

# TOWARD HOW TO ADD AN AESTHETIC IMAGE TO MATHEMATICS EDUCATION

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**Abstract:** The goal of this paper is to suggest how an aesthetic image can be added to mathematics education. Calls for reform in mathematics education (e.g., National Council for Teachers of Mathematics, 1989, 2000) are premised on shifting teacher attention from an absolutist toward a social constructivist philosophy of mathematics and mathematics education. The goal of reform is success for all students of mathematics. In a previous paper, I used ideas from art education to suggest that success for all might be achieved by fostering an aesthetic image of mathematics (Betts & McNaughton, 2003). This paper continues the argument by shifting from questions of why to how.

## Introduction

In 1989, the National Council of Teachers of Mathematics (NCTM) published an influential mathematics curriculum framework document (see National Council of Teachers of Mathematics, 1989) calling for reform in mathematics and success for all mathematics students. This call for reform is based on social constructivism as a philosophy of mathematics and mathematics education. Proponents of social constructivism, as a philosophy of mathematics, view mathematics as fallible, changing and a product of human endeavor (Ernest, 1998). For example, the genesis of mathematical knowledge is a continual cycle of conjecture and refutation, and deduction is not the logic of mathematical discovery (Lakatos, 1976). In other words, mathematical truths are socially constructed via a process of error correction. Social constructivism as a philosophy of mathematics education is based on fostering opportunities for students to experience, discover, discuss and re-construct the socially negotiated nature of mathematics.

In a previous paper (Betts & McNaughton, 2003), I linked social constructivism to an aesthetic image of mathematics and concluded that such an image would be essential for achieving the goal of success for all mathematics students. This link was accomplished by transferring Eisner's (1988) activity framework for arts based education onto current ideas within mathematics education. This framework has four components: production, criticism, cultural/historical place, and judgement (Eisner, 1988), which I summarize below in terms of a social constructivist vision of the activities of mathematicians. Production refers to the invention of mathematics. Criticism is the perception of and response to the qualities of mathematics. Cultural/historical place is the understanding that mathematics doesn't emerge in a vacuum, that there are always cultural and historical influences on and from mathematics. Judgement is the valuing of mathematics and the underlying criteria for making value judgements. This framework provides a theoretical tool for challenging underlying assumptions concerning the nature of math and for finding the connection between aesthetics and mathematics (Betts & McNaughton, 2003).

In this paper, I move toward how to add an aesthetic image to mathematics education. In what follows, I will provide a series of examples, analyzed in terms of Eisner's activity framework, social constructivism and traditional views of mathematics that may short circuit the

aesthetic dimension. These examples can be divided into two categories: seeing mathematics in art or nature (*Artistic as Mathematics*), and seeing art or nature in mathematics (*Mathematics as Artistic*). The final section briefly considers how these ideas may be located within mathematics curriculum.

### **Artistic as Mathematics**

Seeing mathematics in art or nature is the recognition that art and nature can be described mathematically. Descriptors such as harmony, balance and symmetry when applied to works of art or nature can be operationalized using mathematical constructions of these descriptors. Mathematizing art or nature can be further understood with several examples. I will describe seven examples, two being historical, one contemporary, three that endeavor to bring Artistic as Mathematics into the classroom, and one that embodies a current revolution in mathematics.

My first example of Artistic as Mathematics is historically situated in the culture of fifteenth century Italy. The mathematics of fifteenth century Italy was rooted in commerce. Arithmetic was the knowledge of proportion. Geometry was the knowledge of Euclidean forms. Piero della Francesca was an artist in this time period who allowed his paintings to be influenced by the mathematical culture of the time. In particular, proportion and Euclidean forms are elements of his paintings (Baxandall, 1998). Francesca's paintings are interpreted aesthetically using the mathematics of fifteenth century Italy. In this example, Artistic as Mathematics is the culturally and historically driven mathematical interpretation of a work of art.

My second example of Artistic as Mathematics involves the work of an artist from the early 1900's. Jay Hambidge was an artist and designer whose work came into vogue between 1916 and 1923. Hambidge conceptualized his work from a Euclidean perspective. His designs were to be interpreted as mathematical descriptions of beauty (McWhinnie, 1991). Artistic as Mathematics is again an interpretation of mathematical form in beautiful art.

