

How Does the Problem Based Learning Approach Compare to the Model-Eliciting
Activity Approach in Mathematics?

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Abstract

Problem-Based Learning and Model-Eliciting Activities are recommended instructional strategies for teachers in mathematics. The Problem-Based Learning (PBL) approach has become quite widespread and it is used in many grade levels and disciplines. Chronologically, Model-Eliciting Activities, written specifically for instruction in mathematics, were developed after the Problem-Based Learning approach and are not yet as widespread in use as PBL activities. The purpose of this article is to discuss the similarities and differences in the two approaches with an emphasis on implementation and outcomes. Theoretical literature has been used to support the arguments. The authors conclude that the approach that an instructor selects should be based on the needs of the students. To optimize instruction, additional empirical data needs compiled from teachers and researchers.

How Does the Problem Based Learning Approach Compare to the Model-Eliciting Activities Approach in Mathematics?

Two forms of curricula that have shown promise in mathematics instruction are problem based learning tasks (PBL) and model-eliciting activities (MEAs). Problem based learning has existed since at least the 1960s (Lohfeld, et al., 2005) and MEAs have existed since the 1970s (Lesh & Lamon, 1992). Authors describe similarities and differences in the two approaches. The authors' objective is to discuss characteristics of each approach so mathematics educators can decide when to use each instructional approach. After providing history and implementation background on the two approaches, their relationship is explicated.

History and Implementation of each Approach

Problem-Based Learning

The statement that necessity is the mother of all invention is very applicable to PBL. PBL was created to fill a need at McMaster University in Canada in the 1960s (Lohfeld, et al., 2005), although historically Case Western Reserve Academy had a rudimentary model of what is now considered PBL as early as the 1950s (Tan, 2005). In the 1960s, instructors at McMaster's medical school noticed that students had amassed a vast amount of formal/content knowledge, but they had little or no ability to apply their knowledge. Hence, instructors developed an instructional approach that would force students to use their textbook knowledge in case study situations. In turn, students could see real-life scenarios and instructors could adequately assess students' ability to function as practicing doctors. Therefore, it may be accepted that PBL is a curriculum model designed around real-life problems that are ill-structured, open-ended, and ambiguous.

When students engage in a PBL task, several steps are followed. These steps are: meet the problem, define the problem, gather facts about the problem, hypothesize solutions to the problem, research the problem, rephrase the problem, generate alternatives, and advocate solutions to the problem (Fogarty, 1997). Many of these steps align with standards in several disciplines such as English, mathematics, and science (National Council of Teachers of English, 1996; National Council of Teachers of Mathematics, 2000; National Research Council, 1996). The objectives of implementing PBL tasks are varied. For instance, a teacher in one school may use a PBL task to encourage students to investigate standard deviation simultaneous to a teacher in another school using the same PBL task to encourage students to investigate correlation (Chamberlin, in press).

Model-Eliciting Activities

Similar to the PBL approach, the MEA approach was borne out of necessity. For MEAs, the necessity was to have students apply learned mathematical procedures to create mathematical models. The creation of MEAs came about in the mid 1970s to satisfy curricular needs that were unmet by existing mathematics curricula and instruction (Lesh, et al., 1983). Dr. Lesh and his colleagues felt that student development of conceptual models remains a very significant part of the mathematical learning process.

MEAs are mathematical problems created by mathematics educators, professors and graduate students, throughout the United States and Australia, to be used by mathematics instructors. These group activities require students to develop a mathematical model that is a conceptual system allowing students to make sense of certain kinds of mathematical experiences. Moreover, the designed model focuses on

structural characteristics such as elements, operations, and relationships between the elements (Lesh, et al., 2000). Much has been written about MEAs, and to gain a deep understanding of the intricacies of MEAs, readers could complete or implement a MEA. MEAs have also been referred to as Thought Revealing Activities or Case Studies for Kids. Sample MEAs may be retrieved at http://www.edci.purdue.edu/casestudiesforkids/case_studies_table.htm.

A MEA is implemented in several steps (Chamberlin, 2002). First, the teacher reads a simulated newspaper article that develops a context for students. Subsequently, the students respond to readiness questions that are based on the article. Next, the teacher reads the problem statement with the students and makes sure each group understands what is being asked and students subsequently attempt to solve the problem. After creating multiple iterations of the solution and revising when necessary, students present their models to the class. Typically, teachers provide about one hour to solve the problem, but certain MEAs may require up to two periods of class time to complete.

