Mathematics Teaching and Learning in Vietnam

by

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1. Introduction to Educational Framework in Vietnam

1.1 The structure of the educational system in Vietnam is summarised below:

Age	*Year*		
22 +		Colleges, Universities	
21 +			
20 +			
19 +			
18 +			General Certificate of Secondary Education Examination (6 subjects taken)
17 +	12	Senior Secondary School	
16 +	11		
15 +	10		Examination (6 subjects taken)
14 +	9	Junior Secondary School	
13 +	8		
12 +	7		
11 +	6		Examination (Vietnamese, Mathematics)
10 +	5	Primary School	
9 +	4		
8 +	3		
7 +	2		
6 +	1		
5 +	'Young Shoot' education		

Primary schools admit children from 6 years of age calculated according to the year of birth. Grade 1 is the first grade of primary education which comprises five grades (grades 1 - 5).

Junior secondary schools admit children from the age of 15 and consist of three grades (grades 10 - 12).

Senior secondary schools admit children from the age of 15 and consist of three grades (grades 10 - 12).

Gifted children may be admitted into primary or secondary before the specified age at the recommendation of a special council(set up by the head of the District Education Department).

Higher Education (Colleges, Universities and Postgraduate Education): the duration of the course of study is 3 years for colleges, 4 to 6 years for universities.

To get the MA degree, the candidate should have graduated from a university; the duration of study is 2 years. For the Doctor's degree, the duration of study is 3-4 years or more.

1.2 Primary Education in Vietnam

(a) The universalisation of Primary education was a major policy of the Vietnamese State throughout the past 50 years and more. In 1945, the Government issued a Decree on the national education system in which primary education was regarded as fundamental compulsory level for all children from 7 to 13 years of age. However, the outbreak of war prevented its implementation.

In 1989, the Government set up the National Committee responsible for the implementation of the programme of literacy education and universalisation of Primary education.

In 1991 Vietnam promulgated the Law on the universalisation of Primary education and asserting the State's commitment to a compulsory and free primary education for all children.

(b) The network of primary schools

The Vietnamese population is now about 76 million, with 11 million pupils in Primary schools, but there are only 11 683 primary schools. Primary schools exist in all communes down to hamlets. In the cases of communes covering a wide area, school sub-divisions were set up in hamlets to make it easier for small children to attend.

There are two main types of schools:

- *State schools* : organised and managed by the State (98% of pupils attend these schools).
- *Private schools* : organised and managed by individuals.

(c) Teaching Plan

The Primary school year is 33 weeks (from 5 September 1999 to 20 May 2000 in the current year). Sundays and Thursdays are days off. Each lesson lasts 40 minutes.

Since the 1994 -1995 school year, Primary Schools have launched a movement for the teaching of all nine subjects (see table below) and stipulated that in the year 2000, all primary schools should teach all the nine subjects.

In reality, because of poor material facilities, shortage of teachers and difficulty of preparing enough textbooks, in many schools, especially in those places where teaching is given in three shifts a day, only 4 subjects are being taught at present, namely:

- *Vietnamese*
- *Mathematics*
- *Ethics*
- *Nature and Society.*

A number of schools have managed to teach a greater number of subjects, a few of them are teaching all nine subjects.

Weekly Number of Lessons in Primary Education

	Subject	*Grade*				
		1	**2**	**3**	**4**	**5**
1	Vietnamese	12	10	10	8	8
2	Mathematics	3	4	5	5	5
3	Ethics	1	1	1	1	1
4	Labour	1	2	2	3	3
5	Nature and Society	1	1	1	2	2
6	Songs - Music	1	1	1	1	1
7	Fine Art	1	1	1	1	1
8	Physical Education	1	1	1	1	1
9	Gymnastics	1	1	1	1	1

(d) Curriculum and textbooks

After 1975 in Vietnam the two zones - South and North - had two different systems of general education with a duration of 11 years in the North and 12 years in the South.

Therefore, with the third education reform (started in 1982), Vietnam actually has a unified national education and Vietnamese schools had a unified National Curriculum; according to the Law on the universalisation of Primary education (promulgated in 1991) Primary Curriculum defined by Government.

In the 1981-1982 school year, the new textbooks (called 'Reformed' books) were introduced into the first grade and each year subsequently so that in the 1985 -1986 school year, the textbooks were replaced in the 5th grade. Then, for the first time in several decades, in all general schools (Primary and Secondary) in Vietnam, teaching was given according to a single programme and with a single set of textbooks. The set of textbooks of the education reform, after a number of adjustments, were more progressive than the old ones and will be used until the year 2000.

2. Developing the Primary Mathematics Curriculum for 2000

2.1 Introduction to Mathematics Education in Vietnamese Primary Schools

(a) After 1945, Vietnamese general eduction had three education reforms, therefore it had several general education curricula issued by the Educational Ministry.