My third example comes from the work of the Escher. Escher considered himself to be a graphic designer. He created patterns that challenge our notions of perspective and delighted our sense of pattern. His ideas were often inspired by mathematicians and mathematicians have analyzed the symmetry and harmony evident in his work (Escher, 1989). Although Escher did not identify himself with artists, his work is widely considered to be aesthetic and a representation of mathematical ideas. Escher's work represents a contemporary example of Artistic as Mathematics through the interpretation of mathematical form in works of art.

Each of the examples above suggests that the disciplines of art and mathematics can be integrated. Several curricula for the arts base learning on an integration of art and mathematics. A complete catalogue of such integrated curricula is beyond the scope of this paper, but it should be noted that many have elements that are rooted in Artistic as Mathematics interpretations of art. Two examples will serve to illustrate the potential role of Artistic as Mathematics in school curricula.

Armistead (1996) suggests an arts based program that is based on constructivist principles. Students are lead to discover mathematics (and other disciplines) while engaged in arts enriched program activities (Armistead, 1996). The integration of disciplines serves as a catalyst to interpret art by discovering math; Artistic as Mathematics is conceptualized via integration of disciplines.

Nelson and Leutzinger (1979) suggest an integration of art and geometry. They suggest an activity where students ask aesthetic questions about objects, which involve differences and similarities in shape (Nelson & Leutzinger, 1979). Students are encouraged to explore what they

see in objects and interpret seeing geometrically. Artistic as Mathematics is embedded in making connections between what is seen and geometry.

Patterson (2001) has also developed a method of teaching art using geometry and computer technology, and the artwork of Charles Demuth and Charles Sheeler. Demuth and Sheeler are 20<sup>th</sup> century artists who integrated the “geometric patterns of Cubism with the details of their industrial environment” (Patterson, 2001, p. 46). Students are encouraged to critique and identify geometric patterns in the paintings of these artists. The students then used Claris Works to create their own computer-generated pictures by combining various geometric shapes (e.g., cube, cone, sphere, and pyramid). Again, Artistic as Mathematics is promoted in the classroom by making connections between art and geometry.

My final example of Artistic as Mathematics concerns the current revolution in mathematics sparked by Chaos theory. Chaos theory is the mathematics of clouds, fern leaves, fluids, wild life populations, blood cells, a beating heart and anything else that has order masquerading as randomness (Gleick, 1987). In short, Chaos theory is a mathematics of nature. The pictures produced by Chaos theory are pictures of the seemingly complex and disordered objects and systems in nature. Creating these pictures (called fractals) of nature requires a computer to calculate iterations repeatedly using equations as simple as  $x^2 + 1$  or shapes as simple as triangles, which involves mathematics that any high school student could engage in. The surprise is in the realization that the complex can be described using simple models. Although the theoretical mathematics developed in Chaos theory is beyond K-12 mathematics, it provides a unique opportunity for any learner to gain access to sophisticated mathematics, namely, to an understanding of complex and beautiful shapes. For example, activities can be developed using fractals that inspire junior high students to explore, understand and calculate areas of triangles and squares (e.g., Goodman, 2000). The production of fractals is an example of Artistic as Mathematics that could have intriguing implications for mathematics education.

The above examples of Artistic as Mathematics can be analyzed in terms of the activity framework. The activity of production can be achieved by encouraging students to produce their own Escher like (or any other artist) pictures by hand or by using computer technology, to produce their own shapes using techniques from Chaos theory, and to produce their own descriptions of the mathematics in paintings and fractals. The activity of criticism can be achieved by having students perceive and critique the mathematics evident in various works of art. The activity of cultural/historical place is evident in the analysis of the artwork of Francesca, Hambidge, Escher, Demuth, Sheeler and others in terms of mathematical principles such as Euclidean geometry and symmetry. The activity of judgement can be achieved by having learners compare the mathematics in different works of art, and developing their own criteria for assigning value.

### **Mathematics as Artistic**

Artistic as Mathematics could be explored further and is useful to my goal of adding an aesthetic image to mathematics education but has limited value. Mathematics is incapable of completely describing the aesthetic experience of humans while engaging with works of art. For example, to say that music can be described completely by patterns of fractions (e.g., patterns of quarter notes) is to eliminate the role of inspiration and perspiration in the creation of art (Davis & Hersh, 1981). Adding an aesthetic image of mathematics requires an exploration of the inspiration and perspiration in the activity of mathematics. Mathematics can be viewed as an artistic field; this is the essence of Mathematics as Artistic. The remainder of this section will

focus on adding an aesthetic image to mathematics education by exploring Mathematics as Artistic.