Two objectives are accomplished by asking students to complete model-eliciting activities. First, mathematics education researchers may investigate how students develop mathematical or scientific models (Lesh, et al., 2000). Second, MEAs enable assessment specialists to identify a broad base of students with mathematical talent that may go unnoticed (Chamberlin & Moon, 2005; Lesh, et al., 2000). For instance, Chamberlin and Moon describe how MEAs can be used to foster creativity in mathematically gifted students as well as how they can be used to identify creatively gifted mathematicians.

Comparison of PBL and MEA Approach

It is important to issue several caveats prior to the discussion. First, a full explication of all similarities and differences in the two approaches is beyond the scope of this article. Hence, curricular characteristics that are germane to mathematics instruction are covered. Empirical research and theoretical literature for this article has come from all levels of education (elementary, secondary, and tertiary). Second, when possible, statements about each approach are based on empirical articles first and theoretical articles second. The analysis section is broken into two parts. The first section of characteristics addresses similarities in the two approaches and the second section of characteristics addresses differences in the two approaches.

Similarities in PBL and MEA

According to the analysis, MEAs and PBL activities appear to have more in common than they do in opposition. The list generated has led the authors to conclude that while PBL tasks and MEAs are two distinct approaches, they are similar approaches to mathematical problem solving. Similarities in the approaches include: realistic problems, open-ended tasks, higher order thinking, metacognitive coaching, self-directed learning, self-assessment, group work, interconnectedness of disciplines, use with various grade levels, and structure of the problems. Each of these similarities is discussed below.

Realistic problems. PBL problems are realistic (Chin & Chia, 2004; Dunlap, 2005) and authentic (Hubball & Robertson, 2004; Spronken-Smith, 2005). Duch (1996) suggests that the very essence of PBL is real world problems administered to students to promote critical thinking and problem solving skills. One way to inject realism into the task(s) is to reference local sites or geographic landmarks. For instance, a problem

regularly used in the Rocky Mountain region in the United States pertains to the Colorado River (<http://www.udel.edu/inst/problems/colorado/>) which runs through the states of Colorado, Utah, Arizona, Nevada, and California. Other components may be built into problems to add a sense of authenticity. For example, a problem written for a rural area may deal with farming or ranching because many students know about these occupations. With a PBL such as this, students may be asked to create a method of counting cattle over a large quantity of land which would involve sampling and scaling. Similarly, in an industrial area, students may be asked to create a solution to solve over-population, a problem with many mathematical demands, or they may be asked what the ramifications of a changed tax rate might have on the economy.

Analogously, reality is a critical component of MEAs. Making the problem a realistic one is a defining characteristic of MEAs (Lesh, et al., 2000). By creating realistic scenarios, relatively abstract mathematical concepts can be covered in ways that closely relate to students. The use of realistic scenarios may be used to enhance the likelihood that students have increased intrinsic interest in the problem (Chamberlin, 2002). Six principles are followed to develop each MEA and the reality principle is one of the six (Lesh, et al., 2000). As an example of reality, with the MEA entitled On-time Arrival (Chamberlin & Chamberlin, 2001), students are asked to select from a list of five possibilities which airline they would select for their high school Spanish Club trip given sample data (<http://www.edci.purdue.edu/casestudiesforkids/ontimearrival.htm>).

Open-ended tasks. Tasks in both approaches are open-ended and therefore each has more than one correct response for a successful solution (Diefes, et al., 2004; Goodnough, 2003; Lesh, et al., 2003; Moore & Diefes-Dux, 2004). Given a cursory

analysis of PBL and MEA tasks, one may identify the open-ended nature of the respective tasks. Neither type of task may be answered superficially with any sort of mathematical algorithm or pre-existing formula. Moreover, the demands of each question and problem statement clearly suggest that various responses may be appropriate and that there are likely various levels of correctness. A critical feature of each type of task is that an ostensibly unclear solution may be bolstered by a compelling rationale. Providing a rationale is not a component of all types of problem solving, but it is incumbent upon groups to provide one for each type of task.

