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Adding an Aesthetic Image to Mathematics Education

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Abstract

Mathematics educators are calling for reform in mathematics education, usually focussing on “success for all” learners of mathematics (e.g., National Council of Teachers of Mathematics). The assumption that only certain people can understand mathematics is pervasive in society and calls into question the role of mathematics education since not all learners will succeed. The goal of this paper is to explore the possibility of adding an appreciation of the aesthetic nature of mathematics to mathematics education, and to suggest that the goal of success for all cannot be achieved without providing opportunities for students to experience an aesthetic image of mathematics. To explore adding an aesthetic image to mathematics education, we will borrow Eisner’s four aims for discipline-based art education, adapted as a framework based on the activity of doing mathematics, and used to challenge underlying assumptions concerning the nature of math, artistic knowing of mathematics and success for all.

Introduction

Math does make me think of a stainless steel wall – hard, cold, smooth, offering no handhold; all it does is glint back at me. Edge up to it, put your nose against it; it doesn't give anything back; you can't put a dent in it; it doesn't take your shape; it doesn't have any smell; all it does is make your nose cold. I like the shine of it – it does look smart, intelligent in an icy way. But I resent its cold impenetrability, its supercilious glare (Buerk, 1982, p. 19).

The above quote is by a woman who is outstanding in an intellectual field but has a fear of mathematics. We find the quote interesting for three reasons. First, there is an underlying assumption of the nature of mathematics as something that exists (a wall) apart from humans. Second, there is an emotional response, mostly rooted in fear, but still a response with elements that are positive (“...smart, intelligent...”) and negative (“...cold...glare.”). Third, there is a tacit assumption by the quote's author that she cannot understand mathematics, that she would need to be a “math type” to understand mathematics. The assumption that only certain people can understand mathematics is pervasive in society and calls into question the role of mathematics education since not all learners will succeed. The strands in the above quote are relevant to the goal of this paper, namely, to explore adding an aesthetic image to mathematics education.

The above quote is suggestive of several fundamental questions that are relevant to adding an aesthetic image to mathematics education: what is the nature of mathematics, what is the role of knowing through emotional response, and can there be “success for all” in mathematics education? Examining the assumptions underlying answers to these questions provides insight into the connection between aesthetics and mathematics. The questions of the nature of mathematics, the role of knowing through emotional response, and success for all suggest why it is important to add an aesthetic image to mathematics education.

To explore the three fundamental questions stated above, we will make use of a framework based on the activity of doing mathematics (or any discipline) that has four

components: production, criticism, cultural/historical place, and judgement. These components are borrowed from Eisner's (1988) four aims for discipline-based art education and adapted for mathematics education. Production refers to the creation of mathematics. Criticism is the perception of and response to the qualities of mathematics. Cultural/historical place is the understanding that mathematics doesn't emerge in a vacuum, that there are always cultural and historical influences on and from mathematics. Judgement¹ is the valuing of mathematics and the underlying criteria for making value judgements. The components of production, criticism, cultural/historical place and judgement provides a framework for challenging underlying assumptions concerning the nature of math, emotional knowing and success for all, and for finding the connection between aesthetics and mathematics.

In this paper, we will explore the connection between aesthetics and mathematics and the role of this connection in mathematics education. When considering a connection between aesthetics and mathematics, it is necessary to clarify the meaning of aesthetics, the nature of mathematics, the utility of mathematics, and the goals of mathematics education. We will use the activity framework of production, criticism, cultural/historical place and judgement to define aesthetics, clarify the role of the nature and utility of mathematics and of goals for mathematics education, and explore the connection between aesthetics and mathematics. We conclude that mathematics educators can and should add an aesthetic image of mathematics to mathematics education, and that such an image would be essential for achieving the goal of success for all learners of mathematics.