In particular, Primary Mathematics Curriculum was issued in 1950, 1956 and 1981 (so-called 'Reformed Curriculum'). Each curriculum was followed by new mathematics textbooks and other teaching and learning materials.

In addition, during the 1980's under the influence of Davydov's school (former Soviet Union) there was an experimental curriculum for teaching some concepts of modern mathematics for Primary pupils. Unfortunately, this experimental curriculum was quickly judged a failure and created unhelpful debates.

(b) In 1981 the Reformed Curriculum and a set of Mathematics textbooks were developed by a team headed by Professor Phan Van Hoan at the NIES (National Institute of Educational Sciences). These documents restructured and enlarged the topics of the previous curriculum and textbooks for Mathematics, with five main topic fields:

A *Arithmetic: natural numbers; fractions; decimal numbers and operations with numbers*

B *Quantities and measuring of quantities*
The introduction to basic quantities : units for measuring; calculating with these units.

C *Algebraic factors*
Solving equations formed : finding missing numbers in operations of such forms as :

$$x + a = b; \quad x - a = b; \quad a - x = b$$

$$x\,a = b; \quad \frac{x}{a} = b; \quad \frac{a}{x} = b$$

Solving simple inequation formed:

$$x < a, \quad a < x < b$$

D *Geometric factors*
Introduction of simple shapes and space; calculating perimeter, area and volume of some simple shapes and solids.

E *Problem solving*: solving problems with words or stories.

The Reformed Curriculum and set of textbooks (to be used until the year 2000) has achieved **main successes**:

- it has constructed the subject 'Mathematics' with five main topic fields expounded above. Before 1981, in Primary schools there was only the subject 'Calculation' with main targets to train calculating skills in four operations: addition, subtraction, multiplication and division;
- the modernisation of Mathematics teaching content in Primary schools implemented by:
  - presenting traditional Mathematics content under Modern Mathematics influence (but using clear language and symbols of Modern Mathematics),
  - constructing Primary Mathematics Curriculum as the first part of unified Mathematical subject from Primary schools to Secondary schools;
- supplying Primary students with a basic, practical system of knowledge and skills, closely linked to calculating activities (mental and oral, writing calculation and problem solving).

However, in this curriculum there are still some weaknesses:

- there is an imbalance between teaching time for natural numbers and for decimal numbers. Time for teaching natural numbers starts from the beginning of the first year to half of the first term of year 5, but time for teaching decimal numbers only starts at the end of the first term of year 5. Therefore pupils' calculating skills with decimal numbers are weak.
- In most Primary schools, teaching methods are explained and illustrated. The teacher mainly imparts content and knowledge, pupils learning by the examples. The majority of teachers generally prefer explanations, lectures and samples with frequent incidental questioning. The teachers are not monitored, they do not help pupils to create problems and 'occupy' new knowledge.
- Teaching content is not updated.
- Modernisation of the Mathematics teaching content has taken place but there had been no reference to teaching methods and equipment - so some teachers do not understand the ideas for teaching in the new textbooks.

2.2 Teaching Style

(a) In Vietnam almost all teachers use the teaching style:

- Concepts are taught first, followed by lots of practice but with little context. The teacher does 99% of the talking in highly structured lessons on order to complete daily objectives. Much of the learning relies on homework and there are enrichment classes (extra work) after school.
- Arrangement of seating most often seen in primary classrooms is that of pupils in rows.
- In general, the Mathematics lesson time is spent on:

  Teaching whole class: 90% (teacher mainly using explain-illustrated methods to whole class)

  Individual work: 8%

  Group work: 2%
- At every lesson pupils work at the board in front of the class during Mathematics lessons.

- Individual pupil mistakes in Mathematics are usually dealt with individually.
- Addition and multiplication facts are emphasised on learning by heart.
- In every lesson (especially Year 4 and Year 5), homework is set after every mathematics lesson.

(b) In Vietnam, reform of teaching methods in Primary schools began in 1992. In Mathematics education in particular, this has included the following :

- reform of the perception (cognition) of teachers, helping them to be aware of the need to improve teaching methods by the following approaches: to concentrate on learners; to emphasise active learning; to develop pupils' initiative and creativity; to individualise pupils' studies.
- introduction of new forms of teaching: individual learning; cooperative learning; educational games; problem solving; constructivism. Although the teaching can sometimes be monotonous and boring, it is always focused and adheres to the philosophy *concrete, pictorial, abstract* (CPA).
- introduction of new meaning of learning; mainly worksheets and assessment papers, encouraging students to use individual learning instruments.
- improvement of the learning environment.
- improvement of assessment by using tests and assessment papers.

Both teachers and pupils have confirmed good results of the reforms of teaching methods, in particular:

- the effectiveness of Mathematics education has risen remarkably: the number of exercises done has increased threefold. Pupils often complete all the exercises in textbooks in class.
- pupils have studied more actively, confidently and creatively. Teachers have been more active and creative.
- Mathematics classes have been more lively.