An advantage of the open-ended nature of each type of task is that it may precipitate flexibility in mathematical reasoning (Krutetski, 1976) which may eventually lead to creativity in mathematical thinking (Silver, 1997). In addition, open-ended learning often precipitates student ownership (Land & Hannafin, 1997; Oliver & Hannafin, 2000; Tan, 2005). Research from gifted education substantiates these claims in both approaches and this may be tied to positive ratings on affective measures such as interest, motivation, and locus of control (Chamberlin, 2002; Gallagher, 1997; Geerligs, 1994; Hmelo-Silver, 2004).

Students develop higher order thinking skills. The effects of PBL in promoting higher-order thinking (HOT) skills, particularly in tertiary education, are well documented (Nelson, et al., 2004; Tan, 2005). For instance, Palinscar and Herrenkohl (1999) discuss responsibilities of students in PBL as creating theories, engaging in prediction, and seeing how results garnered from PBL research can apply to similar situations.

Like PBL, MEAs help students develop HOT skills in mathematics (Lesh, et al., 2003). In addition to empirical evidence and theoretical writings suggesting the development of HOT skills, an analysis of the types of questions used in each approach substantiates this claim. For instance in the Athletics problem (Chamberlin, in press), students are asked “If it is possible to look at a track athlete’s height and weight, without knowing what event they do, and predict what event the athlete will do best.” Successfully responding to these questions requires the execution of higher level cognitive tasks such as analysis and evaluation (Bloom, 1956).

An analysis of a MEA such as On-time Arrival (Chamberlin & Chamberlin, 2001) also substantiates the claim that successfully designing a mathematical model to solve the problem is contingent upon cognitive tasks such as analysis and evaluation of data. Ultimately, instructors may ask students to apply their model to a subsequent set of data. That is to say, one way to test the model’s generalizability is to apply the model to a different set of data to see if it will withstand the rigors of another set of data and produce consistent results. A final way that higher order thinking is promoted in each approach is by having students justify their thinking through student presentations (Hmelo-Silver, 2004).

Instructor serves as metacognitive coach. The instructor’s role during the PBL approach is that of a metacognitive coach or a facilitator (Hmelo-Silver, 2004; Raucant, 2001; Tan, 2004; Tan, 2005). In this role, the instructor functions to assist the students in learning content and focusing on thinking skills by asking questions. Moreover, the instructor encourages students to learn via a guided discovery approach. Hmelo-Silver (2004) suggests that flexible knowledge is built by using PBL and this may be attributed

in part to the student becoming the instructor and the learner while engaging in PBL. Consequently, a high rate of student autonomy may be expected when the PBL approach is implemented (Hmelo-Silver, 2004).

The same is true of MEAs (Lesh, et al., 2003b). After students read the introductory article as homework, the instructor let groups solve the problems without hints, but the instructor may provide questions to promote student thinking. Hence, the role of the instructor is to guide students rather than to tell them how to create the model. While working with peers, the onus is on students to create mathematical solutions or models and the only student interaction with the instructor is to solicit questions so student autonomy can be expected to be very high. In turn, student autonomy may foster self-directed learning.

Students engage in self-directed learning. Both PBL and MEAs support self-directed learning (Hmelo-Silver, 2004; Lesh, et al., 2000; Savin-Baden & Wilkie, 2004; Tan, 2005). In fact, Hmelo-Silver states that five goals are implicit in each PBL task and self-directed learning is one of those five. She further states that self-directed learning is a distinguishing feature of PBL and explicates the virtues associated with self-directed learning. For instance, students take ownership of their learning and they become critical and reflective thinkers by justifying their answers to peers during presentations. Thinking independently may be what enables students to apply learning to future novel situations. Thus knowledge learned in PBL and MEAs is likely to be transferred more easily than knowledge learned in a traditional setting. Moreover, in PBL, mathematics instructors serving as metacognitive coaches specifically create opportunities for students to see self-directed learning modeled so they understand what is required to foster it.

Similar to the PBL approach, mathematics instructors can expect to see self-directed learning when they implement a MEA. Carlson, et al. (2003) assert that students, “make sense of meaningful situations, invent, extend, and refine their own mathematical constructs (pp.465)” while completing MEAs. Later, the authors discuss the ability of students to engage in self sense-making. As with PBL, an objective of the MEA approach is to allow students the opportunity to take control of their own learning by directing the process. Creating one’s own mathematical model, as is done with MEAs, is one way that this can be accomplished.

