously according to the progress of key-stages.

Second, while the content of written methods is much emphasized and introduced early in the Korean curriculum, the distinct characteristic of the algebra domain of the English curriculum is the emphasis on approximation, mental methods, and calculator use. Because of this, the English curriculum seems to put

less importance or pressure in acquiring written methods, and tends to introduce the content of written methods a little later than the Korean curriculum.

Third, even though the English curriculum focuses relatively less on the number system through whole mathematics education, it introduces even numbers and odd numbers, negative integers, sequences, and so forth earlier than the Korean curriculum. It seems that the English curriculum includes such concepts (terms) from an early key-stage because those numbers are quite often used in a wide range of real-life situations. On the other hand, the content areas which can be considered as fundamental concepts of mathematics (namely, sets, binary systems, absolute/quadratic inequalities, rational/irrational functions, logarithms, matrices, complex numbers, sequences of differences, mathematical induction, fractional/irrational equations, differentiation and integration, and so forth) are not dealt with in the English curriculum, whereas these content areas are all dealt with in the Korean curriculum.

Fourth, while the Korean curriculum emphasizes more formal and abstract mathematical knowledge, based on a sound comprehension of specific mathematical terms and concepts, the English curriculum uses a more flexible approach to number, through the acquisition of rounding, mental methods, calculator use, ratio, proportion, and various sequences. In addition, the English curriculum emphasizes managing mathematical knowledge required to solve mathematics problems related to real life situations, and for this reason some content areas are introduced rather earlier than in Korea and are continuously emphasized.

Finally, in both Korea and England, all mathematical contents dealt with in elementary and secondary schools are given in the national mathematics curricula. But, the significant difference in these curricula is as follows: since in England the contents are given based on the stages covered in two or three years, students are given opportunities to learn mathematics more flexibly in aspects of both depth and speed, according to their own abilities or performance. However, in Korea from the first to the seventh curriculum periods, mathematical contents are given based on a semester system. Because of this reason, it is regrettable that all students in one grade are taught the same mathematical contents regardless of the individual student's mathematical ability or performance. Furthermore, in England, the attainment targets for math (consisting of eight level descriptions of increasing difficulty) set out the knowledge, skills and understanding that students of different abilities and maturities are expected to have by the end of each key stage. In Korea, since 1998, the national curriculum-based assessment standards for mathematics have been developed to implement a criterion-referenced evaluation in schools and to develop exemplary test items based on those standards. But, the standards have yet been established formally or nationally. The assessment standards are expected to be revised and updated, benchmarking the superior models of assessment standard such as attainment targets of England.

Appendix : Content areas of mathematics curriculum in Korea

In all of the tables, the first and second columns refer to grade and main area respectively. The third column represents content numbered according to grade, main area, and instructional order. For example, the content area corresponding to 1.2.1. means that the content area is introduced in the first grade, in the second main area called ‘letters and expressions’ and is the first content area instructed in this specific main area.

♦Elementary School Level

Grade	Main area	Content area
First	1. number and operation	1.1.1. understanding whole numbers up to 100 and of whole numbers up to 1000 (number concepts, comparing and ordering, basic concept of place value, etc.)
		1.1.2. using diverse methods for number counting (for example, counting on in steps of constant size)
		1.1.3. addition and subtraction of one-digit numbers
		1.1.4. addition and subtraction of two-digit numbers (with no regrouping)
		1.1.5. doing addition and subtraction in diverse ways, and explaining them [enriched content]
		1.1.6. problem-solving on addition and subtraction
	2. letters and expressions	1.2.1. understanding the meaning of in addition and subtraction expressions
		1.2.2. understanding problem-solving strategies such as drawing a picture and making an expression
		1.2.3. posing problems according to simple expressions on addition and subtraction [enriched content]
	3. patterns and functions	1.3.1. searching for patterns embedded in sequences of real objects and figures
		1.3.2. arranging real objects and figures according to own rules
		1.3.3. searching for rules embedded in the table of numbers from 1 to 100

Grade	Main area	Content area
Second	1. number and operation	2.1.1. understanding whole numbers up to 1000
		2.1.2. addition and subtraction of two-digit numbers, and problem solving on addition and subtraction, related to real-life situations
		2.1.3. introduction of multiplication
		2.1.4. multiplication table, multiplication of one-digit numbers
		2.1.5. addition and subtraction of three-digit numbers
		2.1.6. problem-solving on addition, subtraction, and multiplication
		2.1.7. doing addition and subtraction in diverse ways, and explaining them [enriched content]
	2. letters and expressions	2.2.1. finding the value in addition or subtraction expressions including the value
		2.2.2. posing problems properly according to given expressions
		2.2.3. converting word problems to expressions
		2.2.4. finding the unknowns in equations of addition, subtraction, and multiplication
		2.2.5. understanding problem-solving strategies such as making a table and working backwards
	3. patterns and functions	2.3.1. searching for patterns embedded in the variation of real objects or figures, and explaining the situation
		2.3.2. searching for rules of counting on in steps of constant size in number sequence including numbers up to 100
		2.3.3. finding out the unknown numbers included in a given number sequence
		2.3.4. searching for various rules embedded in multiplication table (facts)
		2.3.5. constructing 12x12 multiplication table (facts) by the use of patterns [enriched content]