2.3 Development of Primary Mathematics Curriculum for 2000

In January 1996 the Ministry of Education and Training (MOET) set up a Committee for the Development of Primary Curriculum for the year 2000. The Mathematics objectives were developed by a team headed by Professor Do Dinh Hoan at National Institute of Educational Sciences.

Development Primary Curriculum for year 2000 is not only a chance for the reform of teaching content and methods but also a chance for the reform of teaching materials and equipment, training teachers and assessment.

We hope that the Primary Curriculum for the year 2000 inherits and develops the achievements of Mathematics educational tradition in Vietnam, improves Mathematics teaching and learning, using the same effective strategies in Mathematics education in regional countries and the world.

a) The aim of the Primary Mathematics Curriculum for 2000 is to provide pupils with:

- basic applicable knowledge and skills, necessary to their lives in the community and future;
- thinking and studying methods. Develop pupils' problem-solving abilities.

b) Reform of Contents implemented through :

- improving the curriculum: teaching natural numbers mainly in first three years; teaching fractions and operations in Year 4; teaching decimal and operations and percentage and introduction of calculators in Year 5;
- adjusting teaching levels in each year (mainly Years 4 and 5); balancing teaching content between years; avoiding unnecessary difficulties;
- looking forward to the development of students for the year 2000 and raising standards, so revising some relevant content. For example, in Year 1, not only teaching numbers up to 10 as at present, but numbers up to 100 including addition and subtraction; teaching fractions earlier, from Year 2; increasing the realism of problems; teaching more on shape and space such as the cylinder and sphere.

c) Primary Mathematics Curriculum for the year 2000 also aims to reform teaching methods. This includes the following :

- reform of the cognition of teachers. Every teacher could become a good innovator, should improve teaching methods by the following approaches: concentrating on learners; emphasising active learning; developing students' initiative and creativity; individualising students' studies;
- introduction of new forms of teaching individual learning; work in pairs; group work; educational games; problem solving;
- introduction of new means of learning, mainly work sheets and assessment papers; encouraging students to use individual instruments;
- improvement of the learning atmosphere and environment;
- improvement of assessment by using tests and assessment papers.

3. Examination Papers used in Vietnam

The appendix gives sample exam papers for :

(a) End of Year 3 (age 9)

(b) End of Year 5 (age 11)

(c) Certificate of Lower Secondary School (age 15)

(d) National Examination (age 18)

(e) Entrance Examination for National University of Hanoi (age 18)

APPENDIX

(a) Sample Examination paper for Year 3 (Age 9)

VIETNAM
Ministry of Education and Training
National Institute for Educational Sciences
Project 'Assess attainment quality in Primary Education'

Time allowed: 40 minutes

Give the correct responses:

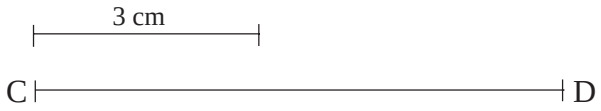
1. The next number after 39 is:

A. 38
B. 49
C. 29
D. 40

2. $79 + 13 - 46 =$

A. 35
B. 36
C. 46
D. 56

3. What is the approximate length of the line C D ?
(Do not use a ruler.)

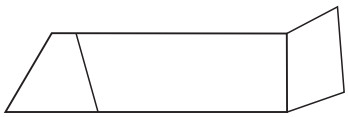


A. About 5 cm
B. About 10 cm
C. About 7 cm
D. About 15 cm

4. $128 \times 7 =$

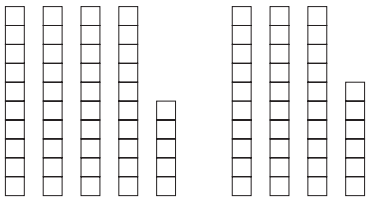
A. 746
B. 796
C. 896
D. 846

5. How many quadrilaterals are shown in the diagram?



A. 2
B. 5
C. 3
D. 4

6. How many squares are there in the diagram below?



A. 80
B. 90
C. 71
D. 81

5. What number is 10 less than 55 ?

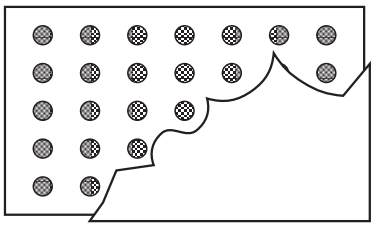
A. 56
B. 65
C. 45
D. 54

8. 5 m 3 cm = $\square$ cm

The number missing from the square is:

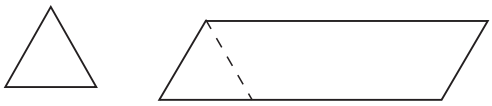
A. 8
B. 53
C. 503
D. 530

9. Lan put some cakes on a tray, with equal numbers in each row, and then covered some of them with a cloth. How many cakes are there on the tray?