Definition of aesthetics

In this section, we will clarify our meaning when using the word aesthetic. The popular image of mathematics is based on logical deduction, whereas the popular image of art is based

on creativity and intuition. A connection between math and aesthetics cannot be clearly established without addressing the opposing popular images of math and art. A definition of aesthetics is important to illuminate popular images that may be misleading and obscure the connection between mathematics and aesthetics. We have borrowed definitions of aesthetics from the perspective of the art critic and adapted these definitions to the discipline of mathematics.

For much of the twentieth century, aesthetic appreciation of works of art has focussed on formal properties such as harmony, color, and rhythm (Eaton, 1998). Eaton suggests that aesthetic appreciation should focus on the affect of the art on the observer, rather than the properties, since aesthetic value is dependent upon the culture and time period. The common element among all aesthetic experience is joy from engaging with the work of art.

Gadamer (1998) challenges the notions that an aesthetic experience is primarily a joyful emotional experience. Gadamer suggests that there is a cognitive element to aesthetic experience, in which the observer's interaction with a work of art is "playful", and the observer's joy is a joy of knowing something more about the world and about ourselves.

The above definitions suggest that the emotion of aesthetics is always joyful. Missing is the role of other emotional responses such as anger and pain. A work of art could evoke strong emotions that are not joyful and may even be painful. Anything that causes emotional disgust can also be art (Escher, 1989). There is also the struggle of creating or understanding a work of art, a struggle that is more likely painful than joyful. Aristotle believed that humans by nature enjoy learning and are curious about the world despite the pain that may be associated with learning and understanding (In Korsmeyer, 1988, p.231).

Synthesizing the above elements gives a working definition of aesthetics in general. An aesthetic experience is an emotional (joyful and/or painful) and insightful experience from an interaction with anything. The next question is determining what relevance this definition has to mathematics. We will explore the question of relevance using the activity framework described earlier: production, criticism, historical/cultural place, and judgement.

First, the act of doing mathematics (production) has an emotional component. From all the frustration, trial and error, dead ends and eventual success, there is a deep satisfaction from the process of searching for new knowledge (Stipek, 2002). Second, every mathematician engages in a critique of mathematics when deciding that a theorem, conjecture or proof is beautiful. Hardy (1992) states that mathematics is beautiful and that there is no enduring ugly mathematics. Third, there are historical and cultural interactions with mathematics in terms of what is valued by mathematicians. The production and justification of mathematics are embedded in personal, cultural and historical context (Lakatos, 1976). Finally, mathematicians make judgements concerning what is beautiful and of value in mathematics, and these judgements are based on assumptions concerning the nature of mathematics (Davis & Hersh, 1981). In summary, mathematicians find mathematics to be beautiful and of value in a cultural and historical context by doing and understanding mathematics.

The activity framework provides us a definition of aesthetics specific to mathematics from our initial general definition. Each activity in the framework is a component of aesthetic activity. Aesthetics is joy and/or pain from the process of doing mathematics and in success from that process, and is insight into the beauty of mathematics and the beauty of the universe via mathematical patterns that is rooted in cultural and historical influences. Mathematics is aesthetic.

Why add an aesthetic image to mathematics education

In this section, we will explore the question of why it would be important to add an aesthetic image to mathematics education. By asking why, we are concerned primarily with the challenge of success for all in mathematics. The importance of recognizing a connection between aesthetics and mathematics is quite simply to bridge the gap between mathematicians and every citizen, perhaps removing some of the mystery associated with mathematics and the activity of mathematicians. We will begin with the importance of aesthetics to education in general and then to mathematics specifically. We conclude that success for all requires that every learner, and not just mathematicians, engage in all aspects of the activity of mathematics (i.e., production, criticism, historical/cultural place, and judgement).

First, aesthetics is important to education in general. To reduce education to fragmented and quantifiable parts, to ignore the aesthetic, is to produce an anxiety in learners toward the world around them (Greene, 1995).

The arts...should become central to the curriculum...thus adding to the modalities by means of which students can make sense of their worlds. With aesthetic experiences a possibility in school, education will be less likely merely to transmit dominant... traditions. Experiences with the arts and the dialogues to which they give rise may give the teachers and learners involved more opportunity for the authentic conversations out of which questioning and critical thinking and, in time, significant inquiries can arise (Greene, 2000, p. 277).

