

Teaching statistics with an interactive tool

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Abstract. The recent diffusion of new information technologies naturally raised some discussions about the effect of Computer Algebra on the “how and what” of teaching mathematics. The present paper provides a Computer Algebra application consisting of an interactive package introducing students to the study of probability and statistics. Even if the package has been realized using Mathematica 3.0, the students do not need to know a programming language. Black boxes philosophy has been merged with step by step solutions to the exercises enabling students to learn the process underlying the solutions.

1. Introduction

The introduction of new technologies has brought on many changes in contents, strategies, and attitudes in education: the computer can be used as a complementary tool for educational training as well as a learning tool on its own.

A key aspect of the use of computers as aids to learning mathematics is its possibility of presenting the subject in an interactive form more accessible to novice exploration than the traditional presentation by marks on paper.

Modern learning theories emphasize the importance of constructivism when integrating technologies in learning. Constructivism based learning is seen as a building process in which learners have an active role and obtain new knowledge by constructing it on the basis of previously acquired knowledge.

A new approach based on information technologies has been experienced at the Department of Information Engineering and Applied Mathematics in the University of Salerno for educating students in statistics.

We have implemented a package with Mathematica 3.0 to enhance the traditional way of teaching statistics through the use of interactive exercises randomly generated. The pedagogical methodologies adopted in the construction of the package can be grouped into four categories: expositive, demonstrative, practical and interactive.

The first is the presentation of the theoretical background needed to practice, and it is based on multimedia hypertext.

In the second the program shows at the same time some examples of package use and typical introductory exercises.

The third and the fourth categories concern the most important phase of the

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program proposes an exercise to the student and he/she has two choices: either s/he wants to do it by hand and then check her/his work through the verification routine; or s/he lacks the “know-how” and needs the automatic-solution routine to solve the exercise.

We would like to spend a few other words to better describe the latter functionalities. The package has been organized in the following way: the material is divided in main arguments. For each argument there is a hyperlink to the related theory and examples; then there is the exercise session. Starting this session the student can choose among various types of exercises, and consequently the program randomly generates a problem (concerning the chosen area) and proposes it to the student. Then, if the student wants to do it by hand, s/he can use the verification tool: the program acts as a tutor, pointing out mistakes and giving suggestions. Otherwise the step by step visualizations of the exercise solution provides a useful tool to increase the knowledge of the student.

The learning tools are interconnected by a hypertextual shell, which ensures a homogeneous appearance and pedagogical continuity.

2. Computer aided learning

During the last 30 years Computer Algebra Systems (CAS) have proved to be a powerful tool in many fields of science and engineering. In mathematics education, one of the most important applications, they have been used in order to facilitate and to encourage the experiments, to illustrate new mathematical contents and to bring a more effective way of teaching abstract mathematical subjects. Such a system may serve as a tutor for students who need to improve their knowledge as well as a challenge for good students. The use of CAS for educational purpose can help students to learn how to deal with mathematics and can increase the certainty of their own knowledge and skills.

Software systems for distance education generally fall into two categories (Buchberger 1990).

White box systems are collections of sample-formulae and/or programs presented in source code in a certain programming language or CAS, sometimes accompanied by black box implementations or some additional functions needed by the examples. The formulae/algorithms complement a textbook by illustrating various notions from it, and are intended to be executed interactively by students. The disadvantage of this type of systems is that students have to become familiar with the programming language which is used, usually requiring a significant effort.

Black box systems are collection of executable programs/routines which implement various algorithms and data structures corresponding to notions presented in the textbook, and which can be executed using menu, input forms

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parameter) etc. The produced results are then presented as numbers, table, graphics, animation, etc. The disadvantage of this type system is that only a very narrow class of problems can be implemented by each program routine, hence illustrating a textbook can become a huge task.

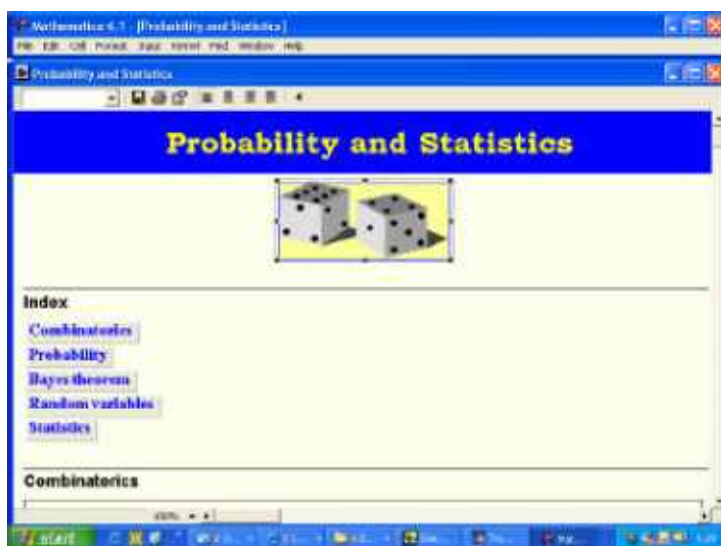
3. An overview of the package

The package is written using the Mathematica programming language and library functions provided by Mathematica. It is structured as following:

- a main menu contains the main topics;
- for each topics theory, exercises, examples and help are available;
- the theory is presented in a hypertextual notebook;
- exercises are randomly generated;
- for each exercise the student has three possibilities: automatic solution, step by step solution and verification routines (we will describe them in more details later).

Although the system is implemented on top of the CAS Mathematica 3.0, the user does not need to learn the programming language of this system, but s/he only needs to click some buttons. The aim of the system is to assist students in learning how to use and to manipulate formal notions, using the language of statistics.

The package introduces the concepts of probability and the terminology required to set up and solve simple probability problems.



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Situations involving equally-likely outcomes, such as with dice, cards, etc., can be tackled by counting outcomes. Methods for counting outcomes in probability problems have been studied. The basic multiplication results, permutations and combinations are all introduced using a variety of examples.

We now go to illustrate the main features of the presented package that is the exercise tools.

First of all the package offers a main menu

Index

[Combinatorics](#)

[Probability](#)

[Bayes theorem](#)

[Random variables](#)

[Statistics](#)

The student can choose one of this main topics and a new menu is available.

[Exercise](#)

[Examples](#)

[Help](#)

[Theory](#)

The “Examples”, “Help” and “Theory” buttons respectively show some examples of the exercises presented in the package, an help for the syntax of the various functions related to the exercises, the main theoretical concepts about the chosen topic.

The “Exercise” button allow the students to practice. The program randomly generates an exercise and propose it to the students.

A population consists of five members : 2, 3, 6, 8, 11.

Let us consider all possible samples of size 2, that can be extracted from the population without repetitions. Determine :

a the sample mean;

b the sample variance.

A box contains 5 balls, numbered from 1 to 5. Two balls are consecutively extracted, without putting back in the box the first extracted ball.

Determine the probability $P(A)$ that both numbers on the extracted balls are even.

Then further functions are available.

[Solution](#)

[Step by step](#)

[Verification](#)

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Students can solve the proposed exercise by hand and then check their result using the following three routines:

1. Solution - it gives the main steps of the solving process and the correct result;

Using the conditional probability formula, we obtain the joint probability of the required events. Thus we have

$$P(A) = \frac{2}{5} \frac{1}{4} = \frac{1}{10}$$

2. Step by step - it gives the step by step solution of the exercise, explaining the reasons and recalling the related theory or needed theorems;

Let P_1 be the event the first number is even and P_2 be the event the second number is even.

We have $A = P_1 \cap P_2$ and then

$$P(A) = P(P_1)P(P_2 | P_1)$$

Since at the first time there are 5 balls in the box,

two of which are even, $P(P_1) = \frac{2}{5}$.

At the second extraction, if the first number was even, 4 balls remain in the box and only 1 is even. We have then

$$P(P_2 | P_1) = \frac{1}{4} \text{ so } P(A) = \frac{2}{5} \frac{1}{4} = \frac{1}{10}$$

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a) There are 10 sample of size 2,
which can be extracted without repetitions.
(this means that we extract two different numbers) :

(2, 3) (2, 6) (2, 8) (2, 11)
(3, 6) (3, 8) (3, 11)
(6, 8) (6, 11)
(8, 11)

The choice (2, 3) is considered the same of (3, 2) .

The corresponding sample means are :

2.5 4.0 5.0 6.5
4.5 5.5 7.0
7.0 8.5
9.5

and the sample mean is :

$$\mu_{\bar{x}} = \frac{\text{sum of all the sample means}}{\text{number of the samples of size 2} = 10} = 6.0$$