Tasks promote self-assessment. The term self-assessment is a misnomer when referring to group-based problems since truly students engage in group assessment. Nevertheless, PBL and MEA tasks are designed to encourage students to assess themselves and to reflect on their thought process. In the PBL literature, self-assessment has been referred to as an iterative process and/or reflective thought (Hmelo & Ferrari, 1997; Hmelo-Silver, 2004; Song, et al., 2003). Specifically, the reflection comes through a careful analysis of what has been learned by conducting research and how this research can be applied to create a solution. A promising addition to PBL for middle school students has been to encourage reflective thinking through journal entries (Puntambekar & Kolodner, 1998).

Similarly, self-assessment occurs in MEAs, because the groups are seeking a refined answer (Diefes, et al., 2004; Lesh, et. al, 2000; Moore & Diefes-Dux, 2004). Doerr and Lesh (2003) caution that when completing MEAs, students rarely arrive at their best answer on their first attempt and they refer to subsequent attempts to provide a refined answer as an iterative process. This iterative process is typical of what applied or

theoretical mathematicians do during the process of solving problems. Self-assessment is such a significant component of MEAs that it is one of the six principles to develop each MEA (Moore & Diefes-Dux, 2004; Lesh, et. al, 2000). Thus, MEA developers have specifically created problem statements that enable students to look back at their work, devoid of teacher assistance, in an attempt to place a value on the mathematical model that has been created. Theoretically, less than appropriate answers will result in subsequent attempts to design an improved model. As Lesh et al. (2000) state, “If students are unable to detect deficiencies in their primitive ways of thinking, then they are not likely to make significant efforts to develop beyond their primitive interpretations” (pp. 618).

Students work in groups. In both types of tasks, students work in groups. With PBL tasks, collaborative work with peers is essential for several reasons (Hmelo-Silver, 2004; Tan, 2005). As an example, Hmelo-Silver suggests that students work in groups to: identify what needs to be learned to solve the problem, what it’s like to function as a team, negotiation and mediation skills, distribute cognitive responsibilities amongst members, and to externalize thinking through explaining ideas to peers. However, it is yet to be empirically proven whether or not PBL is responsible for creating effective collaborators. With respect to group size, one difference in PBL and MEAs is that PBL tasks have between three and fourteen individuals per group with the ideal group size being six to nine (Tan, 2005) and MEAs were specifically created to have three to four individuals per group (Lesh, et al., 2000).

MEA developers have claimed that their tasks are some of the only problem solving tasks that are created specifically for groups and that most problem solving tasks

are designed for individuals and adapted for groups (personal communication with D. Lesh, April 2, 2005). Most MEAs are so in depth that it would be an onerous task for one individual to complete an entire task. An objective of having students work in groups while doing MEAs is to prepare them to enter the workforce. Dark (2003) suggests that in education we expect students to work alone and that does not mimic what happens in the everyday world. In fact, when students ultimately become employed, it may be more common to work with peers than to work alone. Hence, in completing MEAs, students work with peers for many of the same reasons that they do during PBL (i.e. they start to understand what it means to collaborate with peers).

Interconnectedness of disciplines. Students may view disciplines (e.g. mathematics, science, literacy, etc) as interconnected due to demands of PBL and MEAs. While completing PBL tasks, students view the utilitarian purpose of looking at one discipline in relation to others (Majeski & Stover, 2005; Stepien & Pyke, 1997; Vanliet, 2005). This is done purposefully and it is exemplified in analyzing curricular demands in a PBL task such as the Athletics Problem (Duch, 2001). In this task, a student may look at samples of river water and try to ascertain the ecological damage done to the water with the help of a Department of Natural Resources (DNR) representative. This contact of the individual will require formal writing and general communication skills. The data provided by the DNR representative would typically involve analysis in the field of mathematics and science. The communication required to complete all PBL tasks satisfies curricular demands in a host of disciplines and standards.

Similarly, MEAs involve viewing disciplines as interconnected. Though MEAs are written to encourage students to view the interconnectedness of disciplines such as

mathematics, science, and literacy, MEA authors have concentrated their efforts explicitly on students recognizing the interconnectedness of content areas within mathematics (Lesh, et al., 2000). For instance, when creating a model for the On-time Arrival problem (Chamberlin & Chamberlin, 2001), one group may use number sense and another group may use data analysis and probability. The interplay between these mathematics content areas is often accomplished while creating the mathematical model and debriefing on solutions, and the interconnectedness of disciplines such as literacy and mathematics may be recognized while documenting the model.