Eisner (1985) also suggests that there is an aesthetic mode for understanding reality, and that this mode of knowing is as important as any other mode of knowing, including those most closely associated with scientific thinking.

Although the aesthetic mode of knowing is associated with the arts and with emotional response, this popular image is inaccurate. Any practitioner of any discipline (whether arts or science) creates knowledge and then makes qualitative judgements concerning the value and

“rightness” of what has been created (Eisner, 1985). There is insight into the world and value judgements are made, consistent with our definition of aesthetic experience that includes insight together with an emotional response. The aesthetic mode of knowing allows a learner to experience knowledge and reality in a way different from a scientific approach. The aesthetic mode is a way of knowing in it’s own right that allows a learner to experience the nuances of any discipline from a different perspective (Eisner, 1985). To suppress or ignore aesthetics is to minimize how we can experience and interpret the world.

We are hinting that aesthetic experience/knowing should be incorporated into educational experience as a valuable perspective of reality. But the aesthetic way of knowing has been neglected in content areas such as science, social studies and mathematics (McClure & Zitlow, 1991). Aesthetic knowing is virtually ignored in current educational practice (Foshay, 1991). Eisner (1985) lists several reasons why the aesthetic mode of knowing should be an integral component of educational practice:

- an aesthetic component to education can be a motivator, in that the student is moved by the topic of study,
- aesthetic knowledge would challenge the doctrine that textbooks/information is sacred, a challenge needed in any democratic society,
- aesthetic experience provides a reward for learning that would be more sustaining than satisfaction from test scores, namely, the joy of learning, and
- aesthetic knowledge is a way of seeing the big picture of a topic, subject or discipline, a skill highly relevant to an educational system that tends to fragment knowledge and thereby promote short-term learning, and a skill that would promote more meaningful learning.

Thus, we believe that aesthetic experience/knowledge should be incorporated into educational goals for all disciplines, including mathematics.

Second, the importance of aesthetic experience can be further elaborated in the case of the specific activities of mathematicians. In popular culture, mathematics is not usually associated with an aesthetic experience (Eisner, 1985). On the other hand, mathematicians (Hardy, 1992; Davis & Hersh, 1981; Poincaré, 1948) consider aesthetic aspects to be central to their activity. In fact, according to Noddings (1985, p. 117), “mathematicians rely heavily on ... aesthetic modes of knowing.” The question becomes: What is aesthetic about the activities of a mathematician?

Eisner (1985) refers to three components of aesthetic experience/knowing: the aesthetic qualities of the final product, the aesthetic criteria for judging the process, and the satisfaction from the activity or process. These three aesthetic components can be applied to the activities of a mathematician. We have already noted that a mathematician gains satisfaction from the activity of doing mathematics. The activity of a mathematician quite often involves deciding on a conjecture to prove, proving the conjecture and verifying the proof. All of these activities require human judgement, a decision as to what is valuable (Brown, 1996). Deciding on a conjecture to prove is dependent on a decision on what is interesting or worth proving, which are the aesthetic qualities of the final product. The act of proving relies on an agreed upon framework for devising a proof. Verifying a proof must have criteria for establishing validity. According to Brown (1996), in deciding what to prove and in the creation and verification of proof, mathematicians “cannot avoid being guided by a criteria of an aesthetic nature that transcends logic...” (p. 1297), which are the aesthetic criteria for judging the process. Thus, Eisner’s three aesthetic components are fundamental to the activities of a mathematician.

The aesthetic nature of the activities of a mathematician is important for education. Critics of current schooling practices point to the fragmentation of learning in general (e.g., Greene, 1995) and of mathematics specifically (e.g., Romberg & Carpenter, 1986). “This fragmentation of mathematics has divorced the subject from reality and from inquiry” (Romberg & Carpenter, 1986, p. 851). The aesthetic aspect of mathematical inquiry has been stripped from educational practice in favor of sequences of procedural learning outcomes. The aesthetic nature of the activities of a mathematician may provide coherence to mathematical knowledge, and thus, increase learners’ appreciation and understanding of mathematics.

