

THE EFFECT OF COMPUTERS ON TEACHING THE LIMIT CONCEPT¹

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Abstract

In this study, we investigate whether computer support has a contribution to make in teaching by the limit concept. Firstly, 52 students were chosen in a university of Türkiye, who were separated into two groups. The limit concept was instructed by using classical methods to one of the groups whereas using computer support was employed in the other group. With respect to applied exam results, it can be concluded that using computers has a positive effect in visualization for teaching limit concept.

Key words: Limit Concept, Effect of Computer, Teaching of Limit

1. Introduction

The main subject of Calculus is the differentiation and the integration of single variable functions and its applications in engineering and in science. Together with differentiation and integration, several concepts of mathematics are defined by means of limit. That is why limit is one of the most important concepts in mathematics.

Limit of a function at a point stems from the approach to that given point and need to know the behaviour of the function. Limit is a local property. In other words, in this situation, the function values around a point that is important rather than the function value at that point. Possibly behind that point, an element of domain of the function may materialise.

In general the formal ($\epsilon - \delta$) definition is not given in calculus level, whereas the limit intuition is given to the students and the concept is developed by the help of examples and theorems. Consequently the students neglect the conceptual part of limit, and they only consider the computational part of limit which makes it difficult to comprehend the concepts defined by the limit and its applications of the limit [2, 5]. For example, students can find out horizontal asymptote of a function by means of limit but they can

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hardly understand how the function itself behaves like a horizontal asymptote in sufficiently large values of the independent variable.

This problem is not of particular concern in our country Türkiye. But a lot of countries are also facing up the same problem, that is why a number a number of researches are carried out in this particular area [4, 6, 7, 8]. It appears that the computer may have contributed some help to the students in limit and other concepts of calculus. That is why the use of calculus education has been explored [9, 11]. In these researches, together with some positive effects of computers, it is encountered some negative effects as well [10]. To sum up, in this work, our purpose was to discover if the students can comprehend the limit concept and overcome its difficulties by the help of computer.

2. Method

At the first stage of the project, 52 first year mathematics students were chosen in a university of Türkiye. A pre-exam was applied to the students and according to the results they were separated into two groups, approximately at the same level. The groups were called A and B.

The limit concept was instructed by using classical methods to group A in the classroom. The concept taught to group B in a computer laboratory by individual computer has a program prepared by the researchers using MATLAB. Each of the students in this group followed the lesson from his/her computer interactively.

2.1. The Program Prepared For Group B

Step 1:

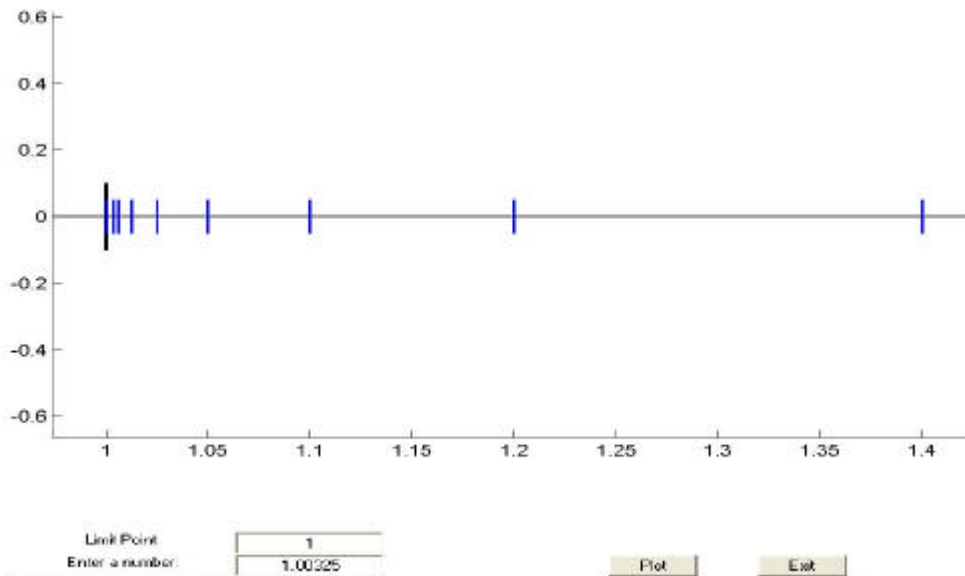


Figure 1: An application of the applet used in Step 1

One of the main reasons of why the limit concept can not be comprehended easily is that students can not understand “approaching a number” [3]. Indeed we aimed in this

step “approaching a number” signifies. The computer applet prepared at this stage displays the entered numbers on the real line. Initially, a number that wanted to be approached, for example a , and another number greater than a are chosen and entered by the students and displayed on real line. Then the students are asked to enter a number greater than a which is less than the previous one. This procedure is repeated many times, so the intention is that the students are made to perceive if such a number can be established each time. Analogically, a similar process was applied with numbers less than a .

