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An Exploration of the Effects of a Practicum-Based Mathematics Methods Course  
on the Beliefs of Elementary Preservice Teachers

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## Abstract

Effects of a practicum-based elementary mathematics methods course on the beliefs of preservice teachers regarding conceptual knowledge in school mathematics were explored using a pre-post design. The intensity of those beliefs was assessed before and after the methods course using the IMAP Web-Based Beliefs Survey, an instrument constructed by the “Integrating Mathematics and Pedagogy” (IMAP) research group at San Diego State University. The IMAP Beliefs Survey assesses belief intensity by asking respondents to react to and analyze written and video cases (Ambrose, Clement, Philipp, & Chauvot, 2004; Philipp, Ambrose, Clement, Sowder, Schappelle, Sowder, Chauvot, & Thanheiser, 2005). Its use with preservice teachers enrolled in mathematics content courses for elementary education majors has been previously reported (Ambrose et al., 2004). In the current study, significant pre-post effects regarding belief changes were obtained in the desired direction, thus suggesting the appropriateness of conducting a future, more extensive experimental investigation of the effects of a practicum-based methods course on teacher beliefs.

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Introduction

Mathematics teacher educators in the United States face critical challenges related to the preparation of future teachers of mathematics. Data from the 2003 *Trends in International Mathematics and Science Study* (National Center for Education Statistics, n.d.) reveal that, despite some recent gains, American students continue to lag behind many of their international counterparts in mathematics competency at both the fourth and eighth grade levels. Ma, in *Knowing and Teaching Elementary Mathematics* (1999), suggested one possible reason for the plethora of troubling data regarding student achievement in mathematics. She described “the vicious circle formed by low-quality mathematics education and low-quality teacher knowledge of school mathematics” (p. 149). In other words, teachers of mathematics, especially those who teach at the elementary level, generally do not possess the knowledge necessary to help empower future generations of adults mathematically. Therefore, without some sort of ongoing, widespread, and powerfully effective intervention, U.S. students appear destined to continued mediocrity in mathematics performance.

Ma (1999) further postulated that teacher preparation might be the force needed to break the above-mentioned “vicious circle” (p. 149). This emphasis on the role of preservice teacher preparation is in some ways rather ironic. Preservice teacher preparation is viewed as a vehicle for improving inservice teacher performance. However, overwhelming evidence from multiple disciplines, including mathematics education, indicates that preservice preparation is significantly enhanced by strong and consistent preservice/in-service teacher partnerships (e.g., Kagan, 1992; Richardson, 1996). In other words, to break the vicious circle that characterizes

inservice teacher performance, those same inservice teachers need to provide laboratories for preservice teachers to explore, investigate, and make sense of conceptual mathematics methodology.

Promoting teacher change is complicated at best. Investigators have emphasized the role of beliefs as an important consideration in encouraging change (e.g., Civil, 1993; Vacc & Bright, 1999). In a review of research regarding teachers' beliefs, Pajares (1992) suggested that teacher change research be conducted from a perspective that focuses on "the things and ways that teachers believe" because "beliefs are the best indicators of the decisions individuals make throughout their lives" (p. 307). Hersh (1986) stated that "one's conception of what mathematics is affects one's conception of how it should be presented" (p. 13). Fullan (2001) emphasized the central role of beliefs in bringing about educational change: "Changes in beliefs and understanding . . . are the foundation [for] achieving lasting reform" (p. 45).

Changing beliefs is generally considered to be challenging in teacher education settings; there is much empirical evidence to support the long-standing assumption that teachers' belief systems tend to be deeply entrenched (Pajares, 1992). Changes in beliefs "challenge the core values held by individuals regarding the purposes of education; moreover, beliefs are often not explicit, discussed, or understood, but rather are buried at the level of unstated assumptions" (Fullan, 2001, p. 44). According to Mewborn (2002), teacher beliefs about mathematics teaching and learning may be embedded within "wider beliefs about education, human relationships, and a person's role in society" (p. 27).