Finally, all learners, and not just mathematicians, should engage in all aspects of the activity of mathematics (e.g., production, criticism, historical/cultural place, and judgement). Assuming that activity in mathematics is the domain of mathematicians only is rooted in the notion of “who decides” (Pickard, 2002). We agree that a child will not likely create anything new based on the judgement of a mathematician. But children can construct new mathematics based on their own judgement (Kamii, 1994). For example, consider the case of children’s developing numerical computation skill. Children can create their own algorithms for adding multi-digit numbers (Fuson, Wearne, Hiebert, Murray, Human, Oliver, Carpenter, & Fennema, 1997), compare the algorithms of their peers, and decide which are “best” based on criteria such as efficiency, ease of use, or clarity. Children can decide on the criteria for the best computational algorithm. In fact, the act of creating and comparing with peers likely produces greater understanding (e.g., National Council of Teachers of Mathematics, 2000; Kamii, 1994). Children build-up increasingly efficient and complicated computational algorithms in a pattern similar to the historical advances of mathematicians, so forcing students to acquire/memorize current algorithms “as is” will be detrimental to their understanding (Kamii, 1994). The

historical and cultural place of mathematics is important for children's developing understanding of computational algorithms (and other concepts). When the idea that mathematicians are the sole deciders of mathematical value is challenged, it becomes clear that children can and should engage in the production, criticism, historical/cultural place and judgement of mathematics.

Children can engage in the same activities as mathematicians.

In conclusion, the importance of recognizing aesthetic experience in mathematics can be seen in several ways. The activities of a mathematician rely on aesthetic considerations. The aesthetic experience is a way of knowing that provides an educational opportunity to motivate students' learning and enjoyment of mathematics. The aesthetic experience in mathematics can be a *modus operandus* for bridging the gap between popular views and the mathematicians' views of mathematics, hence demystifying mathematics and making mathematics accessible to all learners. Every child can and should experience the joy of doing mathematics and the beauty of mathematics (Rogers, 1999). Success for all learners of mathematics requires a recognition that learners can and should engage in the activities of mathematicians.

Aesthetics and the nature of mathematics

Philosophies concerning the nature of mathematics play an important role in adding an aesthetic image to mathematics education. In this section, we will describe two philosophies of mathematics, absolutism and social constructivism, and the implications of each philosophy for mathematics education. We conclude by using the activity framework of production, criticism, cultural/historical place, and judgement to suggest that the social constructivism is better suited than absolutism as a philosophy of mathematics that supports adding an aesthetic image to mathematics education.

The absolutist view of the nature of mathematics has its roots in Platonism. Plato was concerned with “certain knowledge,” knowledge that is pure and abstract, not empirical or encumbered by social issues. For Plato, mathematics was certain knowledge. Mathematical Platonism assumes that the existence of mathematics is independent of human beings or human activity. It is the language of the universe. If aliens from Galaxy X visited earth, mathematics would be the language of communication, since no other form of human knowing is universal (Davis & Hersh, 1981). Mathematics is outside and independent of us; it is discovered or observed not invented (Hardy, 1992). Absolutism describes the nature of mathematics as infallible, unquestionable and unchanging objective truth based on *a priori* knowledge, revealed via reason only, and devoid of social context (Ernest, 1998).