Grade	Main area	Content area
third	1. number and operation	3.1.1. understanding whole numbers up to 1000
		3.1.2. addition and subtraction of three-digit numbers, and their applications
		3.1.3. addition and subtraction of four-digit numbers
		3.1.4. understanding division
		3.1.5. (two-digit numbers) x (one-digit numbers), (two-digit numbers) ÷ (one-digit numbers) (no reminders)
		3.1.6. (three-digit numbers) x (one-digit numbers), (two-digit numbers) x (two-digit numbers), (two-digit numbers) ÷ (one-digit numbers) (including reminders)
		3.1.7. problem-solving on multiplication and division
		3.1.8. posing problems on multiplication and division and solving them
		3.1.9. understanding fractions
		3.1.10. understanding the concepts and relationships of unit fractions and proper fractions
		3.1.11. understanding decimal fractions (down to 10th)
		3.1.12. problem-solving on fractions, decimals [enriched content]
	2. letters and expressions	3.2.1. understanding problem-solving strategies such as looking for a pattern, guess and check
		3.2.2. explaining problem-solving process
	3. patterns and functions	3.3.1. constructing various figures by arranging plane shapes according to own rules
		3.3.2. constructing various figures by arranging two plane shapes [enriched content]

Grade	Main area	Content area
fourth	1. number and operation	4.1.1. understanding whole numbers greater than ten thousand (reading, writing, comparing and ordering)
		4.1.2. understanding decimal numeration system
		4.1.3. addition and subtraction of natural numbers, multiplication of two-digit numbers, division of two-digit numbers, mixed calculations (operations) of natural numbers
		4.1.4. understanding various fractions (proper fractions, improper fractions, and mixed fractions)
		4.1.5. addition and subtraction of fractions with equal denominators
		4.1.6. posing problems by the use of data on large numbers and solving them [enriched content]
		4.1.7. representing fractions according to comparison of two quantities, and
		4.1.8. representing (natural numbers) ÷ (natural numbers) as fractions
		4.1.9. understanding decimals with three decimal places
		4.1.10. comparing and ordering of fractions and decimals
		4.1.11. addition and subtraction of decimals
		4.1.12. solving real-life problems on fractions and decimals [enriched content]
	2. letters and expressions	4.2.1. understanding problem-solving strategies such as making a simplification
		4.2.2. explaining problem-solving process
		4.2.3. posing problems on simple mixed calculations (operations) and solving them [enriched content]
		4.2.4. selecting an appropriate problem-solving strategy for a given problem, and solving the problem using the selected strategy
		4.2.5. solving a problem with two or three strategies and comparing them [enriched content]
	3. patterns and functions	4.3.1. representing patterns of various changes by numbers and explaining them
		4.3.2. rearranging patterns of various changes represented by numbers, using the concrete materials [enriched content]
		4.3.3. guess the rules and representing them through talking or writing
		4.3.4. understanding the correspondence through corresponding table, and explaining its rule
		4.3.5. finding the corresponding rules between two variables, and solving problems according to its rules

Grade	Main area	Content area
fifth	1. number and operation	5.1.1. finding divisors and multiples, common factors and common multiples, greatest common factors, greatest common multiples, and problem-solving through understanding of their relationships
		5.1.2. posing problems on common factors and common multiples, related to real-life situations, and solving them [enriched content]
		5.1.3. understanding equivalent fractions, simplifying fractions by canceling common factors, and converting simple fractions to fractions with a common denominator, and comparing and ordering fractions with different denominators
		5.1.4. addition and subtraction of fractions with different denominators
		5.1.5. multiplication of natural numbers and fractions, multiplication of unit fractions, multiplication of proper fractions, multiplication of mixed fractions
		5.1.6. multiplication of decimals and natural numbers, multiplication of decimals and decimals
		5.1.7. (fractions) ÷ (natural numbers), (natural numbers) ÷ (natural numbers) (quotient is decimal), (decimals) ÷ (natural numbers)
		5.1.8. posing problems on multiplication and division of fractions and decimals and solving them [enriched content]
	2. letters and expressions	5.2.1. selecting an appropriate problem-solving strategy for a given problem, and solving the problem using the selected strategy
		5.2.2. Estimating validity of problem-solving process
		5.2.3. solving a problem using two or three strategies, and comparing them [enriched content]
		5.2.4. posing new problems by changing the conditions for a given problem and solving them [enriched content]
	3. patterns and functions	5.3.1. constructing new figures by slide, flip, and rotation of one figure
		5.3.2. constructing new figures by slide of two kinds of figures [enriched content]