Mathematics as Artistic is the recognition that mathematics has aesthetic qualities. “Aristotle believed that mathematics was the greatest aesthetic” (quoted in Davis & Hersh, 1981, p. 168). “Beauty is the first test: there is no permanent place in the world for ugly mathematics.” (Hardy, 1992, p. 85). Mathematicians appreciate the beauty of a theorem and elegance of a proof, gain pleasure from the discovery of a mathematical concept, and are inspired by the seemingly magical connections in mathematics. The student of mathematics does experience pleasure in the discovery and can learn to appreciate mathematics in similar ways to those of a mathematician, although the appreciation is of school mathematics. The learner of mathematics should develop an appreciation for mathematics, experience the joy of mathematics, and explore the beauty of mathematics (Rogers, 1999). Mathematics as Artistic should not be solely the purview of mathematicians.

The Absolutist philosophy of mathematics promotes the assumption that Mathematics as Artistic can only be the purview of mathematicians. The image of mathematics within popular culture and the mathematics taught in schools is rooted in Absolutist views and propagated by the status quo that describes mathematics as a complete body of knowledge, obscuring what it means to be a mathematician and to do mathematics. A mathematician solves problems, finds patterns, finds models to describe the world, develops abstractions, invents conjectures and then proves and applies these results (Romberg, 1992). There is an art and craft to teaching (Eisner, 1999). Extrapolating Eisner’s definition from the activity of teaching to the activity of doing mathematics, the craft of mathematics would involve facility in performing known skills and routines. From Romberg’s list above, proving results and applying these results in known situations is the craft of mathematics. School mathematics is mostly the application of known skills (e.g., the addition algorithm). The popular image of mathematics is also mostly the craft of mathematics. The art of mathematics is like the activity of an artist: it is the invention of new forms from what is known; it is everything else in Romberg’s list. The art of mathematics is not evident in school mathematics or the popular image of mathematics. It is not surprising that Mathematics as Artistic would be considered accessible only to mathematicians.

A simple example of how Mathematics as Artistic seems inaccessible to non-mathematicians comes from school textbooks. Most mathematical texts are laid out by a simple formula: definition, example, theorem, and proof. The presentation is neat and organized and only inherently illustrates the basics of the craft of mathematics. There is no indication of the inspiration/perspiration of discovery, no indication of how the mathematics was created, or of the actual activities of mathematics (Phillips, 1996). The art of mathematics is entirely missing from textbook presentations. Ironically, the aesthetic demand of the craft of mathematics for efficiency and elegance has worked at cross-purposes in the creation of a limited popular image of mathematics (Davis & Hersh, 1981).

Another example of how Mathematics as Artistic seems inaccessible to non-mathematicians comes from the problematic nature of value judgements in mathematics. Mathematicians are so specialized that very few mathematicians let alone non-mathematicians are qualified to assess the validity of another’s work. Aesthetic appreciation becomes elitist. Put another way, it has been estimated that there are about 200,000 new theorems added to the body of mathematical knowledge yearly (Davis & Hersh, 1981). It is impossible to keep up with mathematical production, let alone establish criteria for beauty in mathematics. Hardy’s claim that there is “no permanent place for ugly mathematics” is immersed in assumptions about who

decides that are exclusive to mathematicians within specific disciplines. The only recourse for criteria of beauty is an absolutist perspective that frees the mathematician of responsibility for his/her discoveries. Mathematics in the absolutist perspective exists *a priori*, and hence is absolved of responsibility for the creation of criteria of beauty. When the nature of mathematics is considered to be Absolute, then Mathematics as Artistic must be the exclusive domain of mathematicians. Only by extensive training can a person truly appreciate mathematics.

Challenging the Absolute view of the nature of mathematics and the resulting popular image of mathematics suggests that Mathematics as Artistic need not be the sole domain of mathematicians. In what follows, I will describe several examples that could be used to make Mathematics as Artistic accessible to non-mathematicians. Inherent in all these examples is the Social Constructivist philosophy and the activity framework.