Step 2:

In this step, an attempt was made to see if the students ascertain where the values of the function $f(x)=x^2$ approach as x tends to 2. The applet prepared for this purpose plots the graphic of a function and mark the values of the function on the y -axis corresponding to entered x -values. In the laboratory application, the students entered numbers close to 2 from the keyboard to their applets, and they realized that the function values approach to 4.

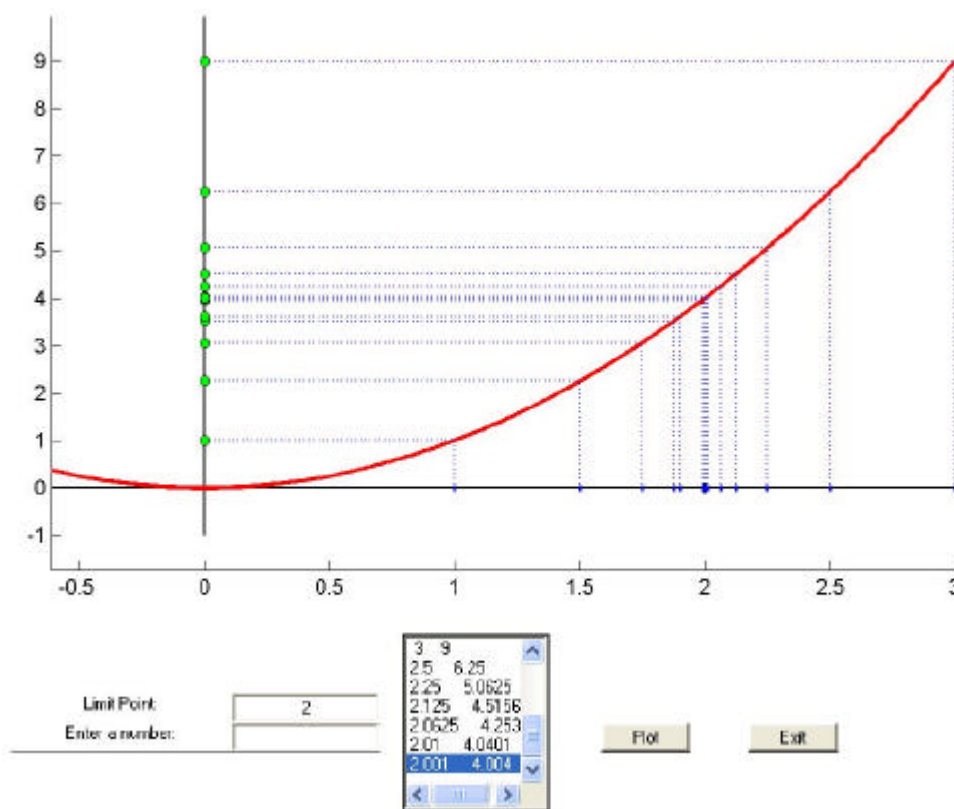


Figure 2: An application of the applet used in Step 2

Next, the definition of the limit of a function at a point was given as “If $f(x)$ is defined for all x near a , except possibly at a itself, and if we can ensure that $f(x)$ is as close as we want to L by taking x close enough to a , we say that the function f approaches the limit L as x approaches a , and we write $\lim_{x \rightarrow a} f(x) = L$.” [1].

Step 3:

The goal of this step is to teach the limit of a function at a point does not have to be the value of the function at that point. The same applet used in Step 2 was applied to find the limit of the function

$$f(x) = \begin{cases} x^2, & x \neq 2 \\ 3, & x = 2 \end{cases}$$

at $x = 2$.