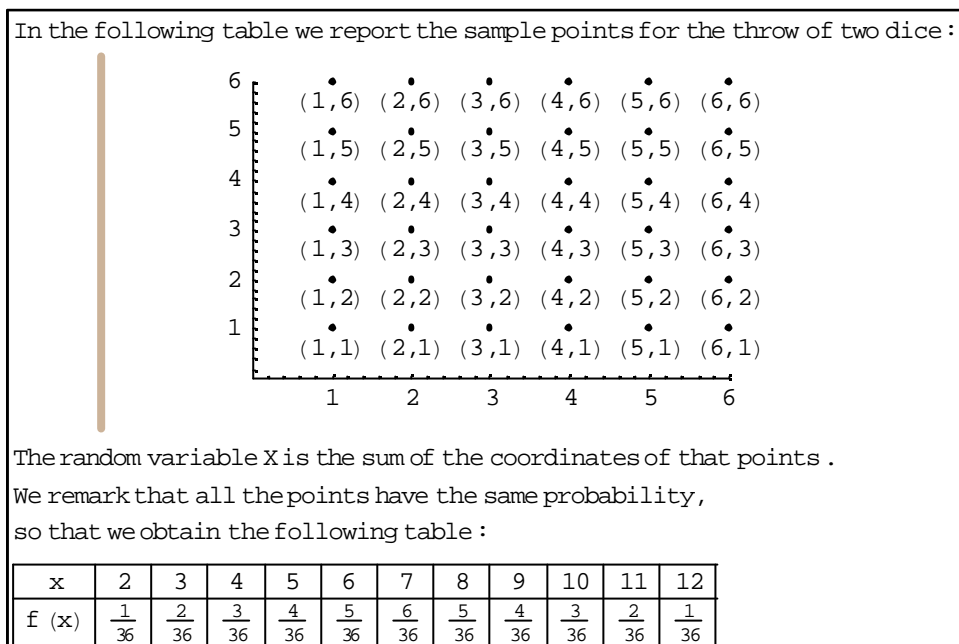
This shows that $\mu_{\bar{x}} = \mu$

b) The sample variance $\sigma_{\bar{x}}^2$ is obtained by the following formula :

$\sum_{i=1}^n (x_i - \bar{x})^2$ where \bar{x} is the sample mean. The final result is :

$$\sigma_{\bar{x}}^2 = 4.05$$

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3. Verification - the students can give their result and the routine checks the correctness; if the given result is not correct, there is an appropriate warning message.

Verification[{6}]

Wrong length of the solution! You have to insert two numbers (separated by a comma).

Verification[{6, 4.05}]

The solution is quite good!

Verification[{6, 7}]

The part a) of the exercise is correct. The part b) is wrong.

Verification[{2.3, 4.05}]

The part b) of the exercise is correct. The part a) is wrong.

Verification[{3.5, 8.6}]

The solution is quite wrong! Try again or have another look at the theory.

Verification[1/10]

The given solution is quite good!

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Verification[2.3]

Warning: the probability is in the interval $[0,1]$!

The given solution is quit wrong! Try again or have another look at the theory.

4. Conclusions

Computer-based-education not is meant to replace the role of the teacher in the classroom. Indeed it is best used in conjunction with traditional techniques. Didactical packages can be used to elucidate the concepts that were learned in the classroom or to foreshadow future subjects. Its use help teachers to achieve maximum comprehension and enjoyment of mathematical subjects taught.

References

- [1] G. Albano, C. D'Apice, G. Gargiulo: "Visualisation of four dimensional curves", Electronic Proceedings ICTMT4 (International Conference on Technology in Mathematics Teaching), Plymouth, Inghilterra, 1999;
- [2] G. Albano, A. Cavallone, C. D'Apice, G. Gargiulo: "Mathematica and didactical innovation: a quadric use case", Electronic Proceedings IMACS Conferences on Applications of Computer Algebra, Madrid, 1999;
- [3] G. Albano, A. Cavallone, C. D'Apice, S. Salerno: "Mathematica and Didactical Innovation", Electronic Proceedings IMS (International Mathematica Symposium), Risc-Linz, Austria, <http://south.rotol.ramk.../~keranen/IMS99/ims99papers/ims99papers.html>, 1999;
- [4] C. D'Apice, R. Manzo, E. Zappale: "Learning power series with computer tools", Proceedings 4th International Derive-TI89/92 Conference, Liverpool, 2000;
- [5] B. Buchberger, Should students learn integration rules?, ACM SIGSAM Bulletin, 24 n. 1, 10-17.
- [6] A. Cavallone, C. D'Apice, M. Marsella, S. Salerno: "A didactical laboratory: image altering", Electronic Proceedings IMACS-ACA Conferences on Applications of Computer Algebra, Madrid, 1999;
- [7] Heli Ruokano, Seppo Pohjolainen: Pedagogical Principles for Evaluation of Hypermedia-Based Learning Environments in Mathematics, J.UCS Volume 4/No.3.