Teachers' prior experiences as students may be particularly influential. Unlike most students choosing to major in other disciplines, potential teachers enter their chosen field with many years of experience as learners in the context in which they will work. Their experience as

students in mathematics classrooms influences their perceptions of what mathematics is, what it means to “do” mathematics, and the role school mathematics plays in society. Thus, the nature of their beliefs regarding how mathematics should be taught may be deeply influenced as well. The difficulty of changing beliefs is further illustrated by the findings of Steinberg, Empson, and Carpenter (2004), who noted that even when teachers embrace new beliefs, they may not necessarily sustain that change or continue to change.

Ernest’s (1989) landmark study demonstrated that the following beliefs commonly possessed by preservice teachers have powerful effects on the decisions they make while teaching mathematics:

1. Mathematics is a set of rules to be learned rather than a body of interrelated and connected concepts.
2. Teaching mathematics consists of telling students how to follow procedures rather than supporting students as they attempt to understand.
3. Learning mathematics is a process of practicing and memorizing rather than a process of reasoning and sensemaking.

These beliefs, which are widely held even today, still shape mathematics education in many classrooms (Fuson, Kalchman, & Bransford, 2005).

Pajares (1992) suggested that teachers are unlikely to change their beliefs unless they find them to be unsatisfactory when challenged. In his review of research on professional growth among preservice and beginning teachers, Kagan (1992) concluded that extensive interaction with school-age students enables preservice teachers to confront their own beliefs and acknowledge the need for change. Wilcox, Schram, Lappan, and Lanier (1991) also reached similar conclusions as they implemented a content and methods course sequence, with

accompanying field experiences, designed to help prospective teachers change their knowledge and beliefs about mathematics and mathematics education. This line of reasoning leads to the implications of Ma's conclusion stated previously. That is, preservice teachers are unlikely to make significant changes in their classroom practice relative to the teaching of mathematics unless they are challenged to confront their own beliefs within the context of extensive field practicum experiences.

Philipp, Thanheiser, and Clement (2002), along with other investigators of the NSF-funded Integrating Mathematics and Pedagogy (IMAP) Project at San Diego State University, promoted significant improvements in preservice teachers' beliefs regarding conceptual mathematics teaching. Their approach paired participation in a math-for-elementary-teachers *content* course with field practicum experiences in a public school setting. They assessed the intensity of those beliefs using the IMAP Web-Based Beliefs Survey. (This instrument is described in greater detail in the following section.) We wondered if a similar approach might also promote belief changes in elementary mathematics *methods* students as measured by the same instrument.

Therefore, we designed this study to explore the effects of a three-credit-hour methods course with a practicum-based component upon the beliefs of preservice elementary education majors regarding conceptual approaches to teaching mathematics. The results should help us determine the viability of conducting future research in which we plan to use an experimental model to investigate more thoroughly possible causal relationships between methods-related field practica and belief changes.

## The Study

We used a simple pre-post design to conduct our exploratory study. We considered using a Likert-type instrument because such instruments have been commonly used to measure teachers' beliefs, but Hoffman and Kugle (1982) suggested that results gained from the use of such instruments may not reveal actual beliefs. In essence, such scales may assess a teachers' ability to "talk the talk" but not necessarily "walk the walk" in terms of applying conceptual teaching approaches as a result of changing beliefs. More recently, researchers have used a variety of means to ascertain preservice teachers' beliefs, including interviews, classroom observations, journals, written reflections, and surveys. Most of this research has been qualitative in nature and typically has involved small samples. Although the sample size for this particular study was not large, we plan to conduct other investigations regarding changing beliefs with larger samples, so our first challenge was to obtain an instrument capable of providing quantitative data on a sizeable scale.

The aforementioned IMAP research group developed an online survey instrument that requires respondents to interpret descriptions of classroom situations or actual videotaped cases. The following is an excerpt from the browse version of the IMAP Web-Based Beliefs Survey available at <http://www.sci.sdsu.edu/CRMSE/IMAP/pubs.html> (IMAP: Integrating Mathematics and Pedagogy Publications/Presentations [Browse the Survey], retrieved June 3, 2005).