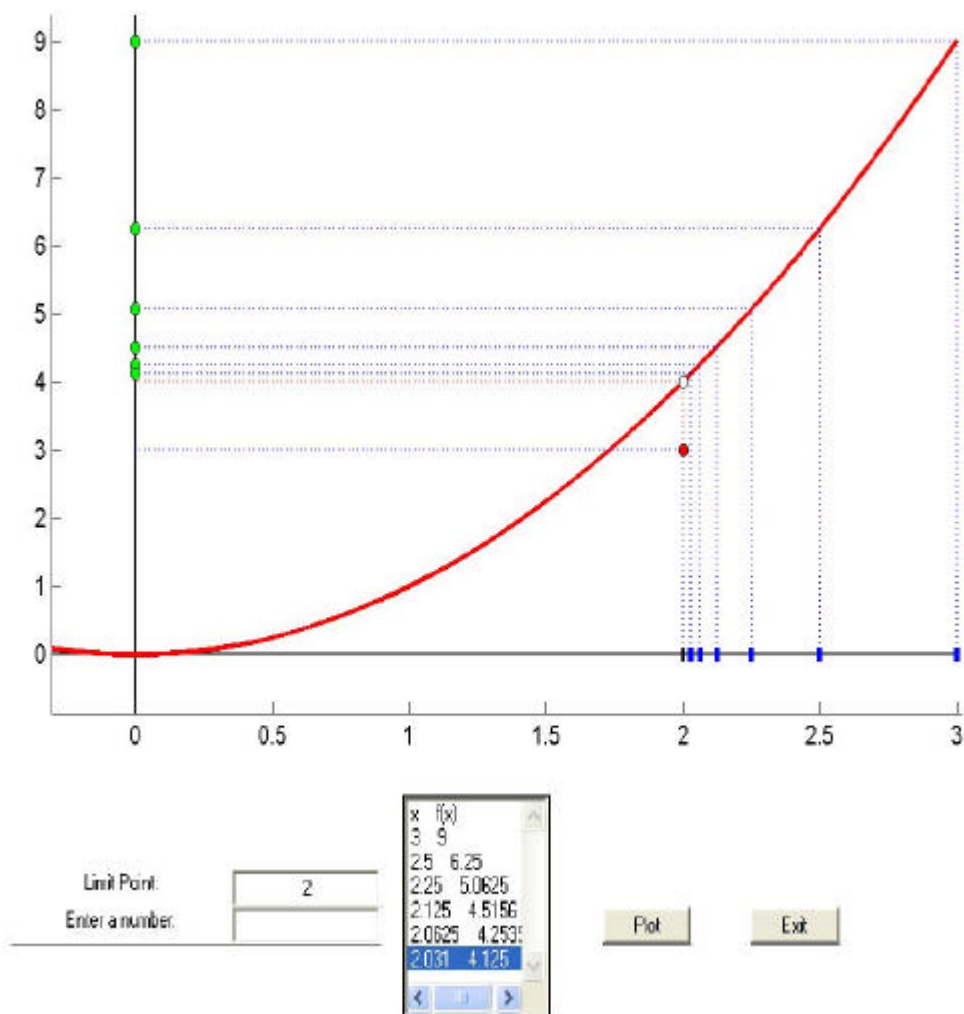


Figure 3: An application of the applet used in Step 3

Step 5:

According to our classroom observations, the students seem to believe that “if a function has a limit at a point then the functions must be defined at that point”. In order to change this misbelief, the limit of the function $f : R \setminus \{2\} \rightarrow R, f(x) = x^2$ when x tends to 2 was examined. It was clarified that a function can have a limit at a point where f is not defined.