A. 30 cakes
B. 28 cakes
C. 22 cakes
D. 35 cakes

10. How many triangles of shape (1) are needed to make shape (2) ?



Shape 1 Shape 2

A. 3 triangles
B. 4 triangles
C. 6 triangles
D. 8 triangles

11. $6 + 6 + 6 + 6 + 6 + 6 + 6 = 18 + \square \times 6$

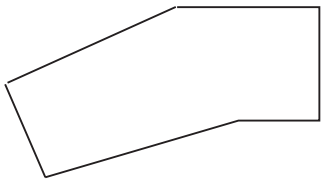
The number missing from the square is:

A. 2
B. 3
C. 4
D. 5

12. There are 263 people. A boat can carry 7 people. How many times does the boat have to cross the river to transport all the people?

A. 37 times
B. 38 times
C. 40 times
D. 39 times

13. How many right angles are there in this diagram?



A. 2
B. 3
C. 4
D. 5

14. One morning a car travelled 345 km. That evening it travelled a distance 5 times less than it travelled in the morning. How far did the car travel that day?

A. 685 km
B. 304 km
C. 414 km
D. 404 km

15. The sum $258 + 365$ gives:

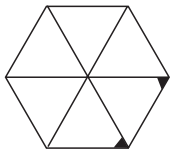
A. 513
B. 613
C. 523
D. 623

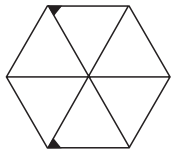
16. Two lengths of cloth are each 8 m long.
A third length is 4 m long.

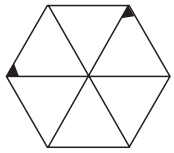
What is the total length of the three pieces of cloth?

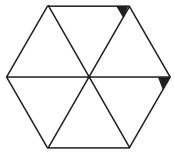
A. 12 m
B. 32 m
C. 16 m
D. 20 m

17. Which shape is a rotation of the shape below:

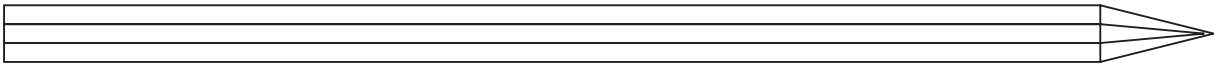
A. 

B. 

C. 

D. 

18. Use a ruler to measure the length of the pencil.



The length of the pencil is:

A. 16 mm
B. 14 cm
C. 17 cm
D. 160 mm

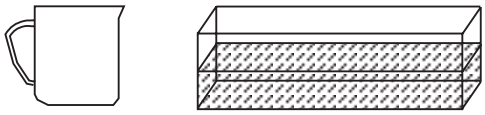
19. The result of the calculation,

$$127 \times 6 + 153 =$$

is:

A. 915
B. 815
C. 775
D. 715

20.



Three jugs of water have filled the glass container to the level shown. How many jugs of water are needed to completely fill the container?

A. 4 cans
B. 5 cans
C. 6 cans
D. 7 cans

21. $125 \div 5 = 5 \times \square$

The missing number is:

A. 25
B. 30
C. 5
D. 20

22. 30 August is a Sunday.

What day of the week is 2 September?

A. Monday
B. Tuesday
C. Wednesday
D. Thursday

23. There are 3 packets of sweets.
The first weighs 124 g.
The second weighs twice as much as the first.
The third is lighter than the second by 10 g.

What is the total weight of the packets of sweets, in grams?

A. 630 grams
B. 137 grams
C. 372 grams
D. 610 grams

24. $27 \div 9 = 18 - \square$

The missing number is:

A. 21
B. 15
C. 14
D. 3

25. A car travels 309 km in 3 hours.

How far does the car travel in 8 hours?

A. 104 km
B. 824 km
C. 804 km
D. 84 km

26. $\square 8 \div 6 = 8$

The missing number is:

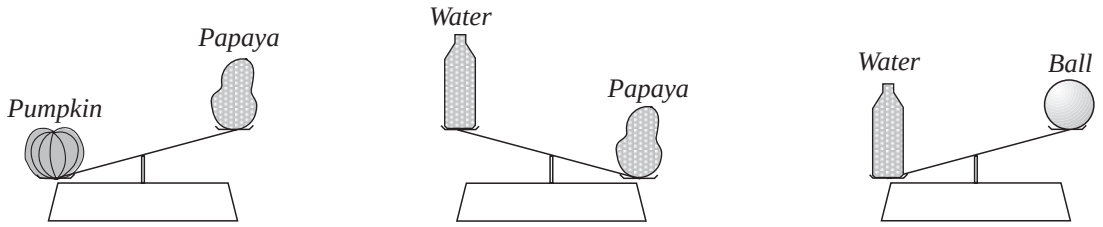
A. 4
B. 5
C. 6
D. 3

27. $30 < 6 \times \square < 40$

The missing number is:

A. 5
B. 6
C. 7
D. 8

28. Which of the items is heaviest:

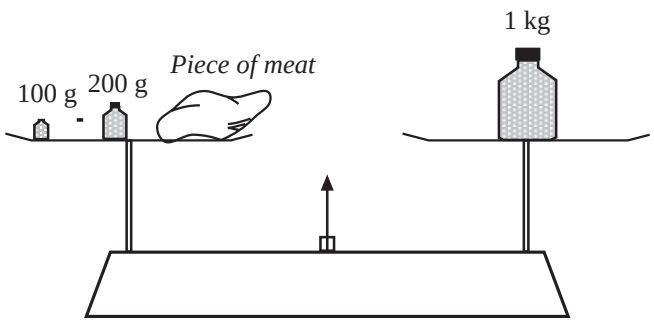


A. Pumpkin
B. Papaya
C. Water
D. Ball

29. Lan went to bed at 9 p.m. and slept for 9 hours.
At what time did Lan wake up the next day?

A. 18:00
B. 5 a.m.
C. 6 a.m.
D. 7 a.m.

30. What is the weight, in grams, of the piece of meat?



A. 1000 grams
B. 700 grams
C. 100 grams
D. 301 grams

APPENDIX

(b) Sample Examination paper for Year 5 (Age 11)

VIETNAM : Test for 11-year-olds in Hanoi (1999)

Year 5

Time : 60 minutes

Calculators NOT allowed

Give the correct responses:

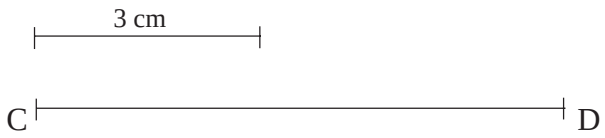
1. The next number after 39 is:

A. 38
B. 49
C. 29
D. 40

2. The largest number in this set (389, 938, 839, 893) is:

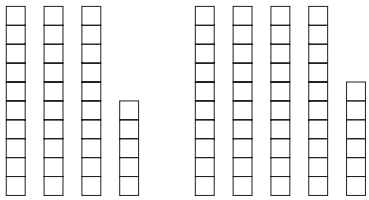
A. 938
B. 839
C. 893
D. 389

3. What is the approximate length of the line C D ?
(Do not use a ruler.)



A. About 5 cm
B. About 10 cm
C. About 7 cm
D. About 15 cm

4. How many squares are there in the diagram below?



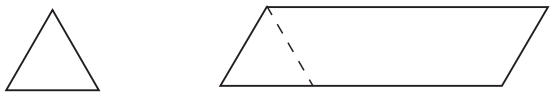
A. 80
B. 90
C. 71
D. 81

5. Which numbers should you write in the square so that
$\square 28 \times 3$ is greater than 700 and smaller than 1000 ?

A. 1
B. 2
C. 3
D. 4

6. Which distance is the greatest?

A. 1 cm
B. 1 km
C. 1 mm
D. 1 m

7. 

Type 1 Type 2

How many triangles of Type (1) are used to make the shape Type (2)?

A. 3
B. 4
C. 6
D. 8

8. 333 $\square$ < 3331

The number missing from the square is:

A. 3
B. 2
C. 1
D. 0

9. The number missing from the square is:

A. 2
B. 3
C. 4
D. 5

10. 2319, 1932, 2139, 2391, 1293

Which of these numbers is greater than 1932 and smaller than 2319 ?

A. 1932
B. 2319
C. 2139
D. 2391

11. Two pieces of cloth are each 8 m long. A third piece is 4 m long. How many metres of cloth are there altogether?

A. 12 m
B. 32 m
C. 16 m
D. 20 m

12. 0.4 equals:

A. four
B. four tenths
C. four hundreds
D. four hundredths

13. $13 \rightarrow 16 \rightarrow 19 \rightarrow 22 \rightarrow \bigcirc$

The number missing from the empty circle is:

A. 23
B. 24
C. 25
D. 26

14. If 9 February is a Tuesday, what day of the week is 30 January?

A. Thursday
B. Friday
C. Saturday
D. Sunday

15. The sum $258 + 365$ gives:

A. 513
B. 613
C. 523
D. 623

16. $112\square \div 49 = 23$

The missing number is:

A. 21
B. 15
C. 14
D. 3

17. $27 \div 9 = 18 - \square$

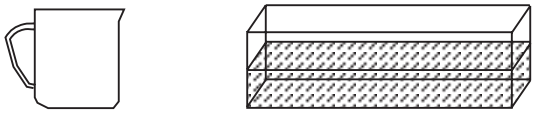
The missing number is:

A. 21
B. 15
C. 14
D. 3

18. The result of $\frac{3}{5} - \frac{1}{3}$ is:

A. $\frac{1}{5}$
B. $\frac{2}{2}$
C. $\frac{2}{5}$
D. $\frac{4}{15}$

19.