To illustrate the possibility of making Mathematics as Artistic accessible to non-mathematicians, consider the example of the golden ratio, namely,  $\frac{1}{2}(1 + \sqrt{5})$ . The ancient Greeks regarded this ratio as nature's ratio. For example, the most aesthetically pleasing rectangle is one whose sides are in the ratio  $\frac{1}{2}(1 + \sqrt{5})$ . This interpretation of beauty is rooted in the cultural assumptions of the time, but this interpretation of the golden ratio is generally not held today.

The story of the golden ratio doesn't end with the Greeks. This ratio has emerged in several seemingly unconnected mathematical fields. For example, the golden ratio is the limit of ratios of successive terms of a Fibonacci sequence, and equivalent to a certain infinite continued fraction. Further, the golden ratio is found in nature (Dalton, 1983), which is perhaps an eerie echo of the Greeks. Mathematicians were surprised to find the golden ratio of relevance in such diverse and unexpected places. The aesthetic experience is amazement. The rush was on to provide an overall theory to explain the presence of the golden ratio in such diverse settings. It wasn't long till mathematicians devised a theory that explained the golden ratio's diverse instances.

That the golden ratio provides an opportunity for learner accessibility to Mathematics as Artistic can be seen in terms of the mathematical activity framework. Students exposed to several different occurrences of the golden ratio in mathematics and nature can experience the amazement of mathematics. Students would likely be so intrigued that they wanted to know why the golden ratio would occur in such diverse places. Students asking this type of question would be no different from the activity of a mathematician; this would be the activity of criticism. I suspect that students could discover some of the occurrences of the golden ratio; this is the mathematical activity of production. Exploring the ancient Greeks' knowledge of the golden ratio would be the activity of cultural/historical place. I even suspect that students could be told about advanced instances of the golden ratio (e.g., infinite continued fractions). Although I would not expect students to truly understand infinite continued fractions, students can still assess the appeal or value of the golden ratio based on simple and difficult occurrences of the golden ratio; this is the activity of judgement. Thus, an aesthetic appreciation of the golden ratio can be rooted in the activity framework.

The golden ratio example hints at the possibility that sophisticated mathematical training is not necessary for accessibility to Mathematics as Artistic. The golden ratio is accessible to school children and there could even be exposure to difficult mathematics (e.g., infinite continued fractions) without complete understanding to add to the aesthetic appreciation of the golden ratio.

Another example that challenges the training assumption and illustrates the possibility of making Mathematics as Artistic accessible to non-mathematicians comes from a classical

theorem of mathematics proved by the ancient Greeks. This theorem is Pythagoras' result that the square root of two is irrational. Hardy (1992) lists this result and a few others as evidence of the beauty of mathematics that all people can recognize. He suggests that anyone who has understood the above theorem will not dispute its beauty. Hardy is practically saying that if you don't find math beautiful then you are dumb! Would the average person "understand" and hence appreciate the above mathematical result, which any mathematician would describe as simple but still beautiful despite its simplicity? I suspect that the average person felt/thought the result was at best mildly interesting and wondered at its value. At worst, there was a complete lack of understanding and an understandable rebellion against "those damn mathematicians who come up with this crap!"

I can see the beauty of Pythagoras' result but I have extensive training in mathematics. It could be said that I have been indoctrinated into the belief that mathematics is beautiful by our training in mathematics. I'd like to think that training helps me to realize the beauty of mathematics: its simplicity, power, utility, multi-connectedness, and other adjectives often used to describe a painting, poem or song. What about the person without sophisticated training in mathematics?

Hardy (1992) seems to think that any intelligent person can understand Pythagoras' result and that understanding is sufficient for an appreciation of beauty. Hardy would suggest that from understanding there is an appreciation of the profoundness of this result, namely that arithmetic is not enough to describe many quantitative situations that occur in nature. I think Hardy's description is only the starting point. The aesthetic appreciation of Pythagoras' result will become accessible by continuing to make connections. Pythagoras' result can be seen as beautiful by its diverse connections, just as the golden ratio is beautiful because of all the connections it makes to mathematics and nature.

I will illustrate my point by building an aesthetic image through connection building to the result of Pythagoras. First, consider the length of the diagonal of a square. Using high school mathematics (e.g., another well known theorem of Pythagoras that in any right-angle triangle  $c^2=a^2+b^2$ ), it can be shown that the length of the diagonal of a unit square is the square root of 2. Before Pythagoras, there was no number that described this length since all numbers were considered to be rational (i.e., fractions or whole numbers). Imagine the mental shift required in embracing the existence of new numbers. The profoundness is in what is missing. Arithmetic is not enough. The ancient Greek mathematicians needed to create a whole new mathematics, the mathematics of real numbers that accounted for the existence of irrational numbers.