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*Leticia has 8 Pokemon cards. She gets some more for her birthday. Now she has 13 Pokemon cards. How many Pokemon cards did Leticia get for her birthday?*

2.1 Do you think a typical first grader could solve this problem? *Note. This problem could be read to the child.*

Yes

No

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You answered that a typical first grader could solve the following problem:

*Leticia has 8 Pokemon cards. She gets some more for her birthday. Now she has 13 Pokemon cards. How many Pokemon cards did Leticia get for her birthday?*

2.2 If a friend of yours disagreed with you, what would you say to support your position?

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Here is another word problem. Again, read it and then determine if a typical first grader could solve it.

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*Miguel has 3 packs of gum. There are 5 sticks of gum in each pack. How many sticks of gum does Miguel have?*

2.3 Do you think a typical first grader could solve this problem? *Note. The problem could be read to the child.*

Yes

No

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You answered that a typical first grader could solve the following problem:

*Miguel has 3 packs of gum. There are 5 sticks of gum in each pack. How many sticks of gum does Miguel have?*

2.4 If a friend of yours disagreed with you, what would you say to support your position?

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For the current study, responses to items were analyzed using rubrics obtained directly from the IMAP group. However, those rubrics are now available at <http://www.sci.sdsu.edu/CRMSE/IMAP/pubs.html> (see “View Manual”). Some rubric scales consist of three points, while others consist of four or five points. Scores obtained from each of the items relating to a specific belief were then used to create an overall belief score through what the IMAP group calls a “rubric of rubrics.” The rubric of rubrics is an algorithm that assigns scores based on the possible combinations of contributing item scores (Ambrose et al., 2004). A sample rubric appears in the Appendix. (The procedures used to create and establish the psychometric soundness of this instrument are discussed in Ambrose et al., 2004).

The IMAP Beliefs Survey measures the level of intensity at which respondents possess the following seven beliefs identified by Ambrose et al. (2004).

#### **Beliefs About Mathematics**

Belief 1. Mathematics, including school mathematics, is a web of interrelated concepts and procedures.

#### **Beliefs About Knowing/Learning Mathematics**

Belief 2. One’s knowledge of how to apply mathematical procedures does not necessarily go with understanding the underlying concepts. That is, students or adults may know a procedure they do not understand.

Belief 3. Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures.

Belief 4. If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn the procedures first, they are less likely ever to learn the concepts.

### **Beliefs About Children's [Students'] Doing and Learning Mathematics**

Belief 5. Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies than their teachers, or even their parents, expect.

Belief 6. The ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics. For example, real-world contexts support children's initial thinking whereas symbols do not.

Belief 7. During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible. (p. 65)

Generally speaking, the higher the rubric score, the higher the level of intensity of the belief as measured by the associated items on the survey (Ambrose et al., 2004; Philipp et al., 2005).

In this exploratory study, we were concerned with the effects of our methods course upon the beliefs of preservice teachers regarding conceptual knowledge in school mathematics. We chose to study the first four beliefs because they relate directly to the nature and emphasis of the mathematics that children are to learn. Specifically, they address beliefs regarding the roles of conceptual and procedural knowledge in school mathematics.

We administered the instrument at the beginning and at the end of the mathematics methods semester to 18 senior level preservice mathematics methods students who had not encountered the instrument previously. These students attended a 4-year, primarily commuter college in the Intermountain West. They were randomly selected from a body of 60 students assigned to two professional preparation cohorts of elementary education majors. The students in the study are primarily Caucasian and at the time of the study were residents of the area in which

the college resides. All were enrolled in the final methods semester prior to student teaching and were assigned to either of two sections of a mathematics methods course, both of which met weekly and were taught by the same professor, who was also the lead researcher for this study.

Students were required to complete the IMAP Beliefs Survey at the beginning and at the end of the 15-week course; however, the results were in no way tied to student evaluation. During the 4 months between survey administrations, students engaged in course activities designed to help them comprehend conceptual teaching approaches and then were assigned in groups of two to apply those approaches in teaching children in public school classrooms. The details of this course-then-application sequence are described in the following paragraphs.