The social constructivist view of the nature of mathematics is based largely on the philosophies of quasi-empiricism (e.g., Lakatos), conventionalism (e.g., Wittgenstein) and radical constructivism (e.g., Glasersfeld) (Ernest, 1991). According to Lakatos (1976), there is a cycle of discovery in mathematics that begins with primitive conjecture, continues with proofs and refutations, and ends with a new beginning at an improved conjecture. This cycle suggests that mathematics is fallible since the creation of new mathematics is based on the discovery of flaws in previous conjectures. Further, deduction is not the logic of mathematical discovery (Lakatos, 1976). Wittgenstein (1978) claims that certainty in mathematics is based on linguistic conventions, but these conventions change and evolve, so that mathematics is also continually in a state of change. Glasersfeld (1988) developed a theory of knowledge where the construction of knowledge is unique for each individual while still constrained by “viability” in the experiential world, and not the passive acquisition or discovery of an external and objective reality. Social constructivism is based on the theses that mathematics is fallible and changing, and that

mathematical creation is by invention, not discovery of preexisting knowledge (Ernest, 1991). Mathematics does not exist independently of human beings. Social constructivism is in opposition to absolutism since mathematics is considered to be fallible, changing and the product of human inventiveness (Ernest, 1998).

The opposing philosophical attitudes of absolutism and social constructivism have different implications for mathematics education. In the following, the different implications of the two philosophies in terms of intelligence, knowledge and learning are explored. Although the absolutist philosophy is what drives current and past traditional mathematics curriculum (Ernest, 1991; Romberg, 1992), we suggest that the social constructivist philosophy is better suited to promoting understanding of mathematics for all learners.

The absolutist philosophy has an influence on current views of intelligence. Mathematics as objective and unchanging *a priori* knowledge makes it easy to view mathematical competence as the highest measure of intelligence, relegating empirical, affective and other ways of knowing to lower levels on a hierarchy of intelligence (Eisner, 1985). Measurement of mathematical ability promotes a notion that mathematics is not for all. Mathematics as objective truth disempowers learners since mathematics is infallible and so is not socially responsible for the failure of students; mathematics can be used as a gatekeeper.

Social constructivism provides an image of intelligence that would not disempower students. When viewed as changing, fallible and a product of human inventiveness, mathematics can no longer set itself apart as the highest level of knowing in an hierarchy of intelligence. Since mathematics can not be a measure of ability, students are free to be successful at engaging in the human activity of doing mathematics.

The absolutist philosophy has an influence on current views of mathematical knowledge. Absolutism suggests a curriculum that is based on the known objective truths of mathematics. Knowledge in mathematics is akin to Dewey's (1916) notion of a "record of knowledge." Mathematics is separated from other disciplines, divided into subjects such as arithmetic and geometry, and further subdivided into topics and lessons, each lesson focussing on a specific fact or skill, thus divorcing mathematics from reality (Romberg & Carpenter, 1986). The absolutist view of knowledge is associated with an emphasis on formal knowledge, namely "back to the basics" and procedural skill (Noddings, 1985). The fragmentation of knowledge and emphasis on basic formal procedures results in a diminished impression of the nature of mathematics. Fundamental mathematical activities such as abstracting, inferring, generalizing, inventing, proving and applying are not emphasized or are even ignored (Romberg & Carpenter, 1986).

Social constructivist epistemology suggests a different emphasis than absolutism. The record of knowledge would be seen as a product of creation and a tool for creating further knowledge. The dynamic and subjective nature of mathematical knowledge reinforces the notion that individuals structure knowledge in meaningful ways (Romberg & Carpenter, 1986). There is room to move away from the massive record of knowledge associated with absolutism and find curricular space for other fundamental mathematical activities such as inventing and generalizing.

The absolutist philosophy has an influence on current views of learning. The emphasis on a massive record of knowledge that is infallible requires a view of learning that ranks the absorption of the work of others as the most important (Romberg & Carpenter, 1986). The meaning making of children is minimized since the perceived infallible and static nature of mathematics is prioritized over any learner's way of knowing. Students store up information

isolated from action and purpose (Dewey, 1916) rather than inquire into the world of mathematics.

The social constructivist philosophy suggests that mathematics education should be inquiry based, since mathematics knowledge is dynamic and fallible. The social context of mathematics knowledge makes it necessary and relevant to provide a context for student learning. Learners are empowered to learn mathematics by creating their own mathematical knowledge. Calls for reform in mathematics education (e.g., National Council of Teachers of Mathematics, 2000/1989; Western Canadian Protocol, 1995; Hope, 1990) promote mathematics instruction that emphasizes students' discovery and construction of mathematics for themselves. These calls for reform of mathematics education are consistent with the social constructivist philosophy of mathematics (Ernest, 1991).