Grade	Main area	Content area
sixth	1. number and operation	6.1.1. understanding relationships of decimals and fractions, and converting fractions to decimals and converting decimals to fractions
		6.1.2. comparing decimals and fractions
		6.1.3. problem-solving through the understanding of relationships between decimals and fractions
		6.1.4. division by a fraction, a decimal
		6.1.5. mixed calculations (operations) of simple fractions and decimals
		6.1.6. posing problems on mixed calculation (operations) of fractions and decimals, related to real-life situations [enriched content]
	2. letters and expressions	6.2.1. comparing various problem-solving strategies and selecting an appropriate strategy for a given problem, and solving the problem by the use of the selected strategy
		6.2.2. organizing problem-solving process and explaining its validity
		6.2.3. posing problems, by finding materials related to real-life situations and solving them
	3. patterns and functions	6.3.1. understanding ratio and rate between two quantities, representing rates by various ways
		6.3.2. finding various examples of rates in real-life situations and solving problems related to them [enriched content]
		6.3.3. understanding proportional expressions and their applications
		6.3.4. representing correspondence of two numbers to an expression by the use of , Δ
		6.3.5. understanding continued ratio and representing relationships between three quantities to continued ratio, and understanding proportional distribution and distributing the given quantities by proportion
		6.3.6. posing problems through the understanding of correspondent relationships represented by expressions, and solving them [enriched content]

♦ **Middle School Level**

Grade	Main area	Content area
seventh	1. number and operation	7.1.1. understanding sets and inclusion relation among sets
		7.1.2. operations of sets
		7.1.3. prime factor decomposition of natural numbers
		7.1.4. finding greatest common factor and least common multiple
		7.1.5. problem solving on greatest common factor and least common multiple, related to real-life situations [enriched content]
		7.1.6. understanding relationships between decimal system and binary system
		7.1.7. understanding the concepts of integers and rational numbers
		7.1.8. operations of integers and rational numbers
	2. letters and expressions	7.2.1. simplifying expressions using letters
		7.2.2. finding numerical values of expressions, calculating expressions of the first degree
		7.2.3. finding solutions of linear equations, understanding properties of equality
		7.2.4. problem-solving on linear equations, related to real-life situations [enriched content]
	3. patterns and functions	7.3.1. understanding relationship between direct and inverse proportions, and representing their relationship as expressions
		7.3.2. understanding the concept of functions
		7.3.3. understanding ordered pair and coordinate, and drawing the graphs of functions
		7.3.4. problem-solving on functions, related to real-life situations
		7.3.5. finding function relation in real-life situations and representing them as expressions [enriched content]

Grade	Main area	Content area
eighth	1. number and operation	8.1.1. converting rational numbers to decimals
		8.1.2. understanding rational numbers and repeating decimals
		8.1.3. comparing repeating decimals [enriched content]
	2. letters and expressions	8.2.1. addition and subtraction of polynomials
		8.2.2. understanding powers and their properties
		8.2.3. (monomial) x (polynomial), (polynomial) ÷ (monomial)
		8.2.4. understanding linear equations with two unknowns
		8.2.5. finding solution of simultaneous linear equations with two unknowns
		8.2.6. applications of simultaneous linear equations with two unknowns
		8.2.7. understanding the properties of linear inequalities and finding their solutions
		8.2.8. solving simultaneous linear inequalities
		8.2.9. problem-solving on linear inequalities or simultaneous linear inequalities
		8.2.10. problem-solving on equations and inequalities, related to real-life situations [enriched content]
	3. patterns and functions	8.3.1. understanding linear functions and drawing their graphs, and understanding of their properties
		8.3.2. understanding relationship between linear functions and linear equations
		8.3.3. finding solutions of simultaneous linear equations through their graphs
		8.3.4. problem-solving on linear functions
		8.3.5. problem-solving on linear functions, related to real-life situations [enriched content]