It is a common theme in mathematics that the mathematicians of a certain era needed to embrace a profound shift in thinking based on the realization that something was missing. This theme provides another connection to the result of Pythagoras. Contemporary mathematics would describe a connection between the result of Pythagoras and the equation  $x^2=2$ . This equation has no solution if there are only rational numbers since its solution is the square root of 2. The result of Pythagoras solved this problem by proving what was missing. A similar profound insight occurred much more recently. The equation  $x^2=-1$  has no solution until mathematicians realized that the mathematics of real numbers is not enough. The profound shift in thought needed to embrace this result is evident in the language to describe the solutions to the equation  $x^2=-1$ . These solutions were called imaginary numbers, as if they didn't really exist, and now are called complex numbers, as if they are much more difficult than real numbers.

Irrational (read insane) numbers were to the ancient Greeks as imaginary numbers are to contemporary mathematics.

The story of what numbers are missing did not end with imaginary numbers. What if imaginary numbers are not enough? Just the question is profound. It would eventually be proved that imaginary numbers were indeed, finally, enough! A student doesn't need to understand this proof (a proof that requires graduate level training) to appreciate the profoundness of asking the question, "What is missing?" A student can appreciate the result of Pythagoras by seeing the connection to more contemporary mathematics. Mathematics as Aesthetics is accessible to the non-mathematician by connection making and the profoundness of the underlying theme of the connections.

That the connection making to Pythagoras' result provides an opportunity for learner accessibility to Mathematics as Artistic can be seen in terms of the mathematical activity framework. Students exposed to the missing numbers question from the perspective of the ancient Greeks and more contemporary mathematicians are engaging in the activity of cultural/historical place. Connection making is a critical activity since the connections promote perceiving the quality of mathematics (e.g., the perennial idea that something is missing) and responding to the quality of mathematics (e.g., it is profound that mathematicians throughout history can be challenged by similar looking situations such as solving  $x^2 = 2$  and  $x^2 = -1$ ). Despite not understanding the final proof that the imaginary numbers are enough, a student can still engage in questions concerning the value of the result and the perceived profoundness of this situation; this is the activity of judgement. Thus, an aesthetic appreciation of the result of Pythagoras can be rooted in the activity framework.

Another approach that facilitates accessibility to Mathematics as Artistic comes from the proving of mathematical results. Often in school mathematics, if a result is proved, only one proof is given and the proof is transmitted by the teacher to the students for absorption. This image of proof is rooted in Absolutism and conceals both the discovery and appreciation of proof. Winicki (1998) suggests that students should be encouraged to devise their own proofs, be exposed to various proofs of the same theorem including the proofs developed by their peers, and be encouraged to decide the aesthetic value of each proof using descriptors such as elegant, surprising, concise and challenging. In particular, Winnicki (1998) encouraged pre-service mathematics teachers to create their own proofs and then critically respond to several distinct proofs of one result. Responses focused on surprise, frustration, unexpected connections, challenge, economy of expression, advantages and limitations of algebraic and visual aspects, and proofs that explain versus proofs that prove (Winicki-Landman, 1998). These responses are reminiscent of previous descriptions by mathematicians of the beauty of mathematics. The difference now is that Mathematics as Artistic is no longer solely the domain of mathematicians. Who decides on the value of a proof are the students!

At the core of these examples of Mathematics as Artistic is a decision concerning what is beautiful and who decides. Absolutists would suggest that only persons with extensive training could decide. Social Constructivists would suggest that students should discover for themselves, hence there is a decision component in that children are ultimately deciding what to discover. Winnicki would go even further by suggesting that students can not only decide, but also decide on the criteria for deciding. Letting students decide has been supported by others. For example, educators can and should provide opportunities for students to respond aesthetically to content from all disciplines (McClure & Zitlow, 1991). Although McClure and Zitlow (1991) are writing from the perspective of Language Arts, there are implications for mathematics education.