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A. 4 cans

B. 5 cans

C. 6 cans

D. 7 cans

20. The result of the calculation,

$$A = 20\,002\,000 + 200\,200 + 2020$$

is:

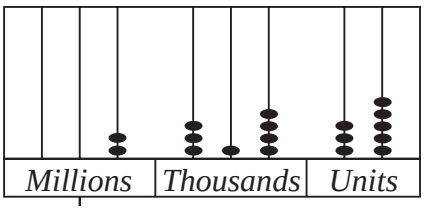
A. A = 6 000 600

B. A = 60 222 000

C. A = 20 204 220

D. A = 20 202 020

21. What number do the beads show:



A. 231 435

B. 2 314 305

C. 288

D. 18

22. 3×23 equals:

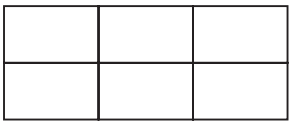
A. 323

B. 233

C. 69

D. 26

23.



How many right angles are shown in the diagram?

A. 12 right angles

B. 24 right angles

C. 18 right angles

D. 6 right angles

24. The result of the subtraction,

$$\begin{array}{r} 6000 \\ 2369 - \\ \hline \end{array}$$

is:

A. 4369
B. 3742
C. 3631
D. 3531

25. The number eighteen point four five is:

A. 108.405
B. 18.405
C. 18.45
D. 108.45

26. Multiplication of 4003 by 7 gives:

A. 300 021
B. 280 021
C. 28 021
D. 30 021

27. The average of the numbers

10, 20, 30, 40, 50, 60, 70, 80, 90, 100

is:

A. 550
B. 110
C. 55
D. 50

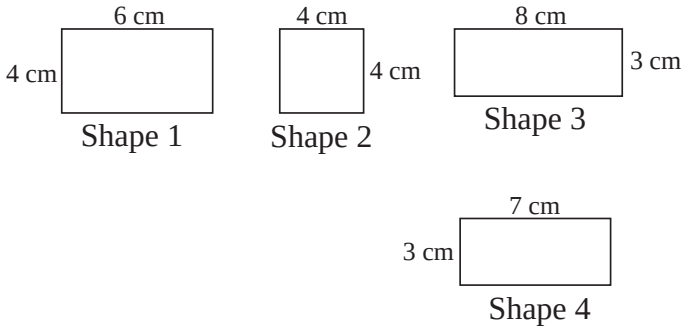
28. One litre of water weights 1 kg.
An empty vase weighs 200 g.

$\frac{1}{2}$ litre of water is poured into the vase.

What is the total weight, in grams, of the vase and the water?

A. 200 gm
B. 1200 gm
C. 700 gm
D. 500 gm

29. Which shape has an area smaller than 20 cm?



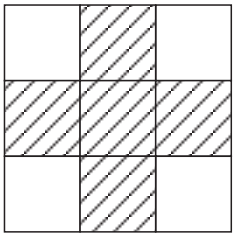
A. Shape 1
B. Shape 2
C. Shape 3
D. Shape 4

30. 303 012 coloured pencils are divided equally into some boxes. 6 pencils are put into each box.

How many boxes of coloured pencils are there?

A. 552
B. 5052
C. 5502
D. 30302

31. What fraction of the diagram is shaded?



A. $\frac{5}{4}$
B. $\frac{6}{9}$
C. $\frac{4}{5}$
D. $\frac{5}{9}$

32. The four months that each have 30 days are:

A. February, March, May, June
B. July, August. September, October
C. September, October, Novmber, December
D. April, June, September, November

33. How many digits are there in the answer to the calculation

$67\,200 \div 80$

A. 5 digits
B. 4 digits
C. 3 digits
D. 2 digits

34. A wire of length 20 cm is made into a rectangle.
One of the sides of the rectangle measures 4 cm.
What is the length of the other side?

A. 5 cm
B. 6 cm
C. 12 cm
D. 16 cm

35. The percentage of the rectangle shaded is:

A. 2%
B. 4%
C. 20%
D. 40%

36.

$$\begin{array}{r} 345.48 \\ 291.07 \\ +\quad 60.21 \\ \hline \end{array}$$

The result of the addition above is:

A. 707.66
B. 707.76
C. 507.76
D. 507.66

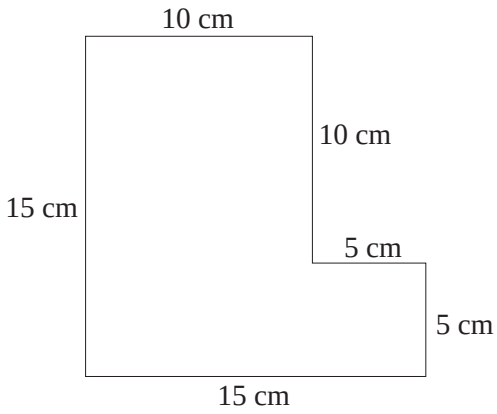
37. In a shop there are 609.6 m of cloth.
The shop sells 253.7 m.
How many metres of cloth are left?