Inherent in our descriptions is our belief that the absolutist paradigm is inadequate to the educational goal of success for all in mathematics. Social constructivism provides a philosophy of mathematics that can promote a true appreciation and understanding of mathematics by all learners. But what does all this have to do with adding an aesthetic image to mathematics education? The opposing philosophical positions of absolutism and social constructivism have different implications for aesthetics in mathematics education, which becomes evident when interpreted using the activity framework (i.e., production, criticism, cultural/historical place, and judgement).

First, consider the activity framework in terms of absolutism. The activities of production, criticism, cultural/historical place, and judgement are minimized by an absolutist philosophy. A record of knowledge is absorbed rather than created. Infallible knowledge is a struggle to understand rather than knowledge that is open for critical interpretation. Mathematics

that is objective and infallible minimizes the meaning of finding connections with historical and cultural influences. Absolutism promotes a view that mathematics is forbidding (e.g., "...[the wall's] cold impenetrability..." [Buerk, 1982, p. 19]). Rather than valuing the beauty of mathematics and allowing learners to create their own criteria of beauty in mathematics, absolutism negates personal engagement. In fact, value judgements are meaningless when mathematics has already been glorified as external and objective. Absolutism is the philosophy that is driving current and past curricula and yet the mathematics done in school bears little resemblance to the activity of mathematicians (Romberg, 1992), or to the activity framework for mathematics.

On the other hand, social constructivism easily supports the activity framework. A philosophy of mathematics knowledge as dynamic, fallible, and amenable to the role of inquiry, supports the activity framework as follows:

- Production - Learners can create or invent mathematics (within the constraints of the discipline). There is joy in the invention of patterns (Davis and Hersh, 1981).
- Criticism - The joy of creation prompts a critical activity in which importance is assigned to a pattern and the importance justifies that aspects of reality can be interpreted by patterns (Davis and Hersh, 1981). Establishing a connection between reality and mathematics is a critical activity since decisions must be made as to the value and usefulness of the connection. That mathematics is fallible makes the critical activity more meaningful by allowing for multiple perspectives and meanings; perspectives and meanings brought to light by learners and not just mathematicians.
- Cultural/historical place - "Mathematics, far from being infallible formal knowledge, is a human enterprise..." (Noddings, 1985, p. 130). A human enterprise is not devoid of cultural

and historical meaning. Mathematics is grounded in historical and cultural influences.

Mathematics is a social product (Romberg, 1992).

- Judgement - Fallible and changing knowledge must be subject to decisions concerning value. Criteria for judging must be established, whether these criteria are rooted in beauty, usefulness, efficiency or something else. That mathematics is fallible and created by human (including learners') activity facilitates the knowing of mathematics by children based on their own criteria for judging and their own judgements about mathematics.

Social constructivism as an underpinning philosophy is not evident in current curricula and yet more adequately supports the activity framework for mathematics than absolutism.

An aesthetic view of mathematics would involve an appreciation of the value and beauty of mathematics. The absolutist view of mathematics as unchanging and devoid of social context seems sterile and threatening. Learners will likely see mathematics as an impenetrable wall, aesthetic only in its incomprehensibility. Mathematics as social constructivism provides an opportunity for learners to view mathematics as beautiful, since they create it themselves. Learners are actively involved in the invention of mathematics (Kamii, 1994), and hence can experience the same aesthetic appreciation (i.e., aesthetic quality of product, process and act) that a mathematician experiences. Through an in depth aesthetic appreciation of mathematics, students can come to understand mathematics and the activity of doing mathematics. In summary, social constructivism is better suited than absolutism as an underpinning philosophy supporting the addition of an aesthetic image to mathematics education.