Consistent with calls for reform (e.g., National Council of Teachers of Mathematics, 2000) that emphasize reasoning and understanding, encouraging students to create an aesthetic response (e.g., poem, song) to a mathematical concept allows students to reflect, listen, savour, explore, contemplate, and then respond rather than just paraphrase or summarize. The descriptors are again reminiscent of the activity of a mathematician, but the activity is no longer exclusionary.

Further, letting students decide can be rooted in the activity framework. Winnicki (1998) and McClure and Zitlow (1991) respectively encourage students to create their own proofs and their own artistic response; this is the production activity. Deciding on which proof and on how to artistically respond is critical activity. The activity of cultural/historical place can be seen in terms of the classroom culture where comparing the proofs or artistic responses of learning peers is no different from the activities that are associated with the culture of mathematicians (Romberg, 1992). The creation of criteria for evaluating the beauty of a proof or artistic response is the activity of judgement. The activities of exploring proofs and responding artistically to mathematics facilitate accessibility to Mathematics as Artistic since these activities can be rooted in the activity framework.

Foshay (1991) described several topics from mathematics that could be used to promote other ways of knowing. These topics could be used to facilitate accessibility to Mathematics as Artistic. For example, consider the topic of arithmetic. The ability to calculate with numbers is considered a basic skill for survival in modern society. The utility of arithmetic has perhaps hampered recognition of the amazing power of numbers. The popular belief is that arithmetic is mundane, but this image is accepted because of utility considerations. An historical and cultural perspective suggests the amazing power of the modern number system (Foshay, 1991). I will develop three examples from arithmetic using the activity of cultural/historical place to illustrate its aesthetic element.

The first example is the modern number notational system. Using 10 symbols (1, 2, 3, 4, 5, 6, 7, 8, 9, 0) it is possible to create an infinite number of distinct numbers. The reason is because of the amazing power of place value notation. Before the development of place value like notational systems, the expression of numerical quantities was laborious and limited expression to small numbers. For example, knowledge of 1000 is impossible when your counting system is limited to the labeling of body parts, as in many tribal cultures. All scientific knowledge hinges on the ability to represent and understand very large (and very small) numbers.

A second example from arithmetic is calculation algorithms. The Roman numeral system can produce very large numbers with a limited number of symbols (so it is an improvement of previous systems), but it still has computational shortcomings (Barrow, 1992). Adding two digit numbers using the modern notation is straightforward compared to the same calculation using Roman numerals.

A third example from arithmetic involves the power and utility of integers. The ancient Greeks had no notion of zero or negative numbers. Without zero, the modern place value system cannot develop. Without negative numbers, the notion of comparing the size of numbers is very difficult. There is also widespread application of negative numbers in science and every day life. It might even be relevant to suggest to students that it is okay to have difficulties with negative numbers, Plato didn't even know about them!

There are many other possible examples of the aesthetic element of arithmetic. Children could explore and compare the number systems of other cultures, such as the theological role of number in Mayan culture. Modern cultural differences could be explored by comparing English

and Asian number words. The point is that the activity of cultural/historical place can provide learners with a very real sense of the amazing qualities of numbers and arithmetic, rather than the image that arithmetic is only a mundane tool for solving common problems.

Foshay (1991) described other mathematical topics that could be used to facilitate an appreciation of Mathematics as Artistic, including uncertainty and faith. The popular image of mathematics makes it surprising that words normally associated with theology also have an expression in mathematics. I will give two brief examples of the connection between uncertainty and faith and mathematics. First, mathematics is faith in pattern making. When a pattern is detected in nature, there is faith that it is really a pattern. The fundamental mathematical activity of abstracting and generalizing hinges on the belief that it is indeed possible to generalize that the pattern is indeed real. Second, mathematics has uncertainty. The Absolutist philosophy certainly contradicts the notion of uncertainty, and this is why the popular image of mathematics does not have an uncertainty component. The uncertainty component is a new realization in mathematics. Contemporary mathematicians recently proved that there are some conjectures that can't be proved (Davis & Hersh, 1981). There are conjectures that humanity will never know whether they are true or false. I feel a sense of awe at the realization that mathematicians also rely on faith and live in an uncertain world. Any learner can be inspired and challenged to know mathematics in a different way when exposed to the faith and uncertainty of the mathematical activity of production.