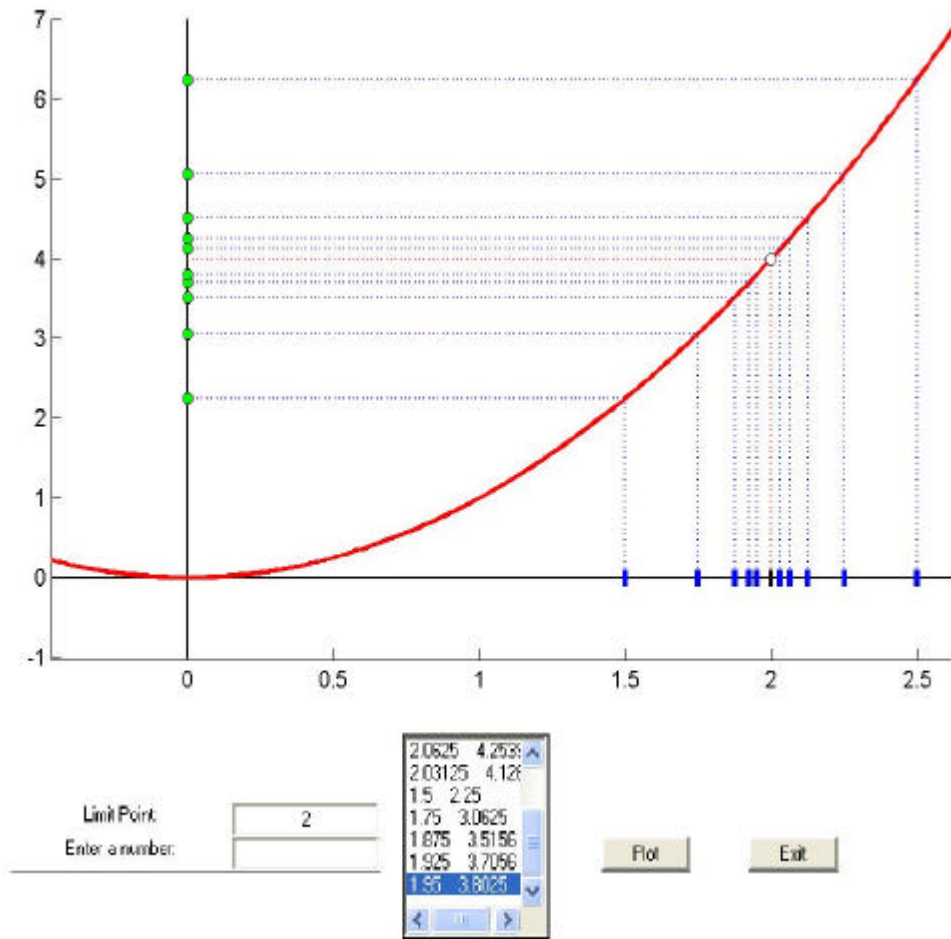


Figure 5: An application of the applet used in Step 5

Step 6:

As a final step, exercises using computer applets were performed to intensify limit concept.

$$\text{Exercise 1 : } f : \mathbb{R} \rightarrow \mathbb{R} \quad ; \quad f(x) = \begin{cases} x^2 & , \quad x \geq 2 \\ -x & , \quad x < 2 \end{cases} , \quad \lim_{x \rightarrow 2} f(x) = ?$$

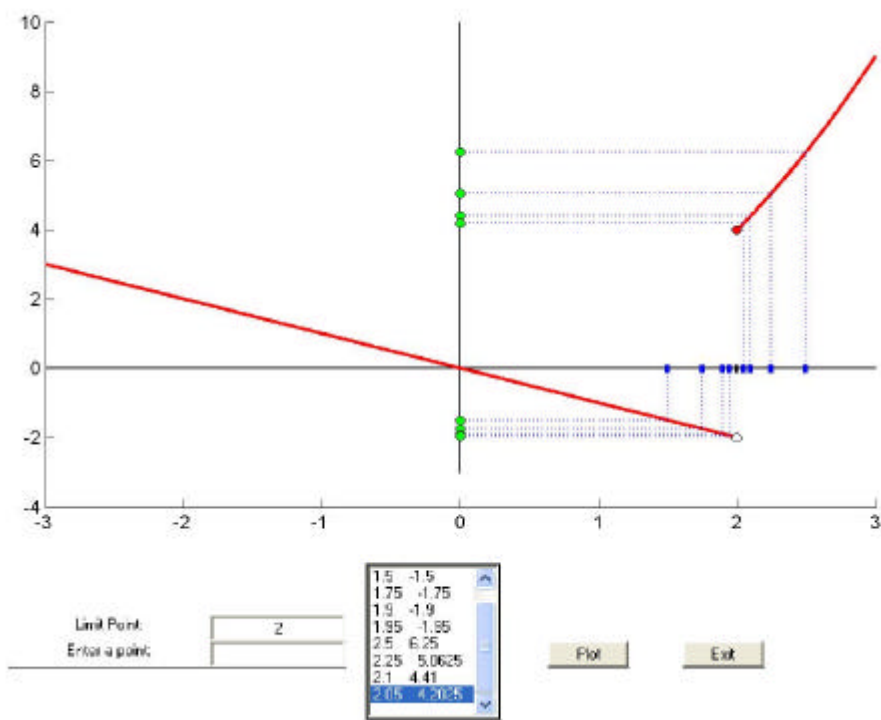


Figure 6: An application of the applet used in Exercise 1

$$\text{Exercise 2: } f : \mathbb{R} - \{3\} \rightarrow \mathbb{R} \quad ; \quad f(x) = \frac{x^2 - 9}{x - 3} , \quad \lim_{x \rightarrow 3} f(x) = ?$$