A. 456.6 m
B. 356.8 m
C. 455.8 m
D. 355.8 m

38. Hung rides his bike at a speed of 20 km/hour.
How long does he take to travel 1 km?

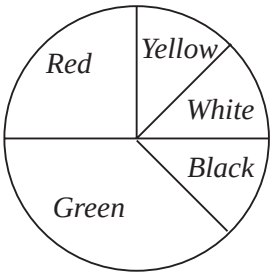
A. 5 minutes
B. 3 minutes
C. 6 minutes
D. 20 minutes

39. The perimeter of this shape is:



A. 45 cm
B. 50 cm
C. 55 cm
D. 60 cm

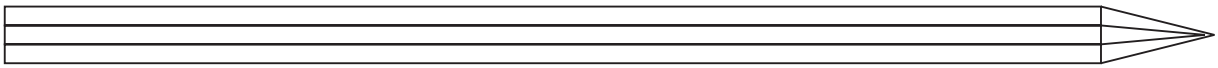
40. The favourite colours of 24 pupils are shown in the pie chart:



How many pupils like the colour *Yellow* best?

A. 2 pupils
B. 3 pupils
C. 4 pupils
D. 6 pupils

41. Measure the length of this pencil with a ruler.



The length is:

A. 16 mm
B. 14 cm
C. 17 cm
D. 160 mm

42. A piece of string is $\frac{3}{5}$ m long.

$\frac{3}{5}$ of the length is cut off.

How many cm of string have been cut off?

A. $\frac{3}{5}$ cm
B. 60 cm
C. 36 cm
D. $\frac{9}{25}$ cm

43. $3.2 + 4.65 \div 1.5 =$

A. 6.783
B. 6.3
C. 5.233
D. 0.969

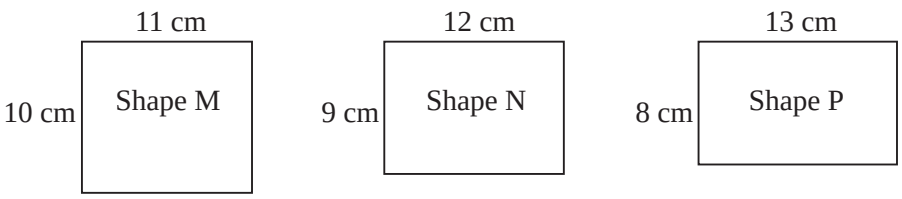
44. The price of a book is 4000 cents.

This price is then reduced by $\frac{1}{5}$.

What is the reduced price of the book?

A. 3000 cents
B. 3800 cents
C. 3200 cents
D. 3600 cents

45.



Which of the following statements is correct:

A. The perimeters are all different.
B. Shape M has the longest perimeter.
C. The perimeter of shape P is 104 cm.
D. The perimeters are all the same length.

46. Hong and Ha agree to meet at 10:40 a.m.

Hong arrives at 10:20 a.m. and Ha is 14 minutes late.

How long does Hong have to wait for Ha?

A. 20 minutes
B. 34 minutes
C. 54 minutes
D. 1 hour 20 minutes

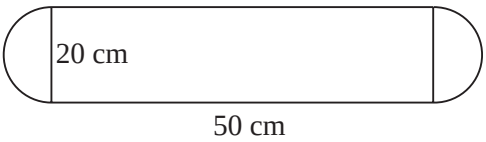
47. $\boxed{1}$ $\boxed{2}$ $\boxed{3}$ $\boxed{4}$

$\frac{\square}{\square}$

How many different proper fractions can you make using the numbers cards shown above?

A. 3
B. 4
C. 5
D. 6

48. The diagram shows a garden.



The area of the garden is:

A. 2814 m^2

B. 454 m^2

C. 3756 m^2

D. 1314 m^2

49. In a class there are 18 girls and 12 boys.
What percentage of the class is boys?

A. 150%

B. 66%

C. 60%

D. 40%

50. The side of a cube is 10 cm. If the size is increased by 10%, by how many cm^3 does the volume of the cube increase?

A. 10 cm^3

B. 21 cm^3

C. 100 cm^3

D. 331 cm^3

APPENDIX

(c) Sample Certificate of Lower Secondary School (age 15)

VIETNAM

Examination for Certificate of Lower Secondary School

School Year (Hanoi) : 1997 - 1998

Subject: MATHEMATICS

Time: 120 minutes

A. Theory

Answer *one* of the two questions.

Question 1 Define what is meant by 'equivalent fractions'.

The following equations are equivalent. Why?

$$2x - 8 = 0 \quad \text{and} \quad (x-3)^2 = 1$$

Question 2 State and prove the second simulated case of triangles.