Aesthetics, utility of mathematics and goals of mathematics education

There is a connection between adding an aesthetic image to mathematics education, the utility of mathematics and the goals of mathematics education. In this section, we will describe common problems concerning perceptions of the utility of mathematics and the goals of mathematics education. Then we will use the activity framework to suggest a possible resolution of the problems described and to provide a connection to aesthetics.

The utility of mathematics is highly debated among mathematicians, mathematics educators, business and government, students, and citizens. The different views of these groups illuminate questions concerning what is of value and who decides what is of value. It would be easy to assume that those citizens with the greatest understanding of mathematics (e.g., mathematicians) should decide what is of value. But, mathematicians account for less than 1% of the population, so the decision process would be elitist and in conflict with the democratic values of our society. Exploring the utility of mathematics from the perspective of various groups suggests the difficulty of resolving the contradiction inherent in elitist decisions and democracy.

Although educational interest groups cannot be precisely separated into distinct camps, Williams (1961) provides a categorization of social groups representing three fundamental viewpoints for the goals of mathematics education. First, the *industrial trainer* perspective emphasizes utilitarian goals and training for the workforce. Mathematics education must be career centred, which usually involves “Back to the Basics” and varying curriculum by future occupation. From this perspective, the only mathematics that is of value is that which trains students to be productive citizens whether in career or in daily life. Davis and Hersh (1981) would call this extreme emphasis on utility Maoism.

The second category described by Williams (1961) is the *old humanist*. This view emphasizes liberal education, and is in stark contrast to the industrial trainer position.

Mathematics education should focus on transmitting the body of pure (not applied) mathematical knowledge, with curriculum streamed according to ability. The utility of mathematics should not be derived from its practical use in day to day activities. A mathematician would quite likely endorse an “abstract” utility of mathematics, of which G. H. Hardy is an exemplar. Hardy is a pure mathematician who viewed school mathematics as trivial and hence defensible only by its day-to-day utility. According to Hardy (1992), the real math of real mathematicians is not useful; its justification is in its beauty.

The final category described by Williams (1961) is the *public educator*, which stresses education for all and/or child centredness. Mathematics education is seen as a vehicle for democratic empowerment and/or self-realization. From this perspective, the content of mathematics curriculum is no longer central as with the industrial trainer and old humanist viewpoint. The utility of mathematics is problematized by questions of which mathematics is valuable for which groups, and hence de-emphasized in favor of individual, social and cultural issues. For example, Phillips (1996) has suggested that students should appreciate the creativity, imagination, artistry and pleasure of doing mathematics, in conjunction with the utility of mathematics. Further, students should be exposed to the heroic efforts of mathematicians. Phillips (1996) questions why great mathematicians such as Gauss, Cayley and Reimann have not been popularized.

In the midst of the three groups described by Williams are the perspectives of students and the public. Students may see mathematics as a subject to “get through,” means to a good career, or perhaps even a useful way of thinking. The public largely believes that it is important

for the stability of modern society that all students study a large quantity of mathematics (Hope, 1990).

The contrast between the values of various educational interest groups illustrates the difficult problem of deciding what mathematics is of value. Romberg (1992) clarifies the problem with the question of how much mathematics is enough mathematics. Most would agree that all students should have training in arithmetic since its applications in society are pervasive. But what about a level of understanding needed to critique the impact of technological developments on society? What about mathematical training that is only relevant to specific occupations, or the more sophisticated training needed even at the high school level for the small minority that will become scientists or mathematicians? What about mathematics as a means for personal growth, or to empower democratic participation of all citizens? The problem of the utility of mathematics prompts mathematics educators and other groups to ponder the goals of mathematics.