$$\text{Exercise 3: } f : \mathbb{R} \rightarrow \mathbb{R} \quad ; \quad f(x) = \begin{cases} x^2 & , \quad x \geq 2 \\ -x & , \quad x < 2 \end{cases} , \quad \lim_{x \rightarrow 1} f(x) = ?$$

$$\text{Exercise 4: } f : (-\infty, 0) \cup (0, \infty) \rightarrow \mathbb{R} \quad ; \quad f(x) = \frac{1}{x^2} , \quad \lim_{x \rightarrow 0} f(x) = ?$$

$$\text{Exercise 5: } f : \mathbb{R} - \{0\} \rightarrow \mathbb{R} \quad ; \quad f(x) = \frac{1}{x} , \quad \lim_{x \rightarrow 0} f(x) = ?$$

$$\text{Exercise 6: } f(x) = \frac{\sqrt{3x^2 - 3x + 2}}{x^2 + 1} , \quad \lim_{x \rightarrow \infty} f(x) = ?$$

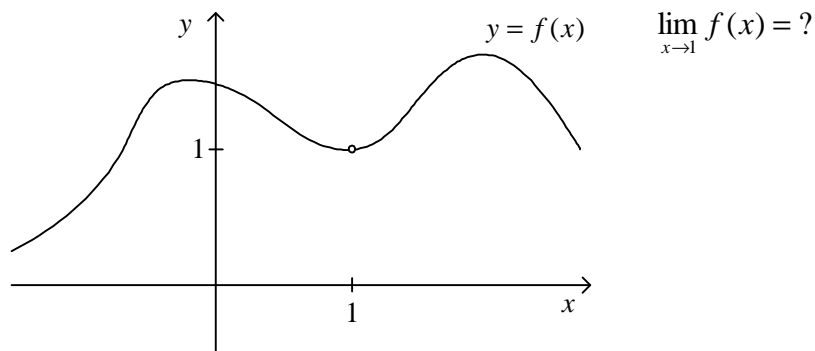
In each exercise the students were required to use the computer applets which show the graph of the function, and were required to compute the limit by entering numbers close to the limit points. The students have obtained the limits of the functions by following function values corresponding to the entered numbers on the screen.

Finally, the properties of limit were instructed to group B with classical methods as in group A. An exam was applied to both groups after two weeks, then the limit concept had to be instructed.

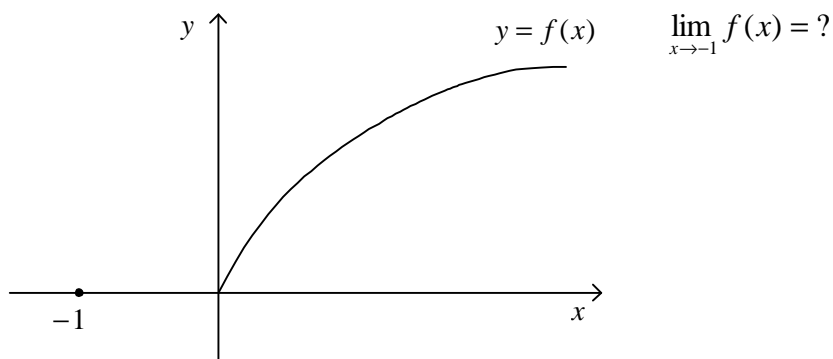
Questions of The Applied Exam

1. Find the limits of the functions f whose graphs are given by the following.

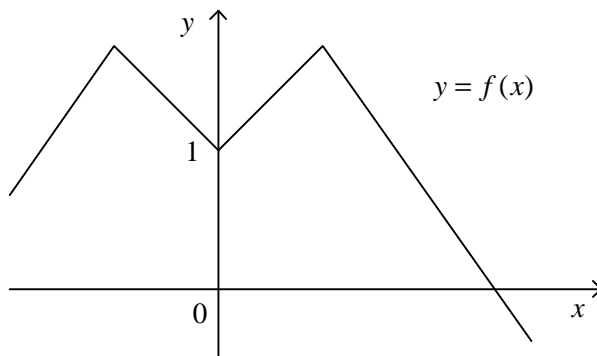
a) $f : \mathbb{R} - \{1\} \rightarrow \mathbb{R}$