The first 2 weeks of the once-a-week, three-semester-hour methods course consisted of introductory, on-campus instruction focused primarily on the assessment of children's thinking. During the following 7 weeks, students were involved in both the methods class and a practicum component. In the methods class, they studied alternative assessment, process standards for learning mathematics (National Council of Teachers of Mathematics, 2000), the role of conceptual and procedural knowledge in mathematics instruction, inquiry-based lesson design, and orchestrating discourse, along with a deep study of the fundamental notions associated with conceptualizing number and operations. They engaged in discussion of readings, group presentations, live and video demonstrations, lectures, and activities in which they experienced and reflected upon conceptual mathematics appropriate for elementary students.

After an hour and a half in the methods classroom, they drove to a public school about 5 minutes away for their practicum experiences. Prior to each week's practicum, the students were assigned to communicate with the classroom teacher about the topic that was to be taught in order to become familiar with the content. Upon arrival, they worked with the mathematics curriculum materials under the supervision of the classroom teacher to understand the lesson for

the day and find ways the methods they were learning in their on-campus class could be integrated in that lesson. The students assumed varying roles during the 45-minute instructional period depending upon the time in the semester. They served as teaching assistants during the first 2 weeks of the practicum experience and taught small groups during the following 3 weeks. During the last 2 weeks, each student had the opportunity to assume lead responsibility for teaching the whole class for one class period. After each practicum experience, the students drove back to campus for a 20-minute debriefing in which their experiences, challenges, and frustrations were discussed.

For the second stage of the practicum, the on-campus portion of the class was not held. Students were assigned to the classrooms in which they would student teach during the following semester. They were in these classrooms full-time for 3 weeks, during which they designed and taught a 7- to 10-day unit of mathematics instruction using the classroom curriculum. The actual units were designed under the supervision of the classroom teacher and the students were supervised during this practicum by various elementary faculty.

In both stages, the students taught the curriculum of the classroom, attempting to implement lessons that focused on conceptual learning as espoused in the methods course. We sought to find adequate placements but did not attempt to find classrooms that were entirely compatible with conceptual mathematics teaching (i.e., classrooms in which mathematics teaching was conducted in accordance with the four beliefs). As expected, the degree of conceptual teaching being conducted in those classrooms by the cooperating teachers varied greatly. Considering these circumstances, the question still remained: Did significant changes in beliefs occur as measured by the IMAP Web-Based Beliefs Survey?

Data obtained from the survey were analyzed using the rubrics that accompany the survey. Pre-post scores relative to the first four beliefs were then derived. One of the two

researchers had been taught by the IMAP team to score data from the survey and had been involved in the validation of its rubrics. This researcher supervised the scoring tasks, with raters following the procedures summarized by Ambrose et al. (2004). (Details of scoring procedures are available under “View Manual” at <http://www.sci.sdsu.edu/CRMSE/IMAP/pubs.html>)

Simple *t*-tests for paired means were performed to ascertain possible statistical significance associated with pre-post differences. Inasmuch as the data obtained from the survey are not ordinal, but rather interval in nature, using distribution-dependent statistical procedures such as *t*-ratios or *ANOVA* for analyses ordinarily would be inappropriate. However, Ambrose et al. (2004) demonstrated the validity of using those distribution-dependent procedures for analyzing data obtained from the survey. They did so by analyzing differences between groups via the survey using a polychotomous log-linear ratio method and then re-analyzing those differences using *t*-ratios. Both analysis procedures revealed the same number of significant differences between the groups they studied. Therefore, because distribution-dependent analyses are more commonly used in quantitative studies and thus are more commonly understood, and because they have been shown to yield the same statistical conclusions with data obtained from the survey, *t*-tests were used for analyses in our study.

### Findings

Statistically significant differences between respondents’ pre- and post-survey scores were obtained for all four beliefs under study, with correspondingly large effect sizes. Table 1 summarizes the associated statistics. The effect size, *d*, which helps to interpret the size of a mean difference by comparing that difference to a standard deviation, was computed by determining the ratio of the mean difference to the average of the pre-treatment and post-treatment standard deviations.