Lists of goals of mathematics education that are found in current curriculum guides usually try to capture the needs of all groups. Goals usually include the following three components: usefulness to daily life, development of higher order reasoning skills, and the aesthetic value of mathematics (e.g., National Council of Teachers of Mathematics, 2000/1989; Western Canadian Protocol, 1995; Manitoba Education and Training, 1995; Saskatchewan Education, 1992; Hope, 1990). The question of value is not at all satisfied by an amalgamation of goals. For example, why teach factoring? The mathematician probably would argue that algebraic manipulation skill is needed for university training. A parent might argue that factoring is pointless since it has no use in everyday life. The mathematics educator might suggest that there are other topics that could be used to teach higher order thinking skills. The

debate concerning the goals of mathematics education seems intractable when considering the varying perspectives from different groups of citizens in society.

Perhaps the utility of mathematics and the goals of mathematics education can best be understood via the activity framework (i.e. production, criticism, cultural/historical place and judgement). Is producing mathematics of value? If we agree with Hardy (1992) that mathematics is beautiful and also see the nature of mathematics from the social constructivist viewpoint, then producing mathematics will be embodied by the pleasure of discovery, where discovery is the invention of mathematics by learners of mathematics. Producing mathematics is of value since learners are engaged in the creation process, a process that may be frustrating but still joyful in that something is discovered, and since what is discovered is beautiful because the learner created it.

Is criticism of mathematics of value? Students who can perceive and respond to the qualities of mathematics must be engaged in making connections that promote a deeper understanding. Further, perceiving and responding to the qualities of something are higher order thinking skills, and skills that could be applied in any occupation. Developing a critical stance is of value since it is a skill for life-long learning.

Is cultural/historical place of mathematics of value? Mathematics is a human activity that students can appreciate, and see its beauty. Historical accounts can motivate the learning of mathematics (e.g., Sobel & Maletsky, 1999). Kamii (1994), supporting Piaget's notion of the compatibility between the historical genesis of knowledge and children's construction of knowledge, provides evidence that children successfully re-invent increasingly sophisticated arithmetic procedures that parallel historical developments. Cross-cultural connections can provide a deeper image of mathematics (e.g., Powell & Frankenstein, 1997). The cultural trends

that have influenced developments in mathematics (e.g., the computer and cryptography developed from code breaking problems in World War II) can provide greater meaning to the activity of doing mathematics. The examples are endless since human interaction with the world is endless. The cultural/historical place of mathematics provides further meaning of the value of mathematics for learners.

Is judgement of mathematics of value? The mathematician must establish criteria for making judgements and then make judgements concerning what to prove and whether a proof is valid. Students can be empowered by being given permission to make judgements concerning mathematics. Judgement is a skill that can be used to decide the value of mathematics by all citizens. Judgment of mathematics is of value since citizens can decide for themselves the value of mathematics.

To summarize, the activity framework provides an alternative view to the question of the utility of mathematics and the goals of mathematics education. We are suggesting that the activity framework provides a litmus test for assessing possible answers to the problem of deciding what mathematics is of value and what should be the goals of mathematics education. The aesthetic nature of mathematics provides an alternative view to the question of what is of value and who decides what is of value – another reason why an aesthetic image should be added to mathematics education.

Conclusion

Calls for reforms in mathematics education are currently predominant, mainly rooted in the concern that mathematics is important for all citizens but that not all citizens are successful at school mathematics. The calls for reform are built on a belief that all students can learn mathematics (NCTM, 2000). We believe that the goal of success for all cannot be achieved

without providing opportunities for students to experience an aesthetic image of mathematics.

Learners can and should engage in the same type of activities as mathematicians, and these types of activities are aesthetic.

We believe that teachers can be supported in an effort to add an aesthetic image to mathematics classroom activities. Activities in mathematics classrooms must be more like the activity of mathematicians. The activity framework that we have described provides a model for describing what mathematicians actually do, and what students can and should do to be successful in mathematics. The activity framework can also provide a framework for developing activities that promote an aesthetic image of mathematics. To this end, mathematics educators should explore teachers' current efforts to incorporate aesthetics into their teaching of mathematics, and fully develop new and innovative instructional lessons, resources and curricula that promote an aesthetic image of mathematics.

Endnotes

1. Eisner referred to judgement as aesthetics. We deal with the semantic contradiction by using a more encompassing definition of aesthetics.

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