b) $f : \{-1\} \cup [0, \infty) \rightarrow \mathbb{R}$

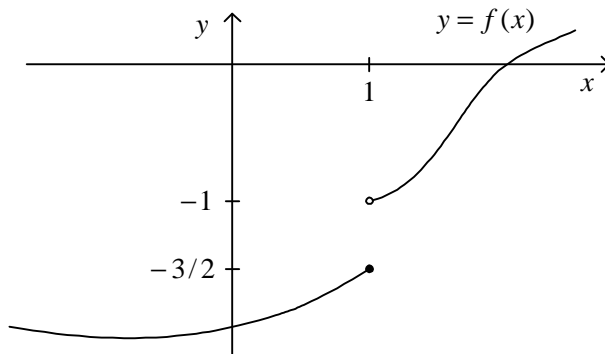


c) $f : \mathbb{R} \rightarrow \mathbb{R}$



$$\lim_{x \rightarrow 0} f(x) = ?$$

d) $f : \mathbb{R} \rightarrow \mathbb{R}$



$$\lim_{x \rightarrow 1} f(x) = ?$$

2. Let $\lim_{x \rightarrow 1} f(x) = \mathbf{p}$. Judge these statements.

a) f is defined at the point $x = 1$.

b) f is continuous at the point $x = 1$.

c) $\lim_{x \rightarrow 1} [f(x) - f(1)] = 0$

d) If g is continuous function and $g(1) = -\mathbf{p}$ then $\lim_{x \rightarrow 1} [f(x) - g(x)] = 2\mathbf{p}$.

3. Let $f(x) = e^{-x} + 2$. $f(0.001) \approx ?$

4. Let $f(x) = \frac{3 - 2x^5}{x(x^2 + 21)(1 - x^2)}$. $f(-2005) \approx ?$

Although both groups had started with 26 students, 25 students from group A and 21 students from group B participated to the exam. The answers of the students were separated into two certain groups, as true and false answers. An answer was accepted as true one if it was given correct result with correct explanation otherwise it was accepted as a false one. The data obtained from the exam results were interpreted by the use of frequency, percentage and t-tests.

3. Conclusions and Discussion:

True answer percentages obtained from the exam results are given in Table 1.

Table 1: True answer percentages of students

	GROUP A (Classroom)	GROUP B (Comp. Laboratory)
Question	True answer (%)	True answer (%)
1	a	36
	b	40
	c	71
	d	44
2	a	16
	b	12
	c	12
	d	80
3	19	28
4	24	28
	$\bar{X}_A = 35,4$	$\bar{X}_B = 46,4$

From Table 1 we obtained the following conclusions:

- The first question has four items, each of them is about finding the limit of a function by the use of the given graphics. It is observed from Table 1, that group B is more successful. It can be said that teaching the limit concept with using computer support has a positive effect in understanding this concept. One of the reasons of this positive effect might be the contributions of computer to the visualization. The students perceive that the explanation on the computer as a real fact, whereas they perceive that the explanations in the classroom as an artificial feature.

- The second question also consists of four items associated the investigation of the relation between the limit of the function at a point and the continuity of the function at that point. It can be seen from Table 1 that the students in both groups have a close true answer percentages for all items. Although the first three items have low percentage, the last item has a high true answer percentage. The difficulty of understanding the relation between limit and continuity at a point is often an encountered problem in our calculus courses. Obtained results shows that the prepared computer applets does not have any considerable effect.
- The third question was concerned with the closeness of the limit value of a function at a point and the function values at near to that point of the limit point. The fourth question is concerned with the task of the limit value at infinity which can be taken as an approximate value of a function in sufficiently great numbers. By these questions, it was expected that group B to be more successful than group A. According to the result obtained from Table 1, group B is 9% and 4% more successful in questions three and four respectively, but both groups are unsuccessful in answering the 3rd and 4th questions. Therefore, these questions demonstrate that the students in both groups do not get in touch with the limit at a point, and the behaviour of the function near the limit point. It is apperent that the (prepared) computer program has no meaningful contribution. Indeed the students attach more importance to operational process than the means of concepts and conceptual relations.
- The mean of the true answer percentages are $\bar{X}_A = 35.4$ in group A and $\bar{X}_B = 46.4$ in group B. In order to verify whether meaningful differences between the means were performed t-test in 5% level of significance, it can be seen from the result that there is no considerable difference between these two averages.

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