

A Recent Encounter with an O.C.

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Current reform in teacher enhancement programs in the U.S. requires new learning in mathematics and pedagogy by in-service mathematics teachers. Recent researchers have said that the central demand for supporting learning through inquiry (for teachers) involved what they termed "openings in the curriculum", better known as O.C.'s [1]. These O.C.'s that arise in many ways can be unanticipated questions, challenges to a solution, or actions/observations made by participating teachers. When an O.C. occurs, it requires the facilitator to make on-the-spot decisions as to how the discourse progresses from this point on.

One such opening occurred recently in one of the courses in the Math ADEPT¹ program for in-service middle-school mathematics teachers at Salisbury University.² With little or no prelude, a teacher forcefully and urgently asked: "What do you say to a ten-year-old student who asks you why, when dividing fractions, multiplication of the dividend by the reciprocal of the divisor yields the quotient?" The student most likely asked, "Why do you turn the second fraction upside down and multiply?" *Why does this give the desired answer and, more importantly, the correct answer?* The teacher added further that the young student went on to ask why we divide fractions anyway and if there were good reasons to do so.

This question posed by this teacher was somewhat removed from the topic being explored, and to be honest, I was somewhat surprised by its timing. I could have repeated to her a famous saying of unknown origin, "Ours is not to reason why; just invert and multiply" [2], and gone on with the topic at hand with some vague promise

that we would discuss her question later. One knows how that goes. Later might never come, and even if it does, a discussion of the question at a later time would lose the urgency in this teacher's voice. I wanted this teacher to have an answer; moreover, I thought her question was an excellent one and certainly one that begs an answer. So I said to this class of 18 in-service teachers, "How would you answer this question?" The teachers formed groups of three to four and went to work. I was quite pleased with the results they gave, so much so that their insights must be shared with others. Their findings are given as results and observations and can be transported to the classroom by teachers who are teaching this content. That's exactly what many of the teachers in this class did.

These results and observations that follow are the thoughts of teachers presented by teachers for teachers:

Result 1: Suppose you want to know the quotient when $\frac{3}{4}$ is divided by $\frac{5}{6}$. Ask the student what the quotient is when the number, 24, is divided by 3. When the student says 8, ask him/her how he knows the quotient is 8. He/she most likely will say the quotient is 8 because 8 multiplied by 3 is 24 or there are eight groups of 3 in 24. Ask the student if he/she knows what $\frac{3}{4}$ divided by $\frac{5}{6}$ is without using a pencil and paper. The student most likely will say no. Tell him someone has told you the quotient is $\frac{a}{b}$ and you would like to check it out to see if $\frac{a}{b}$ is indeed the quotient. Ask him/her how you would do this if you did it the same way as in 24 divided by 3 is 8. The student should then understand that $\frac{a}{b} \times \frac{5}{6} = \frac{3}{4}$. At this point, this equation should be written on a piece of paper (or chalkboard) for the student. Ask the student to multiply both sides of this equation by $\frac{6}{5}$ and simplify the left side of the equation. The student's work

should look like this: $(a/b \times 5/6) \times 6/5 = 3/4 \times 6/5$. Using the associative law of multiplication on the left side of the equation we get $a/b \times 1 = 3/4 \times 6/5$. This, the equation becomes $a/b = 3/4 \times 6/5$. Stress that a/b was what had been given as the solution to the division problem. Notice that the solution, a/b is indeed the fraction $3/4$ multiplied by the reciprocal of the divisor, $5/6$. Of course a/b can be expressed as $9/10$.

Result 2: We want to know the quotient when $3/4$ is divided by $5/6$. We know that there can be only one quotient. We also know that the quotient (to this division problem) when multiplied by $5/6$ yields $3/4$. So $5/6$ multiplied by what number is equal to $3/4$. Suppose we multiply $5/6$ by $(3/4 \times 6/5)$. The product of $5/6$ and $(3/4 \times 6/5)$ is $3/4$. Therefore, $3/4 \times 6/5$ must be the quotient.

Result 3: Tell the young student that every division problem he/she has ever done has been done this way. Maybe he/she did not realize this at the time. For example, 24 divided by 3 is 24 divided by $3/1$ which is equal to $24 \times 1/3$. He/she has multiplied the dividend by the reciprocal of the divisor.

The above three results give some informative comments on the question posed by the teacher; however, the interesting aspect of this O.C. is that in addition to providing insights into the question posed, there were observations that, although they did not answer the question directly, gave some in-depth understanding of the division of fractions.

Observation 1: When one divides $3/4$ by $5/6$, one is really asking how many $5/6$'s there are in $3/4$. Since $5/6$ is larger than $3/4$ there is not even one. Consider a ruler that is a 12 inches long. If we consider $3/4$ foot, we have 9 inches. If we consider $5/6$ foot, we have 10 inches. Also, $3/4$ divided by $5/6$ is the same as the ratio of $3/4$ to $5/6$ and the

quotient ought to be $9/10$ (as seen by the 9 inches and 10 inches on the ruler). So, there is not quite one $5/6$ only $9/10$ of one $5/6$ in $3/4$. That is exactly what $3/4$ multiplied by $6/5$ yields because $3/4 \times 6/5 = 18/20 = 9/10$. (Note: The system of measurement that is used in the U.S. is called the "English system". This system is still used contrary to much publicity a few years ago of converting to the metric system.)

Observation 2: The quotient is desired when $3/4$ is divided by $5/6$. Express both fractions with a common denominator just as you do when adding and subtracting fractions. Then the quotient of the two fractions is just the quotient of the numerators. For example, $3/4 = 9/12$ and $5/6 = 10/12$. Then $3/4$ divided by $5/6$ is the same division problem as $9/12$ divided by $10/12$ which is $9/10$, the quotient of the two numerators.

Observation 3: You want to know the quotient when $3/4$ is divided by $5/6$. Suppose it takes $5/6$ pound of cheese to make a certain pizza. When you go to the market you find that the cheese is sold in chunks that weigh $3/4$ of one pound. You buy one package of cheese. You cannot make the whole pizza because $3/4$ is less than $5/6$. What part of the pizza can you make? Since $3/4 = 9/12$ and $5/6 = 10/12$, we can see that $9/10$ of $10/12 = 9/12$ or $9/10$ of $5/6$ is $3/4$. Therefore, we can make only $9/10$ of the pizza. The quotient of the two fractions should be $9/10$.

Both the results and the observations given above are useful tools to have when teaching division of fractions. The results give plausible reasons for the question posed by the teacher. Observation 2 gives another algorithm for dividing fractions different from the one usually given. This algorithm has pedagogical appeal in that it is quite similar to the one given for addition and subtraction of fractions. Observations 1 and 3 not only give good practical reasons as to why one needs division of fractions, but also

model the whole process with physical objects. Each of these could be used as a hands-on activity with students by providing them with appropriate manipulatives.

In summary, O.C.'s provide rich experiences for any class, as well as the teacher of that class. When guiding instruction in mathematics, teachers everywhere need to exploit these opportunities for exploration and discovery.

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Notes:

¹ Math ADEPT (Allied Delmarva Enhancement Program for Teachers) is a program of interconnected mathematics courses offered for graduate credit to in-service middle-school mathematics teachers in the Delmarva (Delaware, Maryland, Virginia) region of the Eastern Shore (of the Chesapeake Bay).

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References

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