### **Curriculum for Mathematics as Aesthetics**

An aesthetic image is largely missing in school mathematics, which can be seen in terms of curriculum and practice. Historically, mathematics curricula have ignored the aesthetic component of mathematics. There were basically three goals of mathematics education: technological and socio-economic development, maintenance of society, and skills for life as a citizen (Niss, 1996). Although current curricula do add an aesthetic goal to the list, usually in the form of a statement that students should value and appreciate mathematics (e.g., National Council of Teachers of Mathematics, 1989, 2000; Western Canadian Protocol, 1995; Manitoba Education and Training, 1995; Saskatchewan Education, 1992; Hope, 1990), this goal statement is mostly lip service. An analysis of outcome lists in curricula (e.g., Manitoba Education and Training, 1995; Saskatchewan Education, 1992) shows that ALL outcomes are rooted in activities of calculate, identify and describe which are procedural in nature. Performing algorithms will not inspire students to value and appreciate mathematics, let alone experience an aesthetic mode of knowing mathematics.

The practice in schools also does not support students' experiencing an aesthetic image of mathematics. The fragmentation of mathematics into a massive list of specific outcomes forces teachers to suppress (consciously or unconsciously) the aesthetic quality of mathematics. Further, the Absolutist philosophy that drives curricula, textbook structure and the popular image of mathematics is very pervasive. The resources and philosophical underpinnings are not available to support an effort by teachers to add an aesthetic image of mathematics to classroom practice.

Several researchers have made curricular suggestions on how to effectively promote an aesthetic image in school mathematics. Rogers (1999) suggests that NCTM's standards for mathematics education should also include a "Mathematics as Art" strand. Others (e.g., Kenney, 1996; Fellows, 1991) suggest that there should be a greater emphasis on discrete mathematics in school since it has aesthetic appeal, requires very little mathematical training to become

accessible, and is a cutting edge field of current mathematical inquiry that has many real world applications. I think that the Connections principle developed by NCTM (1989, 2000) could be updated to promote aesthetic appreciation of mathematics. Although NCTM recognizes the importance of connection making for improving understanding, the possible aesthetic component has been missed or ignored.

The problem with suggestions for curricular change is that they always involve adding more “stuff.” Curriculum documents are already overloaded and by definition promote a fragmentation that hinders aesthetic appreciation. Egan (1987) has suggested a framework for challenging the fragmentation of learning. First, Egan challenges the underlying assumptions of sequence. Outcomes in mathematics are sequenced according to the assumption that learning proceeds from the concrete to the abstract and from the known to the unknown. These assumptions ignore the powerful and imaginative intellectual abilities that children already have when they come to school (Egan, 1986). Second, Egan suggests that the image of “story telling” challenges sequence assumptions and provides a model for developing classroom activities that tap into children’s abilities. By challenging sequence assumptions and indirectly the assumption that appreciation requires in-depth knowledge, Egan’s model will likely be amenable to aesthetic considerations. The story form embodies artistic forms both in creation and appreciation, so the story form model could be used to promote an aesthetic image of mathematics.

Finally, I believe that teachers can be supported in an effort to add an aesthetic image to classroom activities while still working with current curricula. Activities in mathematics classrooms must be more like the activity of mathematicians. The activity framework that I described in the introduction provides a model for describing what mathematicians actually do, and what students can and should do to be successful in mathematics. The activity framework can also provide a litmus test for developing activities that promote an aesthetic image of mathematics. I have already given several examples in the previous section that passed the litmus test of the activity framework. These activities are not in the form of lesson plans that teachers can immediately use. Converting an idea into an effectively delivered lesson involves the art and craft of teachers, and is beyond the scope of this paper.

### **Conclusion**

All of the examples I have described in this paper suggest ways in which an appreciation of Mathematics as Artistic and Artistic as Mathematics can be promoted among learners of mathematics. In these examples, it becomes evident that the popular image of mathematics, rooted in Absolutism, hinders an aesthetic knowing of mathematics. To add an aesthetic image to mathematics education, all stakeholders must challenge the popular image. A social constructivist philosophy is the starting point to effectively adding an aesthetic image to mathematics education.

Calls for reforms in mathematics education are currently predominant, mainly rooted in the concern that mathematics is important for all citizens but that not all citizens are successful at school mathematics. The calls for reform are built on a belief that all students can learn mathematics and hinge on new views concerning the activity of mathematics. Learners can and should engage in the same type of activities as mathematicians, and these types of activities are aesthetic. Every learner can aesthetically know about mathematics, and that aesthetic knowing is vital to the success of all learners in mathematics.

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