Use with various grade levels. Both types of tasks have been used with various grade levels ranging from late elementary to tertiary education. A slight difference exists in the two approaches with respect to prevalence in secondary and tertiary education. The PBL approach was initially designed for use with tertiary students at McMaster's University Medical School in Canada. Since its inception, PBL has been used largely with undergraduate and graduate students, and it has been successfully adapted to be used from kindergarten to high school as well (Hmelo-Silver, 2004; Tan, 2005). The adaptation of PBL to junior high and high school students has been particularly successful. For instance, PBL is a critical component of the Illinois Mathematics and Science Academy curricula (www.imsa.edu) in the United States. Some individuals claim to have used it in elementary grades, but the cost-effective ratio of adapting such problems to cognitive demands of elementary age students may be in question.

MEAs on the other hand were initially created for use with middle grade and junior high students (Lesh & Lamon, 1992). They have since been adapted for use with undergraduate students in engineering and education, graduate students in mathematics

education, business, and professional development with teachers (Diefes, et al., 2004; Oakes & Rud, 2003). Thus, the similarity is that they are each used in various grade levels, but the adaptation of PBL has occurred from tertiary education to secondary and elementary, and the adaptation of MEAs has occurred from secondary to elementary and tertiary education.

Structure of problem. One word that reappears in virtually every PBL definition is ill-structured (Chin & Chia, 2004; Dochy, et al., 2003; Majeski & Stover, 2005). Terms such as untidy, more than one correct answer, and preceding formal instruction have been used to define the term ill-structured problems. Stepien and Pyke (1997) describe ill-structured by illustrating its antonym well-structured. A well-structured problem is one that is tidy, may have little mystery or complexity, is highly organized, and all of the information needed to solve the problem is present. Often a formula or data that is presented is used to solve the well-structured problem, and limited scrutiny of excess information takes place. An example of a well-structured mathematics problem is implementing the formula for the area of a circle when asked to calculate it. Well-structured problems may enable the student to resist reasoning once the necessary mathematical algorithm is identified.

Though MEAs have never been categorized as ill or well-structured problems, the structure of all problems qualifies the approach as ill-structured. For instance, MEAs have a high level of complexity, more than one mathematical solution, and they often precede formal instruction (Lesh, et al., 2003; Lesh, et al., 2000). Another reason that MEAs are ill-structured is because they are far from simplistic problems and they typically demand intense reasoning on behalf of students. As with the PBL approach,

MEAs are extremely open-ended in that students may create multiple solutions to the problem. Moreover, a formula or model is not provided to students to complete a MEA. As the name implies, Model-Eliciting Activity, students create their own mathematical model to solve the problem.

Differences in PBL and MEA

The PBL and MEA approach have more in common than they do in opposition. Nevertheless, several differences exist in the two approaches and the following section focuses on those differences. At the conclusion of each section, suggestions are provided regarding when each approach may be most effective for teachers.

Time investment in class. The time investment to implement a problem-based learning task is substantial. Conversely, the time required to implement a MEA task is significantly less. According to Coleman (1995), students may need to invest as much as two weeks to complete a PBL task, whereas a MEA may often be completed in about an hour and mathematical solutions may be discussed in another hour. The difference in time requirements may be due to the fact that with PBL tasks, students need to invest extensive time researching the problem whereas in MEAs, students work with the information provided to them on a data page. As an example, to successfully solve a PBL task, students may need to familiarize themselves with academic content, contact outside individuals who are experts in a field such as a statistician, conduct a mini-investigation to gather empirical data, analyze the data, acquaint themselves with empirical and theoretical literature, or some combination of these tasks. Hence, the data gathering and analysis stages, i.e. the stages that enables the PBL group to make an educated

hypothesis, may require over one week to complete. From this point, students often need to generate a solution, troubleshoot the solution, and then present it to peers.

The time investment may not seem excessive given the multiplicity of disciplines (potentially) covered in one PBL task. To some curriculum coordinators and teachers, the approach may be regarded a worthwhile one because it may breed meaningful learning, although Morrison (2004) has questioned its value relative to the time investment for students and teachers. In addition to the time necessary to complete one problem, student presentations may require as much as one week.