B. Compulsory exercises

I Algebra

1. Given the expression:

$$p = \frac{x\sqrt{x} + 25\sqrt{x} - 19}{x + 2\sqrt{n} - 3} - \frac{2\sqrt{x}}{\sqrt{x} - 1} + \frac{\sqrt{x} - 3}{\sqrt{x} + 3}$$

(a) Simplify p.

(b) Calculate the value of p when $x = 7 - 4\sqrt{3}$.

(c) What value of x give a minimum value for p, and what is this value?

2. Solve the following problem by using equations.

The distance between points A and B is 40 km. A motorboat travels from point A to point B and then returns to point A.

The speed of the water is 3 km and the speed of the motorboat is constant.

Calculate the speed of the motorboat if the time taken travelling downstream is less than the time taken travelling upstream by 20 minutes.

II Geometry

A circle, centre (O, R) has a diameter A B.

A chord, C D, intersects A B at E (E ≠ A, E ≠ B).

A tangent (d) of the circle at B intersects lines A C, A D, respectively, at M and N.

(a) Prove that Δ A C B = Δ A B M.

(b) Prove that A C . A M = A D . A N

(c) The tangent of the circle at C intersects (d) at I.
Prove that I M = I B.

(d) Find the position of the chord C D so that Δ A M N is equilateral.

APPENDIX

(d) Sample National Examination (age 18)

VIETNAM

Ministry of Education and Training

National Examination

1998-1999

Subject: MATHEMATICS

Time: 150 minutes

I *(4 marks)*

Given the function

$$y = f(x) = x^3 - (m+2)x + m \qquad (m \text{ parameter})$$

1. Find m so that the function has maximum and minimum.

   Find m so that the product of maximum (or minimum) at $x = -1$.

2. Draw the graph of this function, (c), when $m = 1$.

3. How does (c) intersect the line $y = k$?. Discuss.

II *(2 marks)*

1. Integrate

$$\int_0^{\frac{\pi}{2}} \cos^2 4x \, dx$$

2. Solve the equation

$$A_x^3 + C_x^{x-2} = 14x$$

III *(4 marks)*

In 3-dimensional space with perpendicular coordinate system $Oxyz$, the points D has coordinates (– 3, 1, 2).

The plane (α) contains 3 points, A (1, 0, 11), B (0, 1, 10) and C (1, 1 , 8).

1. Write the equation of the line A C.

2. Write the general equation of plane (α).

3. Write the equation of the sphere with centre D and radius 5.

   Prove that the sphere intersects the plane (α).

APPENDIX

(e) Sample Entrance Examination of National University of Hanoi (Age 18)

VIETNAM

Entrance Examination of National University of Hanoi

1999

Block A - Subject: MATHEMATICS

Time: 180 minutes

A. Compulsory Part

I Given the function

$$y = \frac{x^2 - (m + l)x - m^2 + 4m - 2}{x - 1}$$

1. Find m so that the function has maximum and minimum.
   Find m so that the product of maximum and minimum is minimum value.

2. Draw the graph of this function when $m = 0$.

II 1. Prove that for every m, the system of equations:

$$\begin{cases} x + xy + y &= 2m + 1 \\ xy(x + y) &= m^2 + m \end{cases}$$

have roots. Find m so that the system has only one root.

2. Find m so that the inequality

$$2^{\sin^2 x} + 3^{\cos^2 x} \geq m.3^{\sin^2 x}$$

has roots.

III 1. Solve the trigonometric equation:

$$8\cos^3\left(x + \frac{\pi}{3}\right) = \cos 3x$$

2. Prove that, if A, B, C are angles of triangle A B C and $\cos 2A + \cos 2B + \cos 2C \geq 1$, then $\sin A + \sin B + \sin C \leq 1 + \sqrt{2}$.

IV 1. Given the function

$$f(x) = \begin{cases} \dfrac{\ln(\cos x)}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

calculate its differential at $x = 0$.

2. Prove that, for every a, b, c equation,

$$a\cos 3x + b\cos 2x + c\cos x + \sin x = 0$$

has roots which belong to $[0,\ 2\pi]$.

B. Alternative Part

V a *(For students on the non-streamed curriculum)*

In the plane with perpendicular coordinate system, given set of circle with equation

$$x^2 + y^2 - 2(m+1)x - 2(m+2)y + 6m + 7 = 0$$

(m – parameter)

1. Find set of centres of the circles.

2. Find the coordinates of centre of circle which touches the y-axis.

V b *(For students on the streamed curriculum)*

In the plane with perpendicular coordinate system, given a circle (C) which is determined by equation system:

$$\begin{cases} x^2 + y^2 + z^2 - 4x + 6y + 6z + 17 = 0 \\ x - 2y + 2z + 1 = 0 \end{cases}$$

1. Find the coordinates of centre of circle (C) and calculate its radius.

2. Find the equation of the sphere which contains the circle (C) and which has its centre on the surface

$$x + y + z + 3 = 0$$