Table 1

*Beliefs x Inferential Statistics Matrix*

Belief #	Rubric Scale Range	Pre-treatment Mean	Post-treatment Mean	<i>d</i>	<i>df</i>	<i>t</i>	<i>p</i>
1	0-3	1.56	2.17	.69	17	2.65	0.008
2	0-4	0.61	2.06	1.48	17	5.33	0.000
3	0-3	1.50	2.72	1.52	17	4.89	0.000
4	0-3	0.72	2.00	1.62	17	5.33	0.000

The means obtained from pre-treatment scores regarding Belief 1, “Mathematics involves a web of interrelated concepts and procedures,” and Belief 3, “Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures,” indicate that prior to participating in the course, students’ beliefs tended to be moderately intense. Post-treatment means indicate that students’ beliefs were at a significantly higher level of intensity. In other words, there was a marked change in level of intensity regarding Belief 1 and Belief 3 as measured by the survey.

Means relative to Belief 2, “One’s knowledge of how to apply mathematical procedures does not necessarily match one’s understanding of the underlying mathematical concepts,” and Belief 4, “If students learn mathematical concepts before they learn mathematical procedures, they are more likely to understand the procedures when they learn them,” suggest that students’ beliefs were of a low level of intensity prior to participating in the course. In the case of Belief 2,

the post-treatment mean suggests a resulting position that tended toward a moderate to high level of intensity. In the case of Belief 4, the post-treatment mean suggests that students' beliefs were at a moderate level of intensity. Both of these post-treatment means indicate significant changes in the level of intensity regarding Beliefs 2 and 4 over the semester.

### Conclusions

We can conclude that significant changes in preservice teachers' beliefs occurred; these changes tended toward a conceptual perspective. These changes are similar to those experienced by preservice content students in the research conducted by Philipp et al. (2002). The methods course, with a field practicum as an integral component, comprised the preservice teachers' major mathematics education experiences during the semester of this study. Therefore, we tentatively hypothesize that it was indeed a force, as Ma (1999) postulated, in promoting change in their conceptualizations toward the mathematics that children are to learn.

Kagan's (1992) explanations regarding the power of extended interactions with children as a force for change in beliefs may help to explain why we found changes at a significant level in the beliefs of our methods students. Additionally, Richardson (1996) proposed that preservice preparation is significantly enhanced by strong and consistent preservice-inservice teacher partnerships. Further research is needed to address these and related issues before a strong causal relationship between a practicum-based mathematics methods course and changes in preservice methods students' beliefs regarding conceptual teaching of mathematics can be established. In addition, a larger and more representative sample of preservice teachers is necessary to enable meaningful generalizations of the findings. However, we believe the overall substance of the results obtained in our study justifies continued research in the effectiveness of a practicum-based methods course as a means of promoting change in preservice teacher beliefs. Another rich

line of inquiry, not addressed at all in this study, is the impact of preservice teachers' practicum experiences upon the beliefs of the cooperating teachers in whose classrooms they are placed.

## References

- Ambrose, R., Clement, L., Philipp, R., Chauvot, J. (2004). Assessing prospective elementary school teachers' beliefs about mathematics and mathematics learning: Rationale and development of a constructed-response-format beliefs survey. *School Science and Mathematics, 104*(2), 56-69.
- Civil, M. (1993). Prospective elementary teachers' thinking about teaching mathematics. *Journal of Mathematical Behavior, 12*, 79-109.
- Ernest, P. (1989). The knowledge, beliefs and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching, 15*, 13-34.
- Fullan, M. (2001). *The new meaning of educational change*. New York: Teachers College Press.
- Fuson, K. C., Kalchman, M., & Bransford, J. D. (2005). Mathematical understanding: An introduction. In M. S. Donovan, & J. D. Bransford (Eds.), *How students learn: History, mathematics, and science in the classroom* (217-256). Washington, DC: National Academies Press.
- Hersh, R. (1986). Some proposals for revising the philosophy of mathematics. In T. Tymoczko (Ed.), *New directions in the philosophy of mathematics: An anthology* (pp. 9-28). Boston: Birkhauser.
- Hoffman, J., & Kugle, C. (1992). A study of theoretical orientation to reading in relation to teacher feedback during reading instruction. *Journal of Classroom Interaction, 18*, 2-7.
- IMAP: Integrating Mathematics and Pedagogy. (2002). *IMAP Web-Based Beliefs Survey*. Retrieved April 5, 2004, from <http://www.sci.sdsu.edu/CRMSE/IMAP/pubs.html>
- Kagan, D. M. (1992). Professional growth among preservice and beginning teachers. *Review of Educational Research, 2*, 129-169.

- Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.
- Mewborn, D. S. (2002, April). *Examining mathematics teachers' beliefs through multiple lenses*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA. (ERIC Document Reproduction Service No. ED465526)
- National Center for Education Statistics (NCES). (n.d.). *Highlights from the Trends in International Mathematics and Science Study (TIMSS) 2003*. Retrieved June 1, 2005, from <http://nces.ed.gov/pubs2005/2005005.pdf>
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62*, 307-332.
- Philipp, R. A., Thanheiser, E., & Clement, L. (2002). The role of a children's mathematical thinking experience in the preparation of prospective elementary school teachers. *International Journal of Educational Research, 37*, 195-210.
- Philipp, R. A., Ambrose, R., Clement, L. L., Sowder, J. T., Schappelle, B. P., Sowder, L., Chauvot, J., & Thanheiser, E. (2005). *The effects of early field experiences on the mathematical content knowledge and beliefs of prospective elementary school teachers: An experimental study*. Manuscript in preparation.
- Richardson, V. (1996). The role of attitudes and beliefs in learning to teach. In J. Sikula (Ed.), *Handbook of research on teacher education: A project of the Association of Teacher Educators* (pp. 102-119). New York: Macmillan.

Steinberg, R. M., Empson, S. B., & Carpenter, T. P. (2004). Inquiry into children's mathematical thinking as a means to teacher change. *Journal of Mathematics Teacher Education*, 7(3), 237-267.

Vacc, N. N., & Bright, G. W. (1999). Elementary preservice teachers' changing beliefs and instructional use of children's mathematical thinking. *Journal for Research in Mathematics Education*, 30, 89-110.

Wilcox, S.K., Schram, P., Lappan, G., & Lanier, P. (1991). The role of a learning community in changing preservice teachers' knowledge and beliefs about mathematics education. *For the Learning of Mathematics*, 8(1), 8-13.

## Appendix

## Sample Rubric

A sample rubric (R. Ambrose, personal communication, May 15, 2002) is displayed below, preceded by a belief statement and the item to which it corresponds.

Belief 2: One's knowledge of how to apply mathematical procedures does not necessarily go together with one's understanding of the underlying mathematical concepts. That is, a student or adult may know a procedure that they do not understand.

## Item 8

8. Place the following four problems in rank order of difficulty for children and explain your ordering (you may rank two or more items as being of equal difficulty). NOTE: Easiest = 1.
- a) Understand  $1/5 + 1/8$
  - b) Understand  $1/5 \times 1/8$
  - c) Which fraction is larger,  $1/5$  or  $1/8$ , or are they same size?
  - d) Your friend Jake attends a birthday party at which there are five guests who equally share a very large chocolate bar for dessert. You attend a different birthday party at which there are eight guests who equally share a chocolate bar exactly the same size as the chocolate bar shared at the party Jake attended. Did Jake get more candy bar, did you get more candy bar, or did you and Jake each get the same amount of candy bar?
- 8.1: Explain your ranking of the problem, "Understand  $1/5 \times 1/8$ ."
- 8.2: In question 8.1 we asked you to rank the difficulty of understanding  $1/5 \times 1/8$ . By *understand* were you thinking of the ability to get the right answer?

8.3: If no, about what were you thinking? If yes, were you thinking of anything else?

**Scores**

- 0 Respondents who receive a score of 0 indicate that they only think of the procedure for multiplying fractions although the question asks the respondent to rank the item on the difficulty of *understanding*  $1/5 \times 1/8$ . These students do not distinguish between one's ability to perform this procedure and understanding the underlying concept.
- 1 Respondents who receive a score of 1 are beginning to recognize that the ability to perform procedures does not imply that one understands underlying concepts, but this distinction is fragile. That is, these respondents may mention conceptual understanding in one part of the response but mention only procedures in a different part of the response.
- 2 Respondents who receive a score of 2 recognize that the ability to perform procedures does not imply that one understands the underlying concepts. These respondents consistently distinguish between understanding concepts and performing procedures. (R. Ambrose, personal communication, May 15, 2002)