Many teachers that have implemented MEAs have suggested that the amount of time spent on each project is not excessive and that the investment of time was well worth the investment. A teacher can expect to invest a full class period, i.e. at least one hour, on one MEA. The steps of a MEA are typically: read the problem at home the night before the task is completed, answer the comprehension or readiness questions and discuss them in class the next day, read the problem statement, create a mathematical model to solve the problem, test the model and revise if necessary, and then present the solution to the class. Without student presentations, the task can often be completed in nearly a one-hour class period. To complete one problem and present it to the class generally requires two to three hours for all groups involved given the complexity of the mathematics involved. Only a small amount of work is done at home since interacting with peers is critical to the success of creating a model.

Instructors with limited instructional time interested in implementing one of these two approaches should look closely at the MEA approach. Instructors with limited time for enrichment may investigate PBL tasks, but they should be warned that PBL tasks

require a significant time investment in and out of class for students. Also, implementing PBL tasks will require significantly more time to monitor students' progress to verify that they are investigating appropriate content. Moreover, abbreviating a PBL may alter the problem significantly.

Mathematical models are created as the final solution. To solve PBL tasks, models may be created, but they are not a necessity. For example, the Colorado River Problem (Duch, 2001) may be solved with or without a model. In fact, it may be likely that most students will solve this problem without creating a model of any sort. Moreover, it is not known whether or not students that created a model will develop one based on mathematics.

Although models are not created as the solution to every PBL task, it is important to note that perhaps the most salient of the models created are often mental (Schmidt, 1995) and mathematical (Stepien & Pyke, 1997). Stepien and Pyke cite an instance in which the teacher acts as a metacognitive coach by helping students construct a mathematical model of the life cycle of owls. Certainly, one may speculate that models other than mathematical models may be constructed since the PBL approach encompasses several disciplines.

On the other hand, a successful solution to a MEA always requires the creation of a mathematical model as Lesh, et al. (2000) specify in principles to design MEAs. When students have not created mathematical models, but they have a strong conceptual understanding of the problem, teachers remind students to formulate their answers as a model. An advantage of creating mathematical models is that it promotes a deep understanding of pre-college level mathematics, it mimics what real-life mathematicians

do, and it enables students to transfer their response to a similar situation to see if the model is generalizable (Lesh, et al., 2000). Moreover, creating a model helps students externalize their thinking, which in turn helps students with the cyclical process of modeling: express, test, and revise (Zawojewski, et al., 2003).

The discretion of what approach to use is up to individual instructors. If the creation of mathematical models is an objective of a lesson, or if it aligns closely with district, state, or national standards, then using the MEA approach may be preferential to the PBL approach. However, if the creation of mathematical models is not an objective of the lesson, then the instructional approaches may be considered similar. It is equally important for teachers to understand that students may concentrate more closely on mathematics when completing a MEA and when completing a PBL, teachers may need to focus students' attention specifically on the mathematics at hand. In this sense, MEAs may be preferential to PBL for mathematics instruction particularly when the creation of models is an objective.

Implications for practice

From this analysis of the instructional approaches, two implications may be garnered. First, it is imperative that teachers regularly use non-routine mathematical problem solving tasks in their classroom. PBL and MEAs are strong examples of what is meant by non-routine tasks because they are ill-structured. They are not trite problems that can be easily solved with a mathematical formula. Instead, these problems force the invention of unique solutions. Both types of problems have an important place in mathematics classrooms and help students achieve the goals of mathematics standards related to non-routine problem solving. Significantly, MEAs may be more challenging in

mathematics than PBL tasks and they link to mathematical content areas often better than PBL tasks do. Hence, it may be easier for instructors to identify what mathematics is learned by students with MEAs with greater precision than with PBLs.

Second, there is a need for curriculum developers to create more MEAs so they can be more widely used than they currently are. PBL tasks are fairly widespread and easy for teachers to implement, yet MEAs are in some ways preferable in mathematics classrooms because they take less time than PBL problems and focus directly on important mathematical skills such as creating mathematical models from data sets. However, before MEAs can be recommended for widespread adoption, more of them need to be developed. Approximately 50 MEAs have been developed and most of them are only suitable for implementation at the middle school level. There is a need for the development of MEAs that would be effective in high school and elementary school classrooms. The availability of practitioner-friendly MEA materials with multiple MEAs for different developmental levels would increase the usage of MEAs and the mathematical skills of students.

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