Grade	Main area	Content area
ninth	1. number and operation	9.1.1. understanding square root and its properties
		9.1.2. understanding irrational numbers
		9.1.3. understanding order relation of real numbers on a number line
		9.1.4. four operation of expressions involving square roots, and rationalization of a denominator
		9.1.5. Investigating the ways to find any real number between arbitrary two numbers [enriched content]
	2. letters and expressions	9.2.1. multiplication of polynomials and factorization
		9.2.2. multiplying polynomials by substituting parts of a given expression [enriched content]
		9.2.3. understanding quadratic equations and finding their solutions
		9.2.4. problem-solving on quadratic functions
	3. patterns and functions	9.3.1. understanding quadratic functions and drawing their graphs, understanding their properties (maximum and minimum values)
		9.3.2. finding the values of a, b, c through the understanding of the graphs of $y = ax^2 + bx + c$ [enriched content]

♦ **High School Level**

Grade	Main area	Content area
tenth	1. number and operation	10.1.1. understanding laws of set operations
		10.1.2. understanding inverse, converse, contrapositive
		10.1.3. understanding necessary condition and sufficient condition
		10.1.4. understanding the properties of operations of real numbers, and comparing and ordering real numbers
		10.1.5. understanding complex numbers and their operations, and their properties
		10.1.6. investigating closed or not closed under the four operations in set of any numbers [enriched content]
	2. letters and expressions	10.2.1. operations of polynomials
		10.2.2. understanding identities and reminder theorem
		10.2.3. factorization of polynomials
		10.2.4. divisors and multiples of polynomials
		10.2.5. understanding rational and irrational expressions and their operations
		10.2.6. understanding discriminant of quadratic equations and relationship between root and coefficient
		10.2.7. solving simple cubic equations and biquadratic equations
		10.2.8. solving simultaneous linear equations with 3 unknowns and simultaneous quadratic equations with 2 unknowns
		10.2.9. solving linear inequalities with absolute values
		10.2.10. solving quadratic inequalities and simultaneous quadratic inequalities
		10.2.11. proof of simple absolute inequalities
		10.2.12. problem solving on equations and inequalities, related to real-life situations [enriched content]
	3. patterns and functions	10.3.1. understanding the concept of functions and their graphs
		10.3.2. understanding composite functions and inverse functions, and finding them
		10.3.3. understanding the maximum, minimum values of quadratic functions
		10.3.4. understanding relationship between a graph of quadratic function and a straight line
		10.3.5. understanding relationships between a quadratic function and a quadratic equation, a quadratic inequality
		10.3.6. understanding rational functions and irrational functions and drawing their graphs
		10.3.7. understanding coterminal angles and radians
		10.3.8. understanding trigonometric functions (sine, cosine, tangent) and their graphs
		10.3.9. solving trigonometric equations and inequalities
		10.3.10. understanding the laws of sine and cosine
		10.3.11. finding the area of triangle using trigonometric function
		10.3.12. investigating periodic situations in natural phenomenon and applying them to trigonometric functions [enriched content]

Grade	Main area	Content area
eleventh	1. algebra	11.1.1. understanding powers and radical roots and their properties
		11.1.2. extension of exponents involving real numbers
		11.1.3. understanding logarithms and their properties
		11.1.4. understanding common logarithms, understanding characteristic and mantissa and their application
		11.1.5. understanding matrices and their operation
		11.1.6. finding inverse of 2×2 matrix
		11.1.7. solving simultaneous linear equations using inverse matrix
		11.1.8. finding general terms of arithmetic sequences, geometric sequences, and various sequences and their sums
		11.1.9. understanding mathematical induction and its application to proof
		11.1.10. understanding algorithm and flow-chart
	3. analysis	11.3.1. discriminating convergence and divergence of infinite sequences and finding limits of infinite sequences
		11.3.2. finding limits of infinite geometric sequences
		11.3.3. finding sums of infinite geometric series, and solving diverse problems on infinite geometric series
		11.3.4. understanding the graphs of logarithmic and exponential functions and their properties
		11.3.5. solving logarithmic and exponential equations, and logarithmic and exponential inequalities

Grade	Main area	Content area
twelvth	1. algebra	12.1.1. solving fractional equations and their application
		12.1.2. solving irrational equations and their application
		12.1.3. solving simple cubic inequalities and biquadratic inequalities
		12.1.4. solving fractional inequalities and their application
	3. analysis	12.3.1. understanding limit of functions and their properties, and finding limit values of diverse functions
		12.3.2. understanding continuous functions and their properties
		12.3.3. understanding differential coefficient, and relationship between differentiable and continuity
		12.3.4. finding derivatives of functions
		12.3.5. application of derivatives (tangent line, increase and decrease of a function, extreme value, velocity and acceleration, etc.)
		12.3.6. understanding indefinite integral, and finding integral of scalar product, addition and subtraction of functions
		12.3.7. understanding measurement by subdivision
		12.3.8. understanding basic theorems of definite integral and their application
		12.3.9. application of definite integral (area surrounded with curves, volume of a solid revolution, velocity and